

**Kontsevich, Maxim**

**Geometry in dg-categories.** (English) [Zbl 07425824](#)

Anel, Mathieu (ed.) et al., New spaces in mathematics. Formal and conceptual reflections. Cambridge: Cambridge University Press. 554-592 (2021)

Derived noncommutative geometry attempts to interpret a general triangulated category *without* the symmetric monoidal structure as the category of quasi-coherent sheaves on a noncommutative space. The principal objective in this paper is to review noncommutative analogues of many notions from the commutative geometry and also to show several genuinely new noncommutative phenomena. Homological mirror symmetry was the main source of inspiration to the author.

The table of contents goes as follows.

- 1 Introduction
  - 1.1 Triangulated Categories and Spaces
  - 1.2 Examples of dg-Algebras and dg-Categories
    - 1.2.1 Representation Theory: Quivers, Quantum Groups and So On
    - 1.2.2 Algebraic Geometry
    - 1.2.3 Topology: Chains on Paths, Constructible Sheaves
  - 1.3 Examples of Noncommutative Equivalences
  - 1.4 Comparison with Other Geometries and Further Challenges
- 2 Noncommutative Analogs of Usual Notions
  - 2.1 Families
  - 2.2 Definition of nc-spaces
  - 2.3 Being Affine
  - 2.4 Tensor Products, Opposite Spaces, and Duality
  - 2.5 Finiteness Properties: Smooth, Proper, and So On
  - 2.6 Morphisms
  - 2.7 Recollements: Closed Subsets and Open Subspaces
  - 2.8 Deformation Theory
  - 2.9 De Rham Cohomology
  - 2.10 Calculus Relating Hochschild Homology and Cohomology
  - 2.11 Noncommutative Motives
  - 2.12 Calabi-Yau Structures
  - 2.13 Beyond Affineness
- 3 Exotic Geometry in Noncommutative World
  - 3.1 Large Symmetry Groups, Fourier-Mukai Transform
  - 3.2 Duality of Quotients
  - 3.3 Fractional Calabi-Yau
  - 3.4 Completion at the Support, Koszul Duality
  - 3.5 Smooth Spaces from Singular Schemes, Categories of Singularities and Knörrer Periodicity
  - 3.6 Semiorthogonal Decomposition and Action of Braid Group
  - 3.7 Calculus for Pre-Calabi-Yau Structures
  - 3.8 About Fukaya Categories

Appendix: Technical Definitions

A1 dg-Categories and Semifree Modules

A2 Triangulated dg-Categories

A3 Resolutions and Tensor Products

A4 Compact Objects

A5  $A_\infty$ -Language

For the entire collection see [[Zbl 1466.53002](#)].

Reviewer: [Hirokazu Nishimura \(Tsukuba\)](#)

**MSC:**

[18G35](#) Chain complexes (category-theoretic aspects), dg categories

**Full Text:** [Link](#)