Paré, Robert
Three easy pieces: imaginary seminar talks in honour of Bob Rosebrugh. (English)
Zbl 1469.18002
Theory Appl. Categ. 36, 171-200 (2021).

This paper consists of three sections, each section discussing a topic completely independent from the other two.

§1 establishes the following theorem, though, regrettably, it has already been discovered in [*R. Street*, Theory Appl. Categ. 27, 47–64 (2012; Zbl 1252.18007)].

Theorem. Let $\boldsymbol{A}$ and $\boldsymbol{B}$ be categories. Let $F : \mathrm{Ob}(\boldsymbol{A}) \to \mathrm{Ob}(\boldsymbol{B})$ and $U : \mathrm{Ob}(\boldsymbol{B}) \to \mathrm{Ob}(\boldsymbol{A})$ be object functions with

$$\theta_{A,B} : \boldsymbol{B}(FA, B) \to \boldsymbol{A}(A, UB)$$

bijections abiding by the following octagon diagram

$$\begin{tikzcd}
 & \boldsymbol{A}(A', UB) \arrow[r, "\boldsymbol{A}(f,UB)"] & \boldsymbol{A}(A, UB) \arrow[dr, "\theta^{-1}_{A,B}"] & \\
\boldsymbol{B}(FA', B) \arrow[ur, "\theta_{A',B}"] \arrow[d, "\boldsymbol{B}(FA',g)"'] & & & \boldsymbol{B}(FA, B) \arrow[d, "\boldsymbol{B}(FA,g)"] \\
\boldsymbol{B}(FA', B') \arrow[dr, "\theta_{A',B'}"'] & & & \boldsymbol{B}(FA, B') \\
 & \boldsymbol{A}(A', UB') \arrow[r, "\boldsymbol{A}(f',UB')"'] & \boldsymbol{A}(A, UB') \arrow[ur, "\theta^{-1}_{A,B'}"'] &
\end{tikzcd}$$

Then F and U are functors with $F \dashv U$.

§2 establishes the following three theorems.

Theorem 1. (Schröder-Bernstein for categories). Suppose we have an equivalence of categories $\Phi : \boldsymbol{A} \to \boldsymbol{B}$ with pseudo-inverse $\Psi : \boldsymbol{B} \to \boldsymbol{A}$ and isomorphisms $\alpha : \Psi\Phi \to 1_{\boldsymbol{A}}$ and $\beta : \Phi\Psi \to 1_{\boldsymbol{B}}$. Further assume that Φ and Ψ are both one-to-one on objects. Then there is an isomorphism of categories

$$\Theta : \boldsymbol{A} \xrightarrow{\cong} \boldsymbol{B}$$

and a natural isomorphism $\gamma : \Phi \to \Theta$.

Theorem 2. $\boldsymbol{Set}$ has a functorial choice of pullbacks.

Theorem 3. $\boldsymbol{FinCard}$ does not have a functorial choice of pullbacks.

§3 considers the following two questions.

Problem 1. Let $F : \boldsymbol{A} \to \boldsymbol{B}$ be a functor such that $\boldsymbol{A} \times \boldsymbol{A} \to \boldsymbol{B} \times \boldsymbol{B}$ has a left adjoint. Does F itself have a left adjoint?

Problem 2. Let $\boldsymbol{A}$ and $\boldsymbol{B}$ be categories, E and F idempotent functors on $\boldsymbol{A}$ and $\boldsymbol{B}$ respectively and let $\Phi : \boldsymbol{A} \to \boldsymbol{B}$ be such that $\Phi E = F\Phi$. Then Φ restricts to a functor

$$\boldsymbol{Fix}(\Phi) : \boldsymbol{Fix}(E) \to \boldsymbol{Fix}(F)$$

If Φ has a left adjoint, does $\boldsymbol{Fix}(\Phi)$ also have a left adjoint?

Reviewer: Hirokazu Nishimura (Tsukuba)

MSC:

18A40 Adjoint functors (universal constructions, reflective subcategories, Kan extensions, etc.)
18B35 Preorders, orders, domains and lattices (viewed as categories)
18A30 Limits and colimits (products, sums, directed limits, pushouts, fiber products, equalizers, kernels, ends and coends, etc.)
18N10 2-categories, bicategories, double categories
18D30 Fibered categories

Keywords:

adjoint functor; preorder; functorial pullbacks; 2-category

Biographic references:

Rosebrugh, Robert

Software:

MathOverflow

Full Text: Link

References:

[1] Tim Campion.Can we always make a strictly functorial choice of pullbacks/reindexing? MathOverflow. URL:https://mathoverflow.net/q/279998 (version: 201708-31).

[2] Aurelio Carboni, Stephen Lack, and R. F. C. Walters. Introduction to extensive and distributive categories.J. Pure Appl. Algebra, 84(2):145-158, 1993. · Zbl 0784.18001

[3] Herbert B. Enderton.Elements of set theory. Academic Press [Harcourt Brace Jovanovich, Publishers], New York-London, 1977. · Zbl 0383.03034

[4] Stephen Lack. A 2-categories companion. InTowards higher categories, volume 152 ofIMA Vol. Math. Appl., pages 105-191. Springer, New York, 2010. · Zbl 1223.18003

[5] Peter LeFanu Lumsdaine.Can we always make a strictly functorial choice of pullbacks/re-indexing?MathOverflow.URL:https://mathoverflow.net/q/ (version: 2013-10-11).

[6] Saunders MacLane.Categories for the working mathematician. Springer-Verlag, New York-Berlin, 1971. Graduate Texts in Mathematics, Vol. 5. · Zbl 0705.18001

[7] R. Par´e, R. Rosebrugh, and R. J. Wood. Idempotents in bicategories.Bull. Austral. Math. Soc., 39(3):421-434, 1989. · Zbl 0662.18002

[8] Robert Par´e.Absoluteness Properties in Category Theory. ProQuest LLC, Ann Arbor, MI, 1969. Thesis (Ph.D.)-McGill University (Canada).

[9] Robert Par´e. Contravariant functors on finite sets and Stirling numbers. volume 6, pages 65-76. 1999. The Lambek Festschrift.

[10] Ross Street. Cosmoi of internal categories.Trans. Amer. Math. Soc., 258(2):271-318, 1980. · Zbl 0393.18009

[11] Ross Street. The core of adjoint functors.Theory Appl. Categ., 27:No. 4, 47-64, 2012 · Zbl 1252.18007