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Proof theory of partially normal skew monoidal categories. (English) Zbl 07454906

Spivak, David I. (ed.) et al., Proceedings of the 3rd annual international applied category theory conference 2020, ACT 2020, Cambridge, USA, July 6–10, 2020. Waterloo: Open Publishing Association (OPA). Electron. Proc. Theor. Comput. Sci. (EPTCS) 333, 230-246 (2021)

Substructural logics are logical systems where one or more structural rules such as exchange, weaking and contraction are not allowed. *Affine logics* are substructural with respect to intuitionsitc logics since contraction is disallowed. *Linear logics* are substructural with respect to affine logics since weakening is also disallowed. By dropping the exchange rule as well, one obtains ordered variants of (intuitionistic) linear logics [*V. M. Abrusci*, Z. Math. Logik Grundlagen Math. 36, No. 4, 297–318 (1990; Zbl 0810.03005)].

The principal objective in this paper is to show that the free skew monoidal categories of different degrees of partial normality are to be described as sequent calculi enjoying cut elimination and admitting a deductive description of a root-first search strategy, which define logics weaker than the multiplicateive fragment of the intuitionistic non-commutative linear logic.

It was shown in [*J. Bourke* and *S. Lack*, J. Algebra 506, 237–266 (2018; Zbl 1401.18019)] that *skew monoidal categories* are equivalent to representable skew multicategories, a weakening of representable multicategories [*C. Hermida*, Adv. Math. 151, No. 2, 164–225 (2000; Zbl 0960.18004)]. The authors [Outst. Contrib. Log. 20, 377–406 (2021; Zbl 07440912)] showed that the map constructors and equations of a (nullary-binary) representable skew multicategories are very close to and mutually definable with those of the sequent calculus for the corresponding skew monoidal category. The authors expect that *partially normal* skew monoidal categories are to be analyzed in similar terms, the correct variations of representable skew multicategories being to be systematically derived in the framework of (op)fibrations of multicategories [*C. Hermida*, Fields Inst. Commun. 43, 281–293 (2004; Zbl 1067.18007)] adopted for skew multicategories.

The authors intend to continue the study by broadening its scope to fully skew and partially normal closed and *monoidal closed* categories as well as *prounital-closed* categories where the unit is present in a non-represented way. The *skew closed* categories, the skew variant of closed categories of *S. Eilenberg* and *G. M. Kelly* [in: Proc. Conf. Categor. Algebra, La Jolla 1965, 421–562 (1966; Zbl 0192.10604)], were textitN. Zeilberger [in: Proceedings of the 2018 33rd annual ACM/IEEE symposium on logic in computer science, LICS 2018, Oxford, UK, July 9–12, 2018. New York, NY: Association for Computing Machinery (ACM). 919–928 (2018; Zbl 1453.03012)] first considered in [*R. Street*, J. Pure Appl. Algebra 217, No. 6, 973–988 (2013; Zbl 1365.18008)] while prounital-closed categories were first considered in [`https://ncatlab.org/michaelshulman/show/closed%20category`]. made use of a thin variant of skew monoidal categories in his study of the relation between typing of linear lambda terms and flows on 3-valent graphs. The authors [`https://www.researchgate.net/publication/346598607_Eilenberg-Kelly_Reloaded`] dissected the *Eilenberg-Kelly theorem* about adjoint monoidal and closed structures on a category, revisited by *R. Street* [J. Pure Appl. Algebra 217, No. 6, 973–988 (2013; Zbl 1365.18008)] for the skew situation, to establish it in the general partially normal case. The authors expect as well to study the proof theory of skew braided monoidal categories, as introduced in [*J. Bourke* and *S. Lack*, Theory Appl. Categ. 35, 19–63 (2020; Zbl 1431.18012)].

For the entire collection see [Zbl 1466.68028].

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MSC:

18M45 Categorical aspects of linear logic
03F03 Proof theory in general (including proof-theoretic semantics)
03G30 Categorical logic, topoi

Full Text: Link

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