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Rank-finiteness for G -crossed braided fusion categories. (English) Zbl 1469.18022

Transform. Groups 26, No. 3, 915-927 (2021).

It was in [V. Ostrik, Math. Res. Lett. 10, No. 2-3, 177-183 (2003; [Zbl 1040.18003](#))] that the question whether there are finitely many categories with a fixed number of isomorphism classes of simple objects was first raised. There it was settled affirmatively for rank 2, while it was also settled for weakly integral categories in [P. Etingof et al., Ann. Math. (2) 162, No. 2, 581-642 (2005; [Zbl 1125.16025](#))]. It was around 2003 that Wang conjectured that there are always finitely many *modular* categories of a given fixed rank, which was explicitly verified for rank at most 4. This rank-finiteness conjecture was settled completely in [P. Bruillard et al., J. Am. Math. Soc. 29, No. 3, 857-881 (2016; [Zbl 1344.18008](#))].

The principal objective in this paper is to extend rank-finiteness to the generality of G -crossed braided fusion categories, which includes the significant case of braided fusion categories, not requiring the existence of spherical structure.

To this end, the primary obstacle is the existence of *slightly degenerate* braided fusion categories, which is overcome by analyzing the structure of the Drinfeld centers of slightly degenerate categories. These categories are interesting in their own right, the main open question being whether or not every slightly degenerate braided fusion category \mathcal{C} admits a minimal non-degenerate extension. The authors' analysis of the \mathcal{C} -module subcategories of the Drinfeld center of \mathcal{C} is to be viewed as a step towards settling this question.

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MSC:

[18M15](#) Braided monoidal categories and ribbon categories

[18M20](#) Fusion categories, modular tensor categories, modular functors

Full Text: [DOI](#)

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