Kusraev, A. G.; **Kutateladze, S. S.**

Boolean valued analysis: background and results. (English) ⎡Zbl 07437842⎤

Boolean valued analysis addresses various areas of mathematics by using Boolean valued models $\mathbb{V}^{(\mathbb{B})}$ of set theory. It started with [*E. I. Gordon*, Sov. Math., Dokl. 18, 1481–1484 (1977; Zbl 0406.03064); *G. Takeuti*, Lect. Notes Math. 753, 714–731 (1979; Zbl 0427.03046)]. This paper provides a brief overview of Boolean valued analysis.

For the entire collection see [Zbl 1470.47003].

Reviewer: Hirokazu Nishimura (Tsukuba)

MSC:

06F25   Ordered rings, algebras, modules
46A40   Ordered topological linear spaces, vector lattices

Keywords:

vector lattice; Kantorovich's principle; Gordon's theorem; Boolean valued analysis; Boolean valued representation

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