Faul, Peter F.; **Manuell, Graham**; **Siqueira, José**
2-dimensional bifunctor theorems and distributive laws. (English) Zbl 07451114
Theory Appl. Categ. 37, 1149-1175 (2021)

The following bifunctor theorem is familiar in category theory.

Theorem. Let $\mathcal{B}, \mathcal{C}$ and $\mathcal{D}$ be categories and for each object B in $\mathcal{B}$ and each object C in $\mathcal{C}$, let $L_C : \mathcal{B} \to \mathcal{D}$ and $M_B : \mathcal{C} \to \mathcal{D}$ be functors such that

$$L_C(B) = M_B(C)$$

Then there exists a bifunctor $P : \mathcal{B} \times \mathcal{C} \to \mathcal{D}$ with $L_C = P(-, C)$ and $M_B = P(B, -)$ iff for all morphisms $f : B_1 \to B_2$ in $\mathcal{B}$ and $g : C_1 \to C_2$ in $\mathcal{C}$ we have

$$L_{C_2}(f) M_{B_1}(g) = M_{B_2}(g) L_{C_1}(f)$$

In this case, we have

$$P(B, C) = L_C(B) = M_B(C)$$
$$P(f, g) = M_{B_2}(g) L_{C_1}(f)$$

This paper considers the conditions under which two families of pseudofunctors with a common codomain are to be collated into a bifuncor. Observing similarities between these conditions and distributive laws of monads, the authors establish a version of the bifunctor theorem for lax functors, showing that these generalized distributive laws may be arranged into a 2-category $\mathrm{Dist}(\mathcal{B}, \mathcal{C}, \mathcal{D})$, which is equivalent to $\mathrm{Lax}_{\mathrm{op}}(\mathcal{B}, \mathrm{Lax}_{\mathrm{op}}(\mathcal{C}, \mathcal{D}))$. The collation of a distributive law into its associated bifunctor is shown to extend to a 2-functor into $\mathrm{Lax}_{\mathrm{op}}(\mathcal{B} \times \mathcal{C}, \mathcal{D})$ corresponding to uncurrying via the above equivalence. Subcategories on which collation itself restricts to an equivalence are described. A number of natural categorical constructions are exhibited as special cases on these lines.

Reviewer: Hirokazu Nishimura (Tsukuba)

MSC:

18D05 Double categories, 2-categories, bicategories and generalizations (MSC2010)
18C15 Monads (= standard construction, triple or triad), algebras for monads, homology and derived functors for monads

Keywords:

morphism of bicategories; triple; braiding; curry; exponential

Full Text: Link

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