Ortiz-Morales, Martín; **Sandoval-Miranda, Martha Lizbeth Shaid**; **Santiago-Vargas, Valente**
Gabriel localization in functor categories. (English) ⎡Zbl 1477.18004⎤
Commun. Algebra 49, No. 12, 5273-5296 (2021).

P. Gabriel [Bull. Soc. Math. Fr. 90, 323–448 (1962; Zbl 0201.35602)] introduced the concept of localization within the setting of abelian categories, proposing what is now called a Gabriel filter on a unital ring R while showing that there is a bijective correspondence between the set of Gabriel filters of R and the set of classes of the isomorphisms of Giraud subcategories of the category $\mathrm{Mod}(R)$ of its left unital R-modules. *M. Ortíz-Morales* and *S. Díaz-Alvarado* ["Linear filters and hereditary torsion theories in functor categories", Preprint, `arXiv:1412.0316`] introduced the notion of Gabriel filter for a preadditive category $\mathcal{C}$, establishing that there exists a bijective correspondence between Gabriel filters of $\mathcal{C}$ and hereditary torsion classes of $\mathrm{Mod}(\mathcal{C})$. Similar definitions and results were provided in [*C. E. Parra* et al., Bull. Iran. Math. Soc. 47, No. 4, 1135–1171 (2021; Zbl 1467.13023)].

The principal objective in this paper is to study Gabriel's analogous result for the context of rings with several objects on the lines of [*B. Mitchell*, Adv. Math. 8, 1–161 (1972; Zbl 0232.18009)]. The main result goes as follows.

Theorem. Let $\mathcal{C}$ be a small preadditive category and let $\mathrm{Mod}(\mathcal{C})$ be the category of additive covariant functors from $\mathcal{C}$ to the category of abelian groups $\boldsymbol{Ab}$. Then there exists a bijective correspondence between the left Gabriel filters on $\mathcal{C}$, and the classes of isomorphisms of Giraud subcategories in $\mathrm{Mod}(\mathcal{C})$.

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The results in the paper are however by no means new. They are contained for example in [*N. Popescu*, Abelian categories with applications to rings and modules. Academic Press, London (1973; Zbl 0271.18006), Chapter 4.9 (see pages 221, 222 Theorem 9.1)], cited by the authors where Gabriel filter for preadditive categories is called a left localizing system. There is a much more general result for enriched sheaves [*F. Borceux* and *C. Quinteiro*, Cah. Topologie Géom. Différ. Catégoriques 37, No. 2, 145–162 (1996; Zbl 0883.18006), Theorem 1.5]. The additive case from the paper under review is specialized and quoted in [*W. Lowen*, J. Pure Appl. Algebra 190, No. 1–3, 197–211 (2004; Zbl 1051.18007)].

Reviewer: Hirokazu Nishimura (Tsukuba)

MSC:

18A25	Functor categories, comma categories
18E05	Preadditive, additive categories
16D90	Module categories in associative algebras
16G10	Representations of associative Artinian rings

Keywords:

functor categories; Gabriel filter; Giraud subcategory; localization

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