Marmolejo, F.; **Menni, M.**
The canonical intensive quality of a cohesive topos. (English) Zbl 1469.18004
Theory Appl. Categ. 36, 250-279 (2021).

So far, only two examples of the canonical intensive quality have been calculated explicitly, namely, that of the cohesive topos of finite reflexive graphs [*F. W. Lawvere*, Theory Appl. Categ. 19, 41–49 (2007; Zbl 1123.18001), §V] and that of the pre-cohesive topos of reflexive graphs [*F. W. Lawvere*, Categ. Gen. Algebr. Struct. Appl. 4, No. 1, 1–8 (2016; Zbl 1468.18005)]. The principal objective in this paper is to give an elementary proof of the canonical intensive quality, describing it concretely in other examples of (pre-)cohesion. A synopsis of the paper consisting of 10 sections goes as follows.

- §2 introduces a strengthening of the notion of local map that is opposite to quality types. They are called *intensive* maps. §3 recalls the definition of Leibniz object [loc. cit.], which is used in §4 to prove that every local and hyperconnected geometric morphism $p : \mathcal{E} \to \mathcal{S}$ has an external factorization $p = qs$ with s intensive and q a quality type. Following [*F. W. Lawvere*, Theory Appl. Categ. 19, 41–49 (2007; Zbl 1123.18001)], the direct image of s is called the canonical intensive quality of p. §5 is concerened with what can be said about Leibniz objects under the presence of codiscrete objects. §6 shows that if p is moreover essential, then so is s, thereby establishing a strengthening of a result of Lawvere [loc. cit., Theorem 2] that every pre-cohesive geometric morphism $p : \mathcal{E} \to \mathcal{S}$ has a canonical intensive quality $s : \mathcal{E} \to \mathcal{L}$.
- The rest of the paper (§§7–10) is concerned with examples. It is shown that if $\mathcal{E}$ is a presheaf topos, then so is $\mathcal{L}$, which lifts to Grothendieck toposes while the sites obtained need not be subcanonical. To articulate the subtle passage from $\mathcal{E}$ to $\mathcal{L}$, the authors consider a particular family of bounded cohesive toposes over $\boldsymbol{Set}$ and build subcanonical sites for their associated categories $\mathcal{L}$.

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MSC:

18B25 Topoi
03G30 Categorical logic, topoi
18F99 Categories in geometry and topology

Keywords:

axiomatic cohesion; topos theory; geometric morphisms; intensive quality

Full Text: Link

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