Quijano, Juan Pablo; **Resende, Pedro**
Functoriality of groupoid quantales. II. (English) Zbl 07382619
Appl. Categ. Struct. 29, No. 4, 629-670 (2021)

The principal objective in this paper is to put forward a language for Hilsum-Skandalis maps [*M. Hilsum* and *G. Skandalis*, Ann. Sci. Éc. Norm. Supér. (4) 20, No. 3, 325–390 (1987; Zbl 0656.57015)] and Morita equivalence which is more algebraic in the sense of being closer to what one would expect for ring-like objects such as quantales. Some applications are

- One application concerns Morita theory for pseudogroups. The Morita theory of inverse semigroups has been thoroughly studied [*B. Steinberg*, Houston J. Math. 37, No. 3, 895–927 (2011; Zbl 1236.46049); *J. Funk* et al., J. Pure Appl. Algebra 215, No. 9, 2262–2279 (2011; Zbl 1229.20064)], while pseudogroups carry more topological information than general inverse semigroups, in particular in that their idempotents form locales. It is natural to define Morita equivalence for pseudogroups via the equivalence of categories so that to each pseudogroup S one associates its inverse quantal frame $\mathcal{L}^{\vee}(S)$ [*P. Resende*, Adv. Math. 208, No. 1, 147–209 (2007; Zbl 1116.06014)]. That is to say, two pseudogroups S and T are Morita equivalent iff a biprincipal $\mathcal{L}^{\vee}(S)$-$\mathcal{L}^{\vee}(T)$-bisheaf exists. This leads to a surprisingly elegant notion of equivalence bimodule for pseudogroups [*M. V. Lawson* and *P. Resende*, "Morita equivalence of pseudogroups", Preprint, `arXiv:2011.14335`], which, also surprisingly, is very similar to the Morita equivalence for general inverse semigroups [*B. Steinberg*, Houston J. Math. 37, No. 3, 895–927 (2011; Zbl 1236.46049)].
- The results of this paper are also crucial to [*J. P. Quijano* and *P. Resende*, J. Algebra 566, 222–258 (2021; Zbl 1448.18015)] addressing bi-actions and sheaves on non-étale groupoids after [*M. C. Protin* and *P. Resende*, J. Noncommut. Geom. 6, No. 2, 199–247 (2012; Zbl 1253.06019)] where inverse quantale frames are replaced by pairs $(Q, \mathcal{O})$ in which Q is an inverse quantal frame and $\mathcal{O} \subset Q$ is an ideal coinciding with the quantale of a non-étale groupoid covered by the groupoid of Q.

A synopsis of the paper, consisting of six sections, goes as follows.

- §1 is an introduction, and §6 is a discussion.
- §2 recalls some basic facts concerning the relation between étale groupoids and quantales as well as the relation between (bi-)actions of étale groupoids and quantale (bi-)modules [*P. Resende*, Adv. Math. 208, No. 1, 147–209 (2007; Zbl 1116.06014); J. Pure Appl. Algebra 216, No. 1, 41–70 (2012; Zbl 1231.06020); J. Pure Appl. Algebra 219, No. 8, 3089–3109 (2015; Zbl 1343.06007); *J. P. Quijano* and *P. Resende*, Appl. Categ. Struct. 29, No. 4, 629–670 (2021; Zbl 07382619)].
- §3 discusses technical results about sheaves on locales and quantales which are not found in [*P. Resende*, J. Pure Appl. Algebra 216, No. 1, 41–70 (2012; Zbl 1231.06020); *P. Resende* and *E. Rodrigues*, Appl. Categ. Struct. 18, No. 2, 199–217 (2010; Zbl 1200.18008)].
- §4 provides an overview of definitions and facts concerning principal bundles, Hilsum-Skandalis maps and Morita equivalence for localic étale groupoids.
- §5 achieves the main goal of this paper, which is to study principal bundles, Hilsum-Skandalis maps and Morita equivalence for inverse quantal frames.

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MSC:

18F15	Abstract manifolds and fiber bundles (category-theoretic aspects)
18F20	Presheaves and sheaves, stacks, descent conditions (category-theoretic aspects)
18F70	Frames and locales, pointfree topology, Stone duality
18F75	Quantales
22A22	Topological groupoids (including differentiable and Lie groupoids)
55R10	Fiber bundles in algebraic topology

Keywords:

étale groupoids; inverse quantal frames; sheaves; principal bundles; Hilsum-Skandalis maps; Morita equivalence

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doi:10.1016/j.jpaa.2014.10.004

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