**Gruen, Angus**; **Morrison, Scott**
**Computing modular data for pointed fusion categories.** (English) Zbl 07360772
Indiana Univ. Math. J. 70, No. 2, 561-593 (2021)

*Fusion categories* are a significant arena of investigation, occurring frequently in category theory as well as their applications in physics. An important current arena of investigation in condensed matter physics is *topological phases of matter*, an approach to which is *topological quantum field theories* (TQFTs). The *cobordism hypothesis* [*J. Lurie*, in: Current developments in mathematics, 2008. Somerville, MA: International Press. 129–280 (2009; Zbl 1180.81122)] claims that TQFTs are classified by higher categorical data. In particular, *C. L. Douglas* et al. [Dualizable tensor categories. Providence, RI: American Mathematical Society (AMS) (2021; Zbl 07363185)] identified fully extended TQFTs in $2+1$ dimensions with fusion categories.

It is well known that every *pointed* fusion category is equivalent to $\mathrm{Vec}^{\omega}G$ for some group $G$ and three-cocycle $\omega$. The principal objective in this paper is to study the *Drinfeld centers* $\mathcal{Z}(\mathrm{Vec}^{\omega}G)$ which are *modular tensor categories* arising also as representation categories of the *ribbon quasi-Hopf algebra* $D^{\omega}G$ known as the *twisted Drinfeld double* of a finite group, which first appeared in [*R. Dijkgraaf* et al., Nucl. Phys., B, Proc. Suppl. 18B, 60–72 (1990; Zbl 0957.81670)], having been extensively studied in mathematical physics owing mainly to its appearance in *Dijkgraaf-Witten theory* [*R. Dijkgraaf* and *E. Witten*, Commun. Math. Phys. 129, No. 2, 393–429 (1990; Zbl 0703.58011)]. Especially, in 2+1 dimensions, Dijkgraaf-Witten theory corresponds under the cobordism hypothesis to $\mathrm{Vec}^{\omega}G$, while the invariants of $S^1$ is no other than $\mathcal{Z}(\mathrm{Vec}^{\omega}G)$. A formula for the modular data of $\mathcal{Z}(\mathrm{Vec}^{\omega}G)$ was given without proof in [*A. Coste* et al., Nucl. Phys., B 581, No. 3, 679–717 (2000; Zbl 0984.81055)].

Although the categories $\mathrm{Vec}^{\omega}G$ are relatively straightfoward, the centers $\mathcal{Z}(\mathrm{Vec}^{\omega}G)$ are hard to pin down. *M. Mignard* and *P. Schauenberg* ["Modular categories are not determined by their modular data", Preprint, arXiv:708.02796] identified infinitely many finite groups $G$, for which there are at least two 3-cocyles $\omega$ and $\omega'$ with

$$\mathcal{Z}(\mathrm{Vec}^{\omega}G) \ncong \mathcal{Z}(\mathrm{Vec}^{\omega'}G)$$

though they have equivalent modular data. The smallest examples appear at

$$G = \mathbb{Z}/5\mathbb{Z} \rtimes \mathbb{Z}/11\mathbb{Z}$$

and

$$G = \mathbb{Z}/5\mathbb{Z} \rtimes \mathbb{Z}/13\mathbb{Z}$$

The corresponding sets of modular data in both cases are to be found in the authors' database. It is currently unknown whether the sets and matrices are completely invariant when $G < 55$, concerning which the authors' database is to point to the cases left to check.

Reviewer: Hirokazu Nishimura (Tsukuba)

**MSC:**

18M15 Braided monoidal categories and ribbon categories
16T99 Hopf algebras, quantum groups and related topics
81R50 Quantum groups and related algebraic methods applied to problems in quantum theory

**Software:**

Traces; GAP; IO; HAP; nauty

**Full Text:** DOI

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