**Landsman, Klaas**

**The logic of quantum mechanics (Revisited).** (English) Zbl 07425827

Anel, Mathieu (ed.) et al., New spaces in physics. Formal and conceptual reflections. Cambridge: Cambridge University Press. 85-113 (2021)

The projection lattice $P(A)$ of a von Neumann algebra $A$ is an *orthomodular lattice* [*G. Kalmbach*, Orthomodular lattices. London: Academic Press (1983; Zbl 0512.06011); *L. Beran*, Orthomodular lattices. Algebraic approach. Dordrecht: Kluwer Academic Publishers Group (1985; Zbl 0558.06008); *M. Rédei*, Quantum logic in algebraic approach. Dordrecht: Kluwer Academic Publishers (1998; Zbl 0910.03038)], which *G. Birkhoff* and *J. von Neumann* [Ann. Math. (2) 37, 823–843 (1936; Zbl 0015.14603); Ann. Math. (2) 37, 823–843 (1936; JFM 62.1061.04)] regarded as the logic of quantum mechanics. In an orthomodular lattice, the law of the excluded middle holds while the law of distributivity no longer holds.

The author proposes an *intuitionistic quantum logic*, where the law of distributivity does hold while the law of the excluded middle no longer holds. Semantically the author goes as follows.

- To any unital C*-algebra $A$, the poset $C(A)$ of all unital commutative C*-subalgebras, ordered by set-theoretic inclusion, is of the bottom element $\bot = \mathbb{C}$ but is not of the top element unless $A$ is commutative in which case $\top = A$. The poset $C(A)$ is of arbitrary infima (i.e., meets), but it only has suprema (i.e., joins) of families of elements that mutually commute.
- To any unital C*-algebra $A$, the set

  $$\Sigma_A = \bigsqcup_{C \in C(A)} \Sigma(C)$$

  is defined, where $\Sigma(C)$ is the Gelfand spectrum. $\Sigma_A$ is given the topology making the canonical projection

  $$\pi : \Sigma_A \to C(A)$$

  continuous with respect to the Alexandrov topology on $C(A)$. The topology $\mathcal{O}(\Sigma_A)$ is a Heyting lattice, which the author regards as the logic of quantum mechanics. It is easy to see that

  $$\Sigma_{\mathbb{C}^n} = \bigsqcup_{\pi \in \Pi_n} \pi$$

  where  denotes the the partition lattice of $n$.

Surely the author's intuitionistic quantum logic is affected greatly by the topos-theoretic approach to physics [*C. J. Isham*, in: Deep beauty. Understanding the quantum world through mathematical innovation. Papers based on the presentations at the deep beauty symposium, Princeton, NJ, USA, October 3–4, 2007. Cambridge: Cambridge University Press. 187–205 (2011; Zbl 1236.81013); *C. J. Isham* and *J. Butterfield*, Int. J. Theor. Phys. 37, No. 11, 2669–2733 (1998; Zbl 0979.81018); Int. J. Theor. Phys. 38, No. 3, 827–859 (1999; Zbl 1007.81009); *C. Flori*, A first course in topos quantum theory. Berlin: Springer (2013; Zbl 1280.81001); A second course in topos quantum theory. Cham: Springer (2018; Zbl 1398.81002)]. The author seems to be unaware of [*B. Coecke*, Stud. Log. 70, No. 3, 411–440 (2002; Zbl 0999.03058)], which gave another intuitionistic quantum logic.

For the entire collection see [Zbl 1466.53003].

Reviewer: Hirokazu Nishimura (Tsukuba)

**MSC:**

81-XX Quantum theory
03-XX Mathematical logic and foundations
18-XX Category theory; homological algebra

**Full Text:** Link