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Bruce, Andrew James (LUX-ULUX2-MRU); **Grabowski, Janusz** (PL-PAN)
Odd connections on supermanifolds: existence and relation with affine connections. (English summary)
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Among others, the main results of this paper are as follows:

- If ρ is an odd involution, then the torsion and curvature are tensors (Theorem 2.14).
- The curvature and torsion of an odd connection obey a generalized version of the algebraic Bianchi identity (Theorem 2.28).
- $n|n$-dimensional Lie supergroups always admit an odd connection (Theorem 2.38).
- The example of super-Minkowski space-time leads to the notion of an odd Weitzenböck connection on an $n|n$-dimensional parallelizable supermanifold. It is shown that such connections depend only on the existence of an odd involution and so $n|n$-dimensional parallelizable supermanifolds always admit an odd Weitzenböck connection (Proposition 2.50). It is also shown that an odd Weitzenböck connection is compatible with an odd Riemannian metric (Proposition 2.54).

Odd symplectic/Poisson structures are central to the BV-formalism and its generalizations [S. L. Lyakhovich and A. A. Sharapov, Nuclear Phys. B **703** (2004), no. 3, 419–453; MR2105279; A. S. Schwarz, Comm. Math. Phys. **155** (1993), no. 2, 249–260; MR1230027]. Odd counterparts of superconformal transformations that twist the parity of the standard basis of the module of vector fields on $C^{1|1}$ were first proposed in [S. Duplij, J. Math. Phys. **32** (1991), no. 11, 2959–2966; MR1131674; J. Math. Phys. **38** (1997), no. 2, 1035–1040; MR1434224], leading to an odd generalization of super-Riemann surfaces. The authors ask the following:

- If a good concept of an odd connection exists, do any of the supermanifolds of interest in physics admit such things?
- Is there any relation with supersymmetry as formulated in superspace?

Hirokazu Nishimura

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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.