Morrison, Scott; **Peters, Emily**; **Snyder, Noah**
Categories generated by a trivalent vertex. (English) Zbl 06706083
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This paper is the first in a general program to automate skein-theoretic arguments in quantum algebra and quantum topology, studying skein-theoretic invariants of planar triangular graphs after [*G. Kuperberg*, Int. J. Math. 5, No. 1, 61–85 (1994; Zbl 0797.57008)]. The paper is to be thought of as providing classifications of nondegenerate pivotal tensor categories over $\mathbb{C}$ generated by a symmetrically self-dual simple object X and a rotationally invariant morphism

$$1 \to X \otimes X \otimes X$$

The main result is that the only trivalent categories with

$$\dim \operatorname{Hom}\left(1 \to X^{\otimes n}\right)$$

bounded by

$$1, 0, 1, 1, 4, 11, 40$$

for $0 \leq n \leq 6$ are quantum $SO(3)$, quantum G_2, a one-parameter family of free products of certain Temperley-Lieb categories and the $H3$ Haagerup fusion category. The authors establish similar results where the map $1 \to X^{\otimes n}$ is not rotationally invariant. The main techniques are a new approach to finding skein relations which is to be easily automated with the help of Gröbner bases together with evaluation algorithms making use of the discharging method developed in the proof of the 4-color theorem [*K. Appel* and *W. Haken*, Bull. Am. Math. Soc. 82, 711–712 (1976; Zbl 0331.05106); J. Recreat. Math. 9, 161–169 (1977; Zbl 0357.05043); Ill. J. Math. 21, 429–490 (1977; Zbl 0387.05009); Every planar map is four colorable. Providence, RI: American Mathematical Society (1989; Zbl 0681.05027); *K. Appel* et al., Ill. J. Math. 21, 491–567 (1977; Zbl 0387.05010)].

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MSC:

18M05 Monoidal categories, symmetric monoidal categories
05C10 Planar graphs; geometric and topological aspects of graph theory
57K10 Knot theory

Full Text: DOI arXiv

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