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Join and slices for strict ∞ -categories. (Joint et tranches pour les ∞ -catégories strictes.)

(French. English summary) [Zbl 07362646](#)

Mém. Soc. Math. Fr., Nouv. Sér. 165, 1-203 (2020)

The principal objective in this book is to generalize the formalism of joins and slices to the category $\infty\text{-Cat}$ of strict ∞ -categories. The join defined in the book is compatible with the usual join of categories up to truncation. It is shown that the join gives rise to a monoidal category structure on $\infty\text{-Cat}$, respecting connected inductive limits in each variable, which implies the existence of its right adjoint called the ∞ -categorical slice. Stating some conjectures on the functoriality of the join and the slice with respect to higher lax and oplax transformations, the authors establish some first results in this direction.

A synopsis of the paper consisting of eleven chapters together with three appendices goes as follows.

- **Chapter 1** gives preliminaries on strict ∞ -categories, fixing notations and conventions while recalling some classical constructions.
- **Chapter 2** recalls the theory of augmented directed complexes by *R. Steiner* [Appl. Categ. Struct. 1, No. 3, 247–284 (1993; [Zbl 0804.18005](#))] and *S. E. Crans* and *R. Steiner*, *J. Aust. Math. Soc., Ser. A* 63, No. 1, 47–77 (1997; [Zbl 0883.18003](#)).
- **Chapter 3** aims to redeem the results on commutation of the functor

$$\nu : \mathcal{C}_{\text{da}} \rightarrow \infty\text{-Cat}$$

of Steiner with certain classes of inductive limits.

- **Chapter 4** recalls the category Θ introduced by Joyal [<https://ncatlab.org/nlab/files/JoyalThetaCategories.pdf>].
- **Chapter 5** is concerned with a generalization of a theorem by *B. Day* [Lect. Notes Math. 137, 1–38 (1970; [Zbl 0203.31402](#)); Lect. Notes Math. 420, 20–54 (1974; [Zbl 0367.18008](#)); *J. Pure Appl. Algebra* 2, 1–11 (1972; [Zbl 0236.18004](#))] on biclosed monoidal categories to locally biclosed monoidal categories.
- **Chapter 6** aims to define ∞ -categorical joins and slices, being the central chapter of this book.
- **Chapter 7** shows how the join permits defining easily orientals and nerves of *R. Street* [*J. Pure Appl. Algebra* 49, 283–335 (1987; [Zbl 0661.18005](#))].
- **Chapter 8** aims to demonstrate that the ∞ -categorical join induces an n -categorical join by truncation and that one obtains the usual categorical join for $n = 1$.
- Let \mathcal{C} be an ∞ -category with c an object of \mathcal{C} . Considering c as an ∞ -functor permits defining an ∞ -category $c \backslash \mathcal{C}$, which **Chapter 9** aims to describe explicitly.
- **Chapter 10** is concerned with slices of augmented directed complexes and their properties of functoriality.
- **Chapter 11** addresses results on functoriality of ∞ -categorical slices. Some first results on the functoriality of slices with respect to higher lax and oplax transformations.
- **Appendix A** revisits the tensor product of *J. W. Gray* [Formal category theory: Adjointness for 2-categories. Berlin-Heidelberg-New York: Springer-Verlag (1974; [Zbl 0285.18006](#))]. The authors' exposition is inspired by *R. Steiner* [Homology Homotopy Appl. 6, No. 1, 175–200 (2004; [Zbl 1071.18005](#)), §7].
- **Appendix B** establishes the equivalence between the definition of oplax ∞ -functors in the first chapter and the abstract notion in terms of ∞ -categorical tensor products.
- **Appendix C** aims to redeem the conjectures on functoriality of slices generalizing the results in Chapter 11.

Reviewer: [Hirokazu Nishimura \(Tsukuba\)](#)

MSC:

- 18-02 Research exposition (monographs, survey articles) pertaining to category theory
- 18Nxx Higher categories and homotopical algebra

Cited in 1 Document

Keywords:

Gray ∞ -categories; strict ∞ -categories; monoidal categories; locally biclosed monoidal categories; augmented directed complexes; join; Street's nerve; orientals; Gray tensor product; slices; lax transformations