De Renzi, Marco

Extended TQFTs from non-semisimple modular categories. (English) Zbl 07441196

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The principal objective in this paper is to construct and characterize three-dimensional extened topological quantum field theories (ETQFT) from modular categories, providing a 2-categorical extension of the non-semisimple TQFT's built in [*C. A. Ipanaque Zapata*, "Non-contractible configuration spaces", Preprint, arXiv:1912.02063] and further studied in [*M. De Renzi* et al, "Mapping class group representations from non-semisimple TQFTs", Preprint, arXiv:2010.14852] on the one hand as well as a non-semisimple counterpart to the class of semisimple ETQFT's completely classified in [*B. Bartlett* et al., "Modular categories as representations of the 3-dimensional bordism 2-category", Preprint, arXiv:1509.06811] on the other. The author's consideration here is closely related to the one in [*M. De Renzi*, "Non-semisimple extended topological quantum field theories", Preprint, arXiv:1703.07573] beginning with relative modular categories, though this version of the construction is disengaged from much of the technical challeges characterizing the Costantino-Geer-Patureau-Mirand theory [*F. Costantino* et al., J. Topol. 7, No. 4, 1005–1053 (2014; Zbl 1320.57016)], allowing of an illustration of the main ideas at play in a more straightforward manner.

Some comments on the author's terminology are in order. The term *modular category* is used here in the not necessarily semisimple sense of [*V. V. Lyubashenko*, Commun. Math. Phys. 172, No. 3, 467–516 (1995; Zbl 0844.57016)] distinct from that of [*V. G. Turaev*, Quantum invariants of knots and 3-manifolds. Berlin: Walter de Gruyter (1994; Zbl 0812.57003)]. That is to say, a *modular category* means a finite ribbon category whose braiding abides by an appropriate non-degeneracy condition without semisimplicity required. An *ETQFT* is a symmetric monoidal 2-functor with source a 2-category of cobordisms as well as several possible targets provided by linear 2-categories of some sort [*C. J. Schommer-Pries*, "The classification of two-dimensional extended topological field theories", Preprint, arXiv:1112.1000]. The term 2-*category* has to be understood in the weak sense of [*J. Bénabou*, Lect. Notes Math. 47, 1–77 (1967; Zbl 1375.18001)], where it is called a *bicategory* instead. The target for ETQFT's are provided by the 2-category $\hat{\mathbf{Cat}}_k$ of (additive and idempotent) complete linear categories, functors and natural transformations, while the source 2-category $\check{\mathbf{Cob}}_C$, depending on the modular category C, consists of closed one-dimensional manifolds as objects, two-dimensional cobordisms decorated by marked points with objects of C as labels as 1-morphisms, and three-dimensional cobordisms with corners decorated by so-called *bichrome graphs* labelled with objects and morphisms of C as 2-morphisms, where these decorations have to abide by an admissibility condition requiring the presence of a projective objects of C among the labels of decorations of connected components cobordisms which are disjoint from the incoming boundary.

The main result of the paper (Theorem 7.4 and Proposition 8.2) is

Theorem. If C is a modular category over an algebraically closed field k, then there exists an ETQFT

$$\hat{\mathbf{A}}_C : \check{\mathbf{Cob}}_C \to \hat{\mathbf{Cat}}_k$$

whose circle category obeys

$$\hat{\mathbf{A}}_C\left(\mathbf{S}^1\right) \cong \mathrm{Proj}\,(C)\,.$$

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MSC:

18M15 Braided monoidal categories and ribbon categories
57R56 Topological quantum field theories (aspects of differential topology)
18M45 Categorical aspects of linear logic
81T45 Topological field theories in quantum mechanics

Full Text: DOI

References:

[1] M. ATIYAH, Topological Quantum Field Theories, Publ. Math. Inst. Hautes tudes Sci. 68 (1988), 175-186. http://www.numdam.org/item?id=PMII MR2043362 · Zbl 0692.53053

[2] J. BÉNABOU, Introduction to Bicategories (1967), 1-77. https://doi.org/10.1007/ BFb0074299. · Zbl 1375.18001 · doi:10.1007/BFb0074299

[3] C. BLANCHET, F. COSTANTINO, N. GEER, and B. PATUREAU-MIRAND, Non-Semisimple TQFTs, Reidemeister Torsion and Kashaev's Invariants, Adv. Math. 301 (2016), 1-78. https:// doi.org/10.1016/j.aim.2016.06.003. MR2043362 · Zbl 1412.57025 · doi:10.1016/j.aim.2016.06.003.MR2043362

[4] J. BAEZ and J. DOLAN, Higher-Dimensional Algebra and Topological Quantum Field Theory, J. Math. Phys. 36 (1995), no. 11, 6073-6105. https://doi.org/10.1063/1. 531236. MR2043362 · Zbl 1375.18001 · doi:10.1007/BFb0074299

[5] C. BLANCHET and M. DE RENZI, Modular Categories and TQFTs Beyond Semisimplicity, avail-able at arxiv:2011.12932.

[6] B. BARTLETT, C. DOUGLAS, C. SCHOMMER-PRIES, and J. VICARY, Modular Categories as Representations of the 3-Dimensional Bordism 2-Category, available at arXiv:1509.06811. · Zbl 1412.57025 · doi:10.1016/j.aim.2016.06.003

[7] C. BLANCHET, N. HABEGGER, G. MASBAUM, and P. VOGEL, Topological Quantum Field Theories Derived from the Kauffman Bracket, Topology 34 (1995), no. 4, 883-927. https:// doi.org/10.1016/0040-9383(94)00051-4. MR2043362 · Zbl 0863.18004 · doi:10.1063/1.531236

[8] B. BAKALOV and A. KIRILLOV, Lectures on Tensor Categories and Modular Functors, University Lecture Series, vol. 21, American Mathematical Society, Providence, RI, 2001. https://doi. org/10.1090/ulect/021. MR2043362 · Zbl 0965.18002 · doi:10.1090/ulect/021.MR2043362

[9] F. COSTANTINO, N. GEER, and B. PATUREAU-MIRAND, Quantum Invariants of 3-Manifolds via Link Surgery Presentations and Non-Semi-Simple Categories, J. Topol. 7 (2014), no. 4, 1005-1053. https://doi.org/10.1112/jtopol/jtu006. MR2043362 · Zbl 1320.57016 · doi:10.1112/jtopol/jtu006.MR2043362

[10] M. DE RENZI, Non-Semisimple Extended Topological Quantum Field Theories, available at arXiv:1703.07573.

[11] M. DE RENZI, A. GAINUTDINOV, N. GEER, B. PATUREAU-MIRAND, and I. RUNKEL, 3-Dimensional TQFTs from Non-Semisimple Modular Categories, available at arXiv:1912.02063.· doi:10.1090/ulect/021

[12] , Mapping Class Group Representations from Non-Semisimple TQFTs, available at arXiv: 2010.14852.

[13] M. DE RENZI, N. GEER, and B. PATUREAU-MIRAND, Renormalized Hennings Invariants and 2+1-TQFTs, Comm. Math. Phys. 362 (2018), no. 3, 855-907. https://doi.org/10.1007/ s00220-018-3187-8. MR2043362 · Zbl 1402.57024 · doi:10.1007/s00220-018-3187-8.MR2043362

[14] , Non-Semisimple Quantum Invariants and TQFTs From Small and Unrolled Quantum Groups, Algebr. Geom. Topol. 20 (2020), no. 7, 3377-3422. https://doi.org/10.2140/agt. 2020.20.3377. MR2043362 · Zbl 07335432 · doi:10.2140/agt.2020.20.3377.MR2043362

[15] P. ETINGOF, S. GELAKI, D NIKSHYCH, and V. OSTRIK, Tensor Categories, Vol. 205, Amer. Math. Soc., Providence, RI, 2015. https://doi.org/10.1090/surv/205. MR2043362 · Zbl 1402.57024 · doi:10.1007/s00220-018-3187-8

[16] D. FREED, Higher Algebraic Structures and Quantization, Comm. Math. Phys. 159 (1994), no. 2, 343-398. https://doi.org/10.1007/bf02102643. MR2043362 · Zbl 1402.57024 · doi:10.1007/s00220-018-3187-8

[17] N. GEER, J. KUJAWA, and B. PATUREAU-MIRAND, Generalized Trace and Modified Dimension Functions on Ribbon Categories, Selecta Math. 17 (2011), no. 2, 435-504. https://doi.org/ 10.1007/s00029-010-0046-7. MR2043362 · Zbl 07335432 · doi:10.2140/agt.2020.20.3377

[18] , Ambidextrous Objects and Trace Functions for Nonsemisimple Categories, Proc. Amer. Math. Soc. 141 (2013), no. 9, 2963-2978. https://doi.org/10.1090/S0002-9939-2013-11563-7. MR2043362 · Zbl 0790.58007 · doi:10.1007/bf02102643

[19] A. GAINUTDINOV and I. RUNKEL, Projective Objects and the Modified Trace in Fac-torisable Finite Tensor Categories 156 (2020), no. 4, 770-821. https://doi.org/10.1112/ S0010437X20007034. MR2043362 · Zbl 1248.18006 · doi:10.1007/s00029-010-0046-7

[20] M. HENNINGS, Invariants of Links and 3-Manifolds Obtained From Hopf Algebras, J. London Math. Soc. 54 (1996), 594-624. https://doi.org/10.1112/jlms/54.3.594. MR2043362 · Zbl 0882.57002 · doi:10.1112/jlms/54.3.594.MR2043362

[21] T. KERLER and V. LYUBASHENKO, Non-Semisimple Topological Quantum Field Theories for 3-Manifolds with Corners, Vol. 1765, Lecture Notes in Mathematics, Springer-Verlag, Berlin, 2001. https://doi.org/10.1007/b82618. MR2043362 · Zbl 1280.18005 · doi:10.1090/S0002-9939-2013-11563-7

[22] R. LAWRENCE, Triangulation, Categories and Extended Field Theories, Quantum Topology, Ser. Knots Everything, vol. 3, World Sci. Publ., River Edge, NJ, 1993. https://doi.org/10.1142/ 9789812796387_0011. MR2043362 · Zbl 1445.18011 · doi:10.1112/S0010437X20007034

[23] J. LURIE, On the classification of topological field theories, Current Developments in Mathematics, vol. 2008, Int. Press, Somerville, MA, 2009. https://doi.org/10.4310/cdm.2008.v2008.n1. a3. MR2043362· doi:10.4310/cdm.2008.v2008.n1.a3.MR2043362

[24] V. LYUBASHENKO, Invariants of 3-Manifolds and Projective Representations of Mapping Class Groups via Quantum Groups at Roots of Unity, Comm. Math. Phys. 172 (1995), no. 3, 467-516. https://doi.org/10.1007/BF02101805. MR2043362 · Zbl 0844.57016 · doi:10.1142/9789812796387_0011

[25] , Modular Transformations for Tensor Categories, J. Pure Appl. Algebra 98 (1995), no. 3, 279-327. https://doi.org/10.1016/0022-4049(94)00045-K. MR2043362 · Zbl 0823.18003 · doi:10.4310/cdm.2008.v2008.n1.a3

[26] S. MAJID, Braided Groups and Algebraic Quantum Field Theories, Lett. Math. Phys. 22 (1991), no. 3, 167-175. https://doi.org/10.1007/BF0040354 MR2043362 · Zbl 0745.16019 · doi:10.4310/cdm.2008.v2008.n1.a3

[27] , Braided Groups, J. Pure Appl. Algebra 86 (1993), no. 2, 187-221. https://doi.org/ 10.1016/0022-4049(93)90103-Z. MR2043362 · Zbl 0844.57016 · doi:10.1007/BF02101805

[28] S. MAC LANE, Categories for the Working Mathematician, Graduate Texts in Math-ematics, vol. 5, Springer-Verlag, New

York-Berlin, 1971. https://doi.org/10.1007/ 978-1-4612-9839-7. MR2043362 · Zbl 0232.18001 · doi:10.1007/978-1-4612-9839-7.MR2043362

[29] N. RESHETIKHIN and V. TURAEV, Invariants of 3-Manifolds via Link Polynomials and Quantum Groups, Invent. Math. 103 (1991), no. 1, 547-597. https://doi.org/10.1007/ BF01239527. MR2043362 · Zbl 0725.57007 · doi:10.1007/BF01239527.MR2043362

[30] C. SCHOMMER-PRIES, The Classification of Two-Dimensional Extended Topological Field Theo-ries, available at arXiv:1112.1000. · Zbl 1405.18009

[31] G. SEGAL, The Definition of Conformal Field Theory, Differential Geometrical Meth-ods in Theoretical Physics (Como, 1987), vol. 250, NATO Adv. Sci. Inst. Ser. C Math. Phys. Sci., Kluwer Acad. Publ., Dordrecht, 1988. https://doi.org/10.1007/ 978-94-015-7809-7. MR2043362 · Zbl 0725.57007 · doi:10.1007/BF01239527

[32] V. TURAEV, Quantum Invariants of Knots and 3-Manifolds, De Gruyter Studies in Mathematics, vol. 18, Walter de Gruyter \& Co., Berlin, 1994. https://www.degruyter.com/view/product/ 461906. MR2043362 · Zbl 0812.57003

[33] K. WALKER, On Witten's 3-Manifold Invariants, 1991. http://canyon23.net/math/1991TQFTNotes.pdf. MR2043362

[34] E. WITTEN, Topological Quantum Field Theory, Commun. Math. Phys. 117 (1988), no. 3, 353-386. https://doi.org/10.1007/BF01223371. MR2043362 · Zbl 0656.53078 · doi:10.1007/978-94-015-7809-7

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