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Model Uncertainty, Economic Development, and Welfare Costs of Business Cycles

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Model Uncertainty, Economic Development, and Welfare Costs of Business Cycles *

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Abstract

Some researchers argue that the welfare gains from eliminating consumption fluctuations for the United States are not small once model uncertainty is taken into account. This paper presents new evidence on the welfare gains from eliminating model uncertainty using a data set from a broad range of countries. It quantifies exactly the effect of model uncertainty on the welfare gains using an analytical formula. The results indicate that most countries derive much larger gains from the reduction of model uncertainty compared with the United States. Countries at higher stages of economic development tend to have lower welfare gains because their gains from eliminating model uncertainty become smaller. This relationship does not depend on country size or trade openness.

Keywords: Consumption risk; Detection error probability; Model uncertainty; Welfare costs

JEL Classification: D81; E21; E32

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1 Introduction

Since the seminal work of Lucas (1987), researchers have reconsidered the striking finding that the welfare gains from eliminating consumption fluctuations calculated from US post-World War II data are trivially small. As Lucas (2003) reviews, in the first 15 years after his finding, many researchers have found that modifications of the Lucas calculation can lead to larger, but still small, welfare gains. One exception among these early studies is Tallarini (2000), whose welfare-gain estimates are two orders of magnitude larger than those found by Lucas.

Barillas et al. (2009) focus on the observational equivalence between Tallarini's risk-sensitive and multiplier preferences, and argue that Tallarini's large gains arise from eliminating model uncertainty, not from reducing consumption risk drawn from a known probability distribution. Ellison and Sargent (2015) also argue that the large gains are associated with agents' concerns about model misspecification, and that incorporating idiosyncratic risk into the framework leads to larger gains. These conclusions are based on evidence from US data.

In this paper, we present additional evidence on the welfare gains from eliminating model uncertainty. We examine the following two questions:

(i) Are there large welfare gains from eliminating model uncertainty in a broad range of countries other than the United States?

(ii) If so, how does the gain differ at various stages of economic development, e.g., between rich, emerging, and poor countries?

There is existing evidence on the first question. Engel et al. (2018) show that there are substantial gains in emerging countries. An earlier version of this paper (Okubo (2019)) shows that relatively large gains are observable in other developed and emerging countries compared with the United States.[1] However, as the sample size of countries is small in these studies (15 and

[1] It includes a numerical example based on Barillas et al.'s (2009) parameter settings, a replication of Ellison and

22, including the United States, in Engel et al. (2018) and Okubo (2019), respectively), further investigation is required.

The second question is motivated by the literature on business cycles in emerging and poor countries. Pallage and Robe (2003) follow the tradition of Lucas (1987) and show that there are very large welfare costs of consumption fluctuations in developing countries relative to the United States. Uribe and Schmitt-Grohé (2017) document excess output volatility and higher relative consumption volatility in emerging and poor countries as key business-cycle facts.[2] We aim to reevaluate these differences between poor, emerging, and developed countries through the lens of model uncertainty.

We use annual data on 64 countries for the period 1970–2018. Our welfare-gain calculations are based on an analytical formula, which combines Okubo's (2018a) closed-form solutions of detection error probabilities with the formula proposed by Barillas et al. (2009). The advantage of using this analytical formula is that the effect of agents' fear of model misspecification on welfare gains is quantified exactly. We examine questions (i) and (ii) in two ways. First, we classify the 64 countries into three groups—rich, emerging, and poor—based on a measure of economic development and compare their sample statistics within and between the groups, as well as with the US welfare-gain estimates. Then, we carry out regression analysis with our welfare-gain measures as dependent variables to characterize the relationship between welfare gains and economic development. In this analysis, we control for the effects of two factors—country size and trade openness—that may affect business cycles, as pointed out in the literature (e.g., Kose et al. (2003), Furceri and Karras (2007), di Giovanni and Levchenko (2009, 2012), Haddad et al. (2013), and Uribe and Schmitt-Grohé (2017)).

Sargent (2015) using an analytical solution, and comparisons of results between quarterly and annual data.

[2]More specifically, Uribe and Schmitt-Grohé (2017, p. 9) summarize these facts as "Business cycles in rich countries are about half as volatile as business cycles in emerging or poor countries" and "The relative consumption volatility is higher in poor and emerging countries than in rich countries." They argue that explaining these facts is one of the most important unfinished subjects in business-cycle theory. Indeed, it has led to a large body of literature, as discussed later in Section 4.3.

A summary of our findings is as follows. The welfare cost of model uncertainty is substantial relative to that of consumption risk for a broad set of countries. Most countries have welfare gains from eliminating model uncertainty that are considerably higher than those of the United States. As for the second question, the comparison of the sample statistics between the groups shows that the welfare gains from eliminating model uncertainty are, on average, at least two times higher in emerging or poor countries than in rich countries. This suggests that higher levels of economic development are associated with lower welfare costs of model uncertainty. The regression analysis shows that this negative relationship is robust to controlling for country size and trade openness, and that it is stronger than the negative relationship between the level of economic development and the welfare cost of consumption risk. In addition, it shows that as agents' fear of model misspecification becomes milder, the negative relationship between the level of economic development and the welfare cost of model uncertainty becomes weaker.

There is a limitation to the formula of Barillas et al. (2009) and it deserves special mention at the outset. According to their formula (shown in formula (4) or (5) below), the welfare gain from eliminating model uncertainty is positively linked to the volatility of consumption. This means that cross-country comparisons of the gains from reducing model uncertainty are in fact equivalent to comparing the magnitudes of consumption volatility across countries. Thus, the first part of our findings simply says that countries with higher consumption volatility, which are largely the emerging and poor countries, have higher welfare gains from eliminating model uncertainty, other things being equal. However, the other part of our findings cannot be drawn from such cross-country comparisons. The use of our formula (formula (7) below), which quantifies exactly the effect of agents' fear of model misspecification on welfare gains, allows us to draw these other findings.

The remainder of the paper is organized as follows. Section 2 describes our formula after reviewing the welfare-gain measures proposed by Barillas et al. (2009). Section 3 describes the data and classification of the sample countries. Section 4 presents our findings. We also provide potential explanations for the findings by discussing how they can be related to the existing literature. Section 5 explores the effect of idiosyncratic risk. Section 6 provides a brief conclusion. The separate appendix contains details on parameter settings and estimates for individual countries as well as the regression results that were omitted from the main paper.

2 Framework

2.1 The Barillas–Hansen–Sargent Formula

The notation is the same as that of Barillas et al. (2009). Let $\{\epsilon_t\}$ denote a sequence of random shocks with conditional densities $\pi(\epsilon_t) \equiv \pi(\epsilon_t \mid \epsilon^{t-1}, x_0)$, where $\epsilon^{t-1} \equiv [\epsilon_{t-1}, \epsilon_{t-2}, \ldots, \epsilon_1]$, and x_0 is a given initial state. Two models are assumed for log consumption streams $\{c_t\}$. One is the random walk with drift model:

$$c_t = \mu + c_{t-1} + \sigma_\epsilon \epsilon_t, \quad \epsilon_t \sim \text{i.i.d.} N(0,1). \tag{1}$$

Another is the trend stationary model:

$$c_t = \zeta + \mu t + z_t, \quad z_t = \rho z_{t-1} + \sigma_\epsilon \epsilon_t, \quad |\rho| < 1, \quad \epsilon_t \sim \text{i.i.d.} N(0,1). \tag{2}$$

The key assumption of Barillas et al.'s framework is that a representative agent with log period utility does not completely trust $\pi(\epsilon_t)$ because of model uncertainty. Consequently, the agent imposes a penalty based on relative entropy and chooses some other density $\hat{\pi}(\epsilon_t)$ in proximity to $\pi(\epsilon_t)$. Then, the agent's preferences over consumption streams can be expressed by the following value function recursion:

$$U_t = c_t - \beta\theta \ln E_t \left[\exp\left(-\frac{U_{t+1}}{\theta} \right) \right], \tag{3}$$

where U_t is the value function, β is the discount factor, and θ is the penalty parameter attached to the relative entropy. In their framework, a smaller value of the parameter θ implies a higher degree of agent concern about model misspecification (see Maccheroni et al. (2006) for details).

The Barillas–Hansen–Sargent formula for measuring welfare gains from the removal of consumption fluctuations is based on this value function recursion. The idea is to consider the adjustment to initial consumption that makes the agent indifferent between a deterministic consumption path and risky consumption paths (1) or (2). Additionally, let $\{c_t^d\}$ denote the deterministic consumption path that has the same mean as the risky consumption path. Then, the Barillas–Hansen–Sargent formula is given by

$$\lambda \equiv c_0 - c_0^d = \frac{\beta}{1-\beta}\frac{\sigma_\epsilon^2}{2} + \frac{\beta\sigma_\epsilon^2}{2\theta(1-\beta)^2} \tag{4}$$

for the random walk model, and by

$$\lambda \equiv c_0 - c_0^d = \frac{\beta}{1-\beta\rho^2}\frac{\sigma_\epsilon^2}{2} + \frac{\beta\sigma_\epsilon^2}{2\theta(1-\beta\rho)^2} \tag{5}$$

for the trend stationary model (see Table 3 of Barillas et al. (2009)). The first term on the right-hand side of the formulas corresponds to Lucas's (1987, 2003) welfare-gain measure (multiplied by $\beta/(1-\beta)$ or $\beta/(1-\beta\rho^2)$) under log utility, i.e., the welfare gain from eliminating consumption risk. The second term captures the welfare gain from eliminating model uncertainty.

2.2 Welfare-Gain Calculations Based on Closed-Form Solutions

To implement welfare-gain calculations using the formula above, it is necessary to set the parameter θ, the magnitude of which is associated with the degree of the agent's fear of model misspecification. The strategy for quantifying that degree developed in the literature (see, e.g., Anderson et al. (2003) and Hansen and Sargent (2008)) is to utilize detection error probabilities. The procedure relies entirely on simulation. Barillas et al. (2009) use this simulation-based method in calculating the welfare gain λ. Alternatively, we use an analytical method, which combines Okubo's (2018a) closed-form solutions for the detection error probabilities with the Barillas–Hansen–Sargent formula.

The main point of the method is that the overall detection error probability developed in Hansen and Sargent (2008) (denoted by $p(\theta^{-1})$), which measures the degree to which the agent fears model misspecification, can be represented in a closed form as

$$p(\theta^{-1}) = \begin{cases} \Phi\left(-\dfrac{\sqrt{T}}{2}\theta^{-1}\dfrac{\sigma_\epsilon}{1-\beta}\right) & \text{for the random walk model,} \\ \\ \Phi\left(-\dfrac{\sqrt{T}}{2}\theta^{-1}\dfrac{\sigma_\epsilon}{1-\beta\rho}\right) & \text{for the trend-stationary model,} \end{cases} \tag{6}$$

where $\Phi(\cdot)$ is the standard normal cumulative distribution function and T is the length of time period (more precisely, the number of observations on Δc_t in the current setting) (see footnote 3 of Okubo (2018a) for the intuition of why this formula holds, and see Okubo (2018b) for the derivation). Thus, for a given value of $p(\theta^{-1})$ (denoted by $\overline{p(\theta^{-1})}$), we can write the Barillas–Hansen–Sargent formula as

$$\lambda = \begin{cases} \dfrac{\beta}{1-\beta}\dfrac{\sigma_\epsilon^2}{2} - \dfrac{\beta}{1-\beta}\dfrac{\sigma_\epsilon}{\sqrt{T}}\Phi^{-1}\left(\overline{p(\theta^{-1})}\right) & \text{for the random walk model,} \\ \\ \dfrac{\beta}{1-\beta\rho^2}\dfrac{\sigma_\epsilon^2}{2} - \dfrac{\beta}{1-\beta\rho}\dfrac{\sigma_\epsilon}{\sqrt{T}}\Phi^{-1}\left(\overline{p(\theta^{-1})}\right) & \text{for the trend-stationary model,} \end{cases} \tag{7}$$

where $\Phi^{-1}(\cdot)$ is the inverse function of $\Phi(\cdot)$.

The advantages of using formula (7) are twofold.[3] First, it facilitates calculations because there is no need to implement simulation.[4] Second, it makes the results easier to interpret in two aspects. The first is that the formula clarifies that the welfare gain from eliminating model uncertainty is a function of the volatility parameter σ_ϵ and the length of time period T. For example, formula (7) tells us that the length of the sample period used for analysis must be common to all countries in making cross-country comparisons. In contrast, formulas (4) and (5) do not provide any such guidance. The second aspect is that the formula clarifies how the degree of the agent's fear of model misspecification affects the welfare gain from eliminating model uncertainty through the term $\Phi^{-1}\left(\overline{p(\theta^{-1})}\right)$. To see this, let $\overline{p(\theta^{-1})} = 0.5$. This means that there is no fear of model misspecification. Then, the second term on the right-hand side of formula (7) is clearly zero because

[3] Okubo (2019) shows that there is another advantage of formula (7), as it allows us to compute standard errors of λ, λ_{fear}, and λ_{nofear} by applying the delta method.

[4] In the simulation-based method, the overall detection error probability function (i.e., the values of $p(\theta^{-1})$ for various values of θ^{-1}) must be computed by simulation. See Chapter 9 of Hansen and Sargent (2008) or Section 2 of Okubo (2018a) for an exposition of the simulation procedure. The use of formula (6) enables us to skip this step.

$\Phi^{-1}(0.5) = 0$. Thus, the welfare-gain measure λ coincides with Lucas's (1987, 2003) calculation (multiplied by $\beta/(1-\beta)$ or $\beta/(1-\beta\rho^2)$). In the case of $0 < \overline{p(\theta^{-1})} < 0.5$ (which means that the agent fears model misspecification), the term $\Phi^{-1}\left(\overline{p(\theta^{-1})}\right)$ takes a negative value. Hence, the second term is positive as long as the agent fears model misspecification. It determines an increment in the welfare gain associated with a change in an amount of model uncertainty. For ease of notation, we refer to the first and second terms on the right-hand side of formula (7) as λ_{nofear} and λ_{fear}, respectively.

As is clear from a comparison between the two measures in formula (7), if ρ is close to one, then the measure for the trend stationary model is approximately equal to that for the random walk model. Otherwise, because $\beta/(1-\beta\rho^2) < \beta/(1-\beta)$ and $\beta/(1-\beta\rho) < \beta/(1-\beta)$, λ_{nofear} and λ_{fear} (the first and second terms, respectively, on the right-hand side) for the trend stationary model are smaller than those for the random walk model, other things being equal.

In our welfare-gain calculations, we must give the overall detection error probability, $\overline{p(\theta^{-1})}$. Following the literature,[5] we choose $\overline{p(\theta^{-1})} = 0.05$, 0.10, and 0.20 in our empirical analysis.

3 The Data

The data are from the World Bank's World Development Indicators (WDI) database. The WDI data are annual and the sample period that we use is 1970–2018.[6] One advantage of using this data is that they cover many more countries relative to those from other international databases. For every country, we require data on private consumption, total population, and purchasing power parity (PPP)-converted GDP. After removing the countries with missing data for the sample period, our sample consists of 64 countries. A list of the countries included is provided in Appendix A and

[5]For example, Barillas et al. (2009, p. 2405) state: "... we think it is sensible for a decision maker to want to guard against possible misspecications whose detection error probabilities are 0.2 or even less." Meanwhile, Ljungqvist and Sargent (2018, p. 584) state: "From our own experience fitting models to data, a person whose specication doubts include perturbed models with a detection error of .25 or .1 or even .05 could be said to have a plausible amount of model uncertainty."

[6]The WDI data are available from 1960. However, for many countries including developed ones, data on private consumption start from 1970. Thus, we set 1970 as the beginning of the sample period.

a detailed description of the data is given in Appendix B.

For each country, we use real private consumption per capita as consumption in the model. In the WDI data, consumption is measured as total consumption expenditure for all countries. Unfortunately, separate data on nondurable and durable consumption expenditure are not available. Hence, an important caveat to our analysis is that estimates of the volatility parameter σ_ϵ may become somewhat higher than those based only on expenditure on nondurables and services because of the lumpiness of durable consumption.

We classify the countries in the sample into three groups (i.e., poor, emerging, and rich) by stages of economic development. Following Uribe and Schmitt-Grohé (2017), we use the geometric average of PPP-converted GDP per capita in 2011 US dollars over the period 1990–2018 as the measure of development. We define poor, emerging, and rich countries with the geometric average of PPP-converted GDP per capita up to 3,000 dollars, between 3,001 and 25,000 dollars, and more than 25,000 dollars, respectively. Our sample contains 10 poor countries, 28 emerging countries, and 26 rich countries.

As pointed out by Uribe and Schmitt-Grohé (2017), among others, the choice of thresholds for the classification is somewhat arbitrary. Another classification adopted by the World Bank is also applicable. It divides countries into four groups (i.e., high, upper-middle, lower-middle, and low income) based on gross national income per capita.[7] Table 1 shows the relationship between the two classification schemes using a cross tabulation. Combining the upper-middle and lower-middle income groups into one middle income group, we can see that our three groups (rich, emerging, and poor) almost coincide with the high, middle, and low income groups based on the World Bank classification. Thus, we concentrate only on our PPP-converted GDP-based classification below.

[7] The thresholds for the four groupings are updated every year. For more information on that method, see the World Bank data help desk (https://datahelpdesk.worldbank.org/).

Table 1: Relationship between Two Classifications

	The World Bank classification by income				
	High	Upper-middle	Lower-middle	Low	Total
Rich	26	0	0	0	26
Emerging	3	16	9	0	28
Poor	0	0	4	6	10
Total	29	16	13	6	64

Note: The number of countries is shown in each cell. The classification (i.e., rich, emerging, and poor) is based on the geometric average of PPP-converted GDP per capita in 2011 US dollars over the period 1990–2018. Following Uribe and Schmitt-Grohé (2017, Chapter 1), we define poor, emerging, and rich countries as those for which the geometric average of PPP-converted GDP per capita is up to 3,000 dollars, between 3,001 and 25,000 dollars, and more than 25,000 dollars, respectively. See Appendix A for details of the sample countries.

4 Results

4.1 Welfare Gains in Rich, Emerging, and Poor Countries

We begin by looking at a simple scatterplot between welfare gains and a measure of economic development. In what follows, we set the discount factor and the sample size to $\beta = 0.950$ and $T = 48$, respectively. The choice of the discount factor follows Ellison and Sargent (2015). The values of the volatility and autoregressive parameters, σ_ϵ and ρ, are set to maximum likelihood estimates presented in Appendix Table B. To save space, we provide results only for the case of a detection error probability of 0.10.

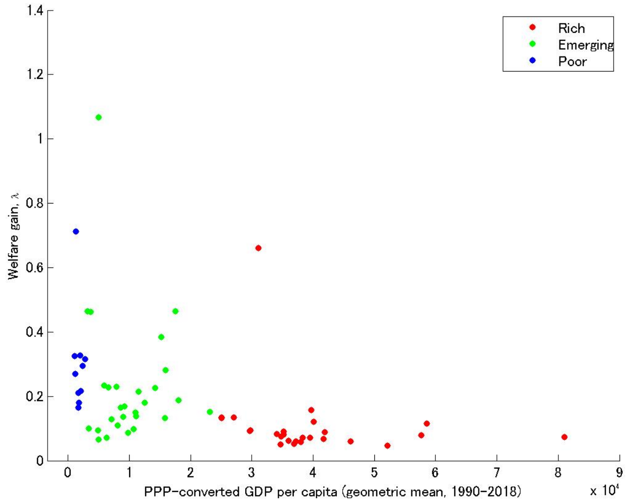
Figures 1 and 2 plot the welfare gains against the geometric average of PPP-converted GDP per capita over the period 1990–2018. Panels (a), (b), and (c) of each figure correspond to the cases where the vertical axis indicates λ, λ_{fear}, and λ_{nofear}, respectively. Figure 1a, which is based on the random walk model, shows that there is a negative relationship between the welfare gains and the measure of economic development: rich countries, i.e., countries at higher stages of economic development, have lower welfare gains from eliminating consumption fluctuations. The comparison between Figures 1b and 1c also indicates that this negative relationship arises mainly from the negative relationship between the welfare gain from eliminating model uncertainty and the measure of economic development. Figure 2 shows that these results are also observable for the welfare gains based on the trend stationary model.

As is clear from Figure 1, there are three extreme values in the random walk case. In the following analysis, we present results both including and excluding the three countries (Congo, Puerto Rico, and Togo) that produce those extreme cases to illustrate their influence. Furthermore, there are two inappropriate cases in the trend stationary model, Bolivia and the Philippines, as their autoregressive parameters are above one, as confirmed in Appendix Table B. Figure 2c shows that, of the two cases, one leads to a negative value of λ_{nofear}. Therefore, for the trend stationary model, we present results both including and excluding these two countries.

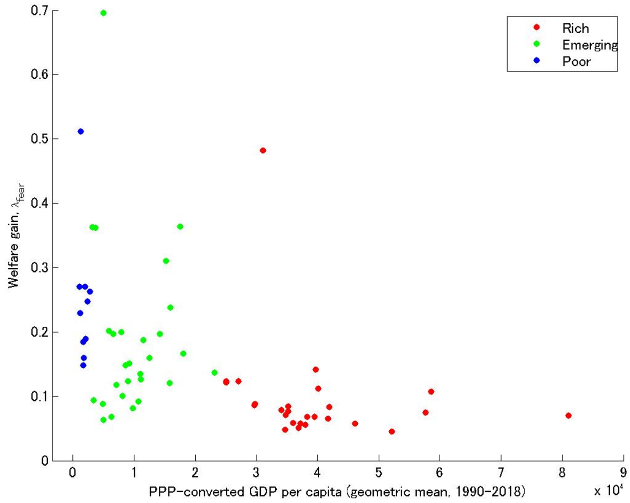
Table 2 reports the mean and standard deviation of the welfare gains for each group as well as estimates for the United States. Panels A and B of Table 2 show the results from the random walk and trend stationary models, respectively. The second column for each group (i.e., columns (3), (5), (7), and (9)) reports the results for the sample excluding the extreme or inappropriate cases. The standard deviations are shown in square brackets.

Table 2 reveals three interesting facts. First, comparisons within groups show that the welfare gain from eliminating model uncertainty, λ_{fear}, is considerably higher than that from eliminating consumption risk, λ_{nofear}. For example, the average λ_{fear}-to-λ_{nofear} ratio at $\overline{p(\theta^{-1})} = 0.10$ for all countries is approximately 5 (= 0.160/0.031) and 7 (= 0.141/0.020) in the random walk model for the samples including and excluding outliers, respectively, whereas it is almost 10 (= 0.053/0.005 and 0.049/0.005 for the two samples) in the trend stationary model.

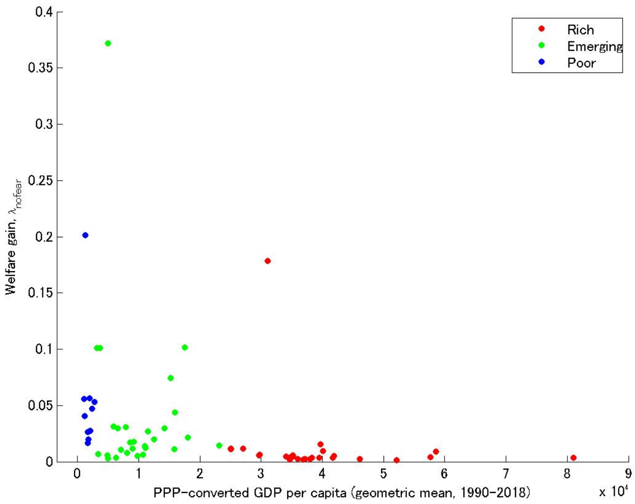
Second, comparisons between groups show that the welfare gains, λ and λ_{fear}, are at least two times higher on average in emerging or poor countries than in rich countries. For the case of $\overline{p(\theta^{-1})} = 0.10$ in Panel A, for example, the average total welfare gain for rich countries is $\lambda = 0.108$ and 0.086 for the samples including and excluding outliers, respectively, whereas it is $\lambda = 0.229$ and 0.198 for emerging countries and $\lambda = 0.302$ and 0.256 for poor countries. This fact is true for the other values of detection error probability, irrespective of whether the random walk or trend



(a) Welfare gains from eliminating consumption fluctuations, λ

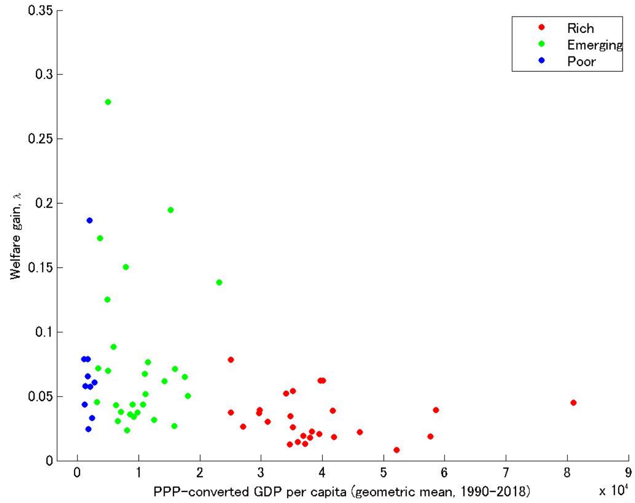


(b) Welfare gains from eliminating model uncertainty, λ_{fear}

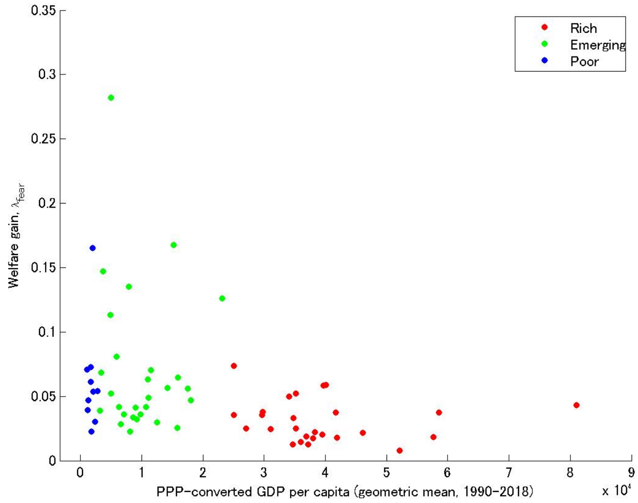


(c) Welfare gains from eliminating consumption risk, λ_{nofear}

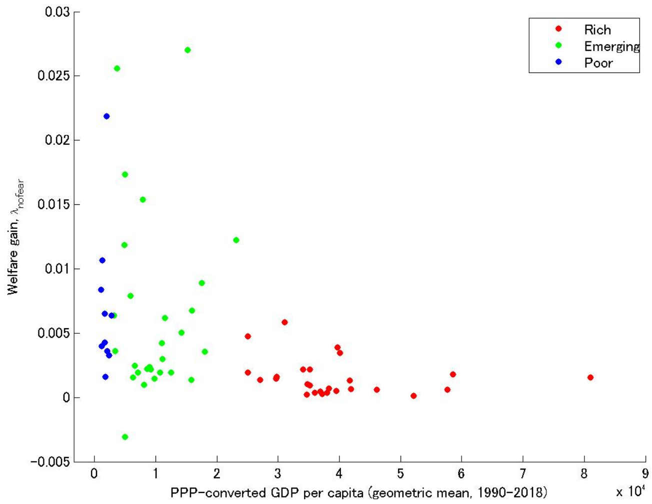
Figure 1: Relationship between welfare gains based on the random walk model and PPP-converted GDP per capita under the 10% detection error probability



(a) Welfare gains from eliminating consumption fluctuations, λ



(b) Welfare gains from eliminating model uncertainty, λ_{fear}



(c) Welfare gains from eliminating consumption risk, λ_{nofear}

Figure 2: Relationship between welfare gains based on the trend stationary model and PPP-converted GDP per capita under the 10% detection error probability

Table 2: Welfare Gains in Rich, Emerging, and Poor Countries

	Detection error prob	US	Mean and standard deviation							
			All		Rich		Emerging		Poor	
		(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
A. Results based on the random walk model										
λ	5%	0.077	0.237	0.201	0.136	0.109	0.283	0.247	0.372	0.318
			[0.212]	[0.130]	[0.140]	[0.038]	[0.239]	[0.144]	[0.186]	[0.078]
	10%	0.060	0.192	0.161	0.108	0.086	0.229	0.198	0.302	0.256
			[0.178]	[0.106]	[0.116]	[0.030]	[0.201]	[0.119]	[0.156]	[0.064]
	20%	0.041	0.136	0.113	0.075	0.059	0.164	0.140	0.217	0.181
			[0.136]	[0.078]	[0.088]	[0.021]	[0.156]	[0.088]	[0.121]	[0.047]
λ_{fear}	5%	0.074	0.206	0.180	0.123	0.104	0.243	0.218	0.317	0.280
			[0.157]	[0.106]	[0.106]	[0.034]	[0.170]	[0.114]	[0.133]	[0.062]
	10%	0.058	0.160	0.141	0.096	0.081	0.189	0.170	0.247	0.218
			[0.122]	[0.082]	[0.083]	[0.027]	[0.132]	[0.089]	[0.103]	[0.048]
	20%	0.038	0.105	0.092	0.063	0.053	0.124	0.112	0.162	0.143
			[0.080]	[0.054]	[0.054]	[0.018]	[0.087]	[0.058]	[0.068]	[0.032]
λ_{nofear}		0.003	0.031	0.020	0.012	0.006	0.040	0.028	0.054	0.038
			[0.058]	[0.025]	[0.034]	[0.004]	[0.071]	[0.030]	[0.054]	[0.016]
# of countries			64	61	26	25	28	27	10	9
B. Results based on the trend stationary model										
λ	5%	0.028	0.073	0.067	0.042	–	0.098	0.085	0.086	–
			[0.062]	[0.050]	[0.023]	–	[0.076]	[0.057]	[0.056]	–
	10%	0.022	0.058	0.053	0.033	–	0.077	0.068	0.069	–
			[0.049]	[0.040]	[0.018]	–	[0.060]	[0.046]	[0.045]	–
	20%	0.015	0.040	0.037	0.022	–	0.053	0.047	0.048	–
			[0.034]	[0.028]	[0.012]	–	[0.041]	[0.032]	[0.032]	–
λ_{fear}	5%	0.028	0.069	0.063	0.040	–	0.091	0.079	0.079	–
			[0.059]	[0.045]	[0.022]	–	[0.073]	[0.051]	[0.051]	–
	10%	0.022	0.053	0.049	0.031	–	0.071	0.061	0.062	–
			[0.046]	[0.035]	[0.017]	–	[0.057]	[0.039]	[0.040]	–
	20%	0.014	0.035	0.032	0.021	–	0.047	0.040	0.041	–
			[0.030]	[0.023]	[0.011]	–	[0.037]	[0.026]	[0.026]	–
λ_{nofear}		0.001	0.005	0.005	0.002	–	0.007	0.007	0.007	–
			[0.006]	[0.006]	[0.001]	–	[0.007]	[0.007]	[0.006]	–
# of countries			64	62	26	–	28	26	10	–

Note: Column (1) shows the welfare gains for the United States. In column (1), there are cases in which the sum of the estimates λ_{nofear} and λ_{fear} is not completely equal to the estimate of λ due to rounding errors. Columns (2)–(9) show the mean and standard deviation of the welfare gains for each group. The standard deviation is in square brackets. The second column for each group (columns (3), (5), (7), and (9)) shows the results for the sample excluding outliers. The symbol "–" denotes that the mean and standard deviation are the same as those in the first column for each group.

stationary model is used.

Third, comparisons between column (1) and the others indicate that many other countries have welfare gains that are considerably higher than those of the United States. As shown in Appendix Tables C.1 and C.2, there are only four countries with smaller welfare gains than the United States in the random walk model and nine countries in the trend stationary model.

Our estimated value of λ_{nofear} for the United States in column (1) is comparable with Lucas (2003), who reports a welfare gain from eliminating consumption risk of 0.0005 using annual US data for the period 1947–2001. If we use another parameter estimate ($\sigma_\epsilon = 0.022$) mentioned in footnote 5 of Lucas (2003), then his welfare-gain estimate becomes 0.0002 ($\approx (0.022)^2/2$). For direct comparison with these estimates by Lucas (2003), for example, in the random walk case, we must divide λ_{nofear} by 19 ($= 0.950/(1-0.950)$), as indicated by formula (7). This adjustment produces a welfare-gain estimate of approximately 0.0002 ($\approx 0.003/19$).

One fact that emerges clearly from the figures and table is the negative relationship between the welfare gains and the extent of economic development, although it does not indicate any causation. In the next subsection, we further examine this negative relationship.

4.2 Evidence from Cross-Sectional Regressions

At least two factors must be controlled to establish the negative relationship between welfare gains and the extent of economic development noted in the previous subsection. The first is that a larger country size (i.e., a larger population) may be associated with lower output volatility (see, e.g., Furceri and Karras (2007) for empirical evidence and di Giovanni and Levchenko (2012) for a detailed discussion of this mechanism). This implies that, in our context, a larger country size may reduce the welfare cost of business cycles. The second factor is that differences in international trade openness may affect business cycles because even a shock common to all countries can have different impacts across countries through trade (see, e.g., Kose et al. (2003), di Giovanni and

Levchenko (2009, 2012) and Haddad et al. (2013)). Uribe and Schmitt-Grohé (2017) examine the effects of these two factors on business cycles using regressions based on the standard deviation of output. They conclude that there are both country-size and trade-openness effects, but that they are less statistically significant than economic development levels. In this subsection, we estimate the same type of regression for our sample.

The key to our specification is that the standard deviation of output, which is a typical measure of business cycles in the literature, is replaced with our welfare-gain measures. The regression that we run across countries is

$$\lambda_i = \beta_0 + \beta_1 \cdot \ln(\text{PPP\_ GDP}_i) + \beta_2 \cdot \ln(\text{pop}_i) + \beta_3 \cdot \text{openness}_i + u_i, \quad i = 1, \ldots, 64, \tag{8}$$

where λ_i is the welfare gain for country i, PPP_ GDP_i is the geometric average of PPP-converted GDP per capita over the period 1990–2018 for country i, pop_i is country i's total population in 2018 (the end of the sample period), openness_i is country i's trade openness in 2018, and u_i is the error term.[8] As the measure of trade openness, we use the ratio of exports plus imports to GDP (see Appendix B for more details). In the literature, countries with higher values of this measure are interpreted as being more open to trade. We also estimate regressions in which $\lambda_{fear,i}$ and $\lambda_{nofear,i}$ are used as the dependent variable. There are three cases for each of λ_i and $\lambda_{fear,i}$ because we consider three different values for the detection error probability (i.e., $\overline{p(\theta^{-1})} = 0.05$, 0.10, and 0.20). As before, we report results only for the case of $\overline{p(\theta^{-1})} = 0.10$ below.

An important caveat to this specification is that there may be a negative and linear relationship between the country-size and trade-openness variables (see, e.g., Alesina and Wacziarg (1998) and Ram (2009) for empirical evidence and discussions).[9] This point has not been considered in Uribe and Schmitt-Grohé's regression. To see this effect, we consider three types of regressions with log

[8] Here, we use variations in 2018 for the country-size and trade-openness variables. We also tried using the averages over the sample period. However, as the results were similar to those reported below, we have omitted them from the paper.

[9] When we implement a regression of the trade-openness variable on a constant and the country-size variable as in Alesina and Wacziarg (1998), the coefficient of the country-size variable is negative and significant at the 1% level.

population, openness, and both of these control variables (as in (8)) for each of the three dependent variables, λ_i, $\lambda_{fear,i}$, and $\lambda_{nofear,i}$, as well as a base regression with no such control variables. Hence, 12 regressions are run for each of the random walk and trend stationary models.[10]

The regression results for the random walk model are presented in Table 3. Panel A of the table reports the results from all 64 countries, whereas Panel B gives the results for 61 countries, after excluding the three outliers mentioned in the previous subsection. The first regression for the total welfare gain λ, reported in column (1) in Panel A, shows that the slope for the log of average PPP-converted GDP per capita is negative and significant at the 1% level. Columns (2)–(4) confirm that including additional variables that control for country size and trade openness mitigates an upward bias in the slope estimate of interest. They also show that the coefficient on the trade-openness variable is not statistically significant if it is added together with the country-size variable.

The results for λ_{fear} (columns (5)–(8)) and λ_{nofear} (columns (9)–(12)) indicate that the slope for the log of average PPP-converted GDP per capita is steeper in the λ_{fear} regressions than in the λ_{nofear} regressions. The slope estimates range from -0.059 to -0.051 in the λ_{fear} regressions, whereas they fall in a range from -0.020 to -0.017 in the λ_{nofear} regressions. In other words, higher levels of economic development (higher income per capita) tend to be associated with lower welfare costs of model uncertainty, and this negative relationship is stronger for welfare costs of model uncertainty than for welfare costs of consumption risk.

[10] We do not include dummy variables for regions in the regressions. Note that differences in stages of economic development are closely associated with the regions to which countries belong. This holds true for our sample. According to the classification of regions by the World Bank, of the 26 rich countries in our sample, 24 countries are located in three regions ("Europe and Central Asia," "East Asia and Pacific," and "North America"). Of the 28 emerging countries, 17 countries are in the two regions of "Latin America and the Caribbean" and the "Middle East and North Africa," while the 10 poor countries are in the two regions of "South Asia" and "Sub-Saharan Africa." There is no overlap of regions between the rich and poor countries and the overlaps between the emerging and poor countries and between the rich and emerging countries are very restricted. Thus, the inclusion of such region-dummy variables controls for differences in economic development. If we include the region-dummy variables in equation (8), the coefficient of the log of average PPP-converted GDP per capita becomes insignificant.

Table 3: Cross-Sectional Regressions of Welfare Gains on Variables that Measure Economic Development, Country Size, and Openness (Random Walk Model)

	Dependent variable											
	λ				λ_{fear}				λ_{nofear}			
Regressor	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
A. Results based on all 64 countries												
ln(PPP_GDP)	-0.068	-0.075	-0.079	-0.076	-0.051	-0.056	-0.059	-0.057	-0.017	-0.019	-0.020	-0.019
	(0.017)	(0.017)	(0.021)	(0.020)	(0.011)	(0.011)	(0.014)	(0.013)	(0.006)	(0.006)	(0.008)	(0.007)
ln(pop)		-0.047		-0.045		-0.033		-0.032		-0.014		-0.013
		(0.014)		(0.013)		(0.009)		(0.009)		(0.005)		(0.004)
openness			0.052	0.008			0.035	0.004			0.017	0.004
			(0.030)	(0.028)			(0.019)	(0.018)			(0.012)	(0.010)
SER	0.161	0.148	0.158	0.149	0.108	0.098	0.106	0.099	0.055	0.052	0.054	0.052
R^2	0.198	0.331	0.235	0.331	0.238	0.378	0.274	0.378	0.113	0.223	0.150	0.224
B. Results based on 61 countries excluding outliers												
ln(PPP_GDP)	-0.053	-0.058	-0.058	-0.057	-0.043	-0.046	-0.047	-0.046	-0.011	-0.012	-0.012	-0.011
	(0.009)	(0.008)	(0.010)	(0.009)	(0.007)	(0.006)	(0.007)	(0.007)	(0.002)	(0.002)	(0.003)	(0.002)
ln(pop)		-0.026		-0.027		-0.020		-0.021		-0.006		-0.007
		(0.008)		(0.009)		(0.006)		(0.007)		(0.002)		(0.002)
openness			0.021	-0.005			0.017	-0.003			0.004	-0.002
			(0.010)	(0.014)			(0.007)	(0.011)			(0.002)	(0.003)
SER	0.088	0.081	0.088	0.081	0.067	0.061	0.067	0.062	0.021	0.020	0.021	0.020
R^2	0.329	0.445	0.346	0.446	0.350	0.463	0.368	0.464	0.249	0.369	0.263	0.371

Note: Heteroskedasticity-robust standard errors are in parentheses. The welfare gains, λ and λ_{fear}, are those for a detection error probability of 0.10. PPP_GDP is the geometric average of PPP-converted GDP per capita over the period 1990–2018, a measure of economic development, pop is the total population in 2018, and openness is the trade openness measured by the ratio of exports plus imports to GDP in 2018. See Appendix B for the details of each variable. The coefficient on the constant term in the regression is not reported. SER is the standard error of the regression.

We find the same results for the other cases of $\overline{p(\theta^{-1})} = 0.05$ and 0.20, although we do not report the results here to save space (see Appendix Tables D.1 and D.2 of the separate appendix). In addition, we obtain an additional finding that the slope for the log of average PPP-converted GDP per capita becomes much steeper for $\overline{p(\theta^{-1})} = 0.05$; conversely, it is flatter for $\overline{p(\theta^{-1})} = 0.20$. This finding suggests that as agent fear of model misspecification becomes milder, the negative relationship between levels of economic development and the welfare gains of eliminating model uncertainty becomes weaker.

A point to note concerning the regression results presented so far is that the sample size is only 64 in our regressions; thus, the three outliers lying far above the other values may make the regression line steeper. Panel B of Table 3 confirms that excluding the three countries with extreme values of λ (i.e., Congo, Puerto Rico, and Togo) reduces the slope estimate for the log of average PPP-converted GDP per capita in terms of the absolute value. Except for this reduction in the magnitude of the slope, however, the regression results in Panel B are in line with those in Panel A. Thus, the previous findings hold in the regressions without the outliers.

Do they also hold in the trend stationary model? Table 4 presents regression results based on the welfare-gain calculations in the trend stationary model. Panel A gives results for all 64 countries, whereas Panel B gives results for 62 countries, excluding the two countries (i.e., Bolivia and the Philippines) for which the autoregressive parameters are above one. Table 4 and the results shown in Appendix Tables E.1 and E.2 confirm the same three findings. First, the λ, λ_{fear}, and λ_{nofear} regressions all produce negative slopes for the log of average PPP-converted GDP per capita. Second, the negative slopes are steeper in the λ_{fear} regressions than in the λ_{nofear} regressions. Third, as the agent's fear of model misspecification becomes milder, the negative slopes become flatter. Thus, the regression results of Table 4 lend additional support to the negative relationship between levels of economic development and the welfare gains of eliminating model uncertainty.

Table 4: Cross-Sectional Regressions of Welfare Gains on Variables that Measure Economic Development, Country Size, and Openness (Trend Stationary Model)

	Dependent variable											
	λ				λ_{fear}				λ_{nofear}			
Regressor	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
A. Results based on all 64 countries												
ln(PPP_GDP)	-0.015	-0.015	-0.017	-0.017	-0.013	-0.013	-0.015	-0.015	-0.002	-0.002	-0.002	-0.002
	(0.005)	(0.005)	(0.005)	(0.005)	(0.004)	(0.004)	(0.005)	(0.005)	(0.0005)	(0.0005)	(0.0006)	(0.0006)
ln(pop)		-0.002		0.000		-0.001		0.001		-0.001		-0.001
		(0.004)		(0.005)		(0.004)		(0.005)		(0.0004)		(0.0004)
openness			0.009	0.009			0.008	0.009			0.001	0.0001
			(0.004)	(0.005)			(0.004)	(0.005)			(0.0007)	(0.0006)
SER	0.046	0.047	0.046	0.047	0.043	0.044	0.043	0.044	0.005	0.005	0.005	0.005
R^2	0.129	0.132	0.143	0.143	0.114	0.114	0.126	0.127	0.148	0.229	0.168	0.229
B. Results based on 62 countries excluding outliers												
ln(PPP_GDP)	-0.012	-0.013	-0.014	-0.014	-0.010	-0.011	-0.012	-0.012	-0.002	-0.002	-0.002	-0.002
	(0.004)	(0.004)	(0.005)	(0.004)	(0.004)	(0.004)	(0.004)	(0.004)	(0.0006)	(0.0006)	(0.0006)	(0.0006)
ln(pop)		-0.004		-0.003		-0.003		-0.002		-0.001		-0.001
		(0.003)		(0.003)		(0.003)		(0.003)		(0.0004)		(0.0004)
openness			0.009	0.005			0.007	0.005			0.001	0.0003
			(0.004)	(0.003)			(0.004)	(0.003)			(0.0007)	(0.0006)
SER	0.038	0.038	0.038	0.038	0.033	0.033	0.033	0.033	0.005	0.005	0.005	0.005
R^2	0.127	0.152	0.148	0.158	0.118	0.137	0.138	0.144	0.160	0.227	0.183	0.228

Note: See the note to Table 3.

Finally, a few caveats are in order. The coefficients themselves (especially their magnitudes) in the regressions cannot be compared directly between the random walk and trend stationary models. As shown in (7), the formulas for λ, λ_{fear}, and λ_{nofear} differ between these models. The main message here is that the negative relationship suggested in the previous subsection holds even after adding control variables, irrespective of whether the random walk or trend stationary model is used. However, we do not intend to push the existence of a causal effect from levels of economic development to welfare gains too hard.

4.3 Interpreting the Difference between Rich, Emerging, and Poor Countries

The findings presented so far include: (i) the gains from eliminating consumption fluctuations in emerging and poor countries are larger than in rich countries, and (ii) emerging and poor countries have larger gains from eliminating model uncertainty, which is the main source for (i).

Why does this difference arise? As formula (7) indicates, the gain from reducing model uncertainty is proportional to the volatility of consumption. In fact, the difference is attributable to the cross-country difference in the magnitude of the standard deviation of consumption shocks, σ_ϵ (see the last three rows of Appendix Table B). Therefore, the question can be rephrased as a familiar one: why is consumption in emerging and poor countries more volatile than in rich countries?

A large literature exploring business cycles in emerging countries provides at least two views on this question. One is that emerging countries are subject to permanent and/or more volatile exogenous shocks, including permanent and transitory productivity shocks, as well as shocks to interest-rates, preferences, country premiums, and terms-of-trade (see, e.g., Aguiar and Gopinath (2007), Neumeyer and Perri (2005), Uribe and Yue (2006), García-Cicco et al. (2010), Fernández-Villaverde et al. (2011), Chang and Fernández (2013), Fernández and Gulan (2015), Ben-Zeev et al. (2017), and Schmitt-Grohé and Uribe (2018)). Another view, which is not mutually exclusive, concerns the existence of financial frictions in emerging economies (see, e.g., García-Cicco et al.

(2010), Chang and Fernández (2013), Álvarez-Parra et al. (2013), Hevia (2014), and Fernández and Gulan (2015)). The main message from this line of research is that financial frictions in emerging economies amplify the impact of the exogenous shocks that they encounter.

In light of these views, our findings, summarized in (i) and (ii) above, may be interpreted as follows: (i) there are larger welfare gains in countries with more financial frictions (i.e., emerging and poor countries) than in countries with less financial frictions (i.e., rich countries); (ii) the larger gains arise because the influence of model uncertainty is amplified by the large consumption volatility in emerging and poor countries that is a result of financial frictions.[11] Thus, the existing literature suggests that we may be able to focus on financial frictions rather than on economic development. However, as the issues are complicated, a reinterpretation that focuses on financial frictions is left to future research.[12]

5 The Effect of Idiosyncratic Risk Revisited

Our findings reinforce those of Barillas et al. (2009). As mentioned in the Introduction, another position put forward by Ellison and Sargent (2015) is that a model incorporating both idiosyncratic risk and agent fear of model misspecification yields large welfare-gain estimates even under log preferences. Therefore, a question that arises is to what extent the inclusion of idiosyncratic risk affects the contribution of model uncertainty to welfare gains. This section compares the welfare gains computed with the closed-form solution of this paper and the version obtained in Ellison and Sargent (2015) to see the effect of idiosyncratic risk.

[11] Note the second term of formula (7) can be regarded as the volatility σ_ϵ multiplied by the term $\Phi^{-1}\left(\overline{p(\theta^{-1})}\right)$.

[12] According to this line of thinking, the following questions arise: why do emerging and poor countries lack financial markets to diversify shocks and relevant policies to counter such shocks? Are the weak shock absorbers in emerging and poor countries the result of their fragile financial systems and their economic and political institutions? Answering this question is one of the most important subjects in open-economy macroeconomics, but it is beyond the scope of this paper. We hesitate to draw further implications about such questions from our welfare-gain estimates.

Following Ellison and Sargent (2015), we assume the following consumption process:

$$\begin{aligned} c_t^i &= c_t + \delta_t^i, \\ \Delta c_t &= \sqrt{\epsilon} w_{1t}, \\ \Delta \delta_t^i &= \sqrt{\epsilon} w_{2t}, \end{aligned} \tag{9}$$

$$\begin{bmatrix} w_{1t} \\ w_{2t} \end{bmatrix} \sim N\left(\begin{bmatrix} g - \tau_1^2/2 \\ -\tau_2^2/2 \end{bmatrix}, \begin{bmatrix} \tau_1^2 & 0 \\ 0 & \tau_2^2 \end{bmatrix} \right),$$

where c_t^i is log consumption for an individual i and δ_t^i is an individual-specific (i.e., idiosyncratic) shock. The value function recursion that Ellison and Sargent (2015) used is

$$\tilde{U}_t = (1-\beta)c_t^i - \frac{1}{\sigma} \ln E_t \left[\exp\left(-\sigma\beta\tilde{U}_{t+1} \right) \right], \tag{10}$$

where $\tilde{U}_t$ and σ in (10) are related to U_t and θ in (3) as $U_t = \tilde{U}_t/(1-\beta)$ and $\sigma = 1/\beta\theta(1-\beta)$, respectively. In their paper, σ is called the robustness parameter.

The crucial point here is that specification (9) can be rewritten as a random walk model, $c_{t+1}^i = c_t^i + \sqrt{\epsilon}(w_{1t+1} + w_{2t+1})$. This means that we can derive a closed-form solution for welfare gains in the same way as for (7). Note that the case of no idiosyncratic risk that we have discussed so far corresponds to $c_t^i = c_t$ and $c_{t+1} = c_t + \sqrt{\epsilon} w_{1t+1}$. Therefore, using the notation of Ellison and Sargent (2015), our welfare-gain measure can be expressed by replacing σ_ϵ^2 with $\epsilon\tau_1^2$ (i.e., the variance of aggregate shocks $\sqrt{\epsilon} w_{1t+1}$) as

$$\lambda = \frac{\beta}{1-\beta}\frac{\epsilon\tau_1^2}{2} - \frac{\beta}{1-\beta}\frac{\sqrt{\epsilon\tau_1^2}}{\sqrt{T}}\Phi^{-1}\left(\overline{p(\sigma)}\right). \tag{11}$$

In computing welfare gains under the model with idiosyncratic risk, the definition of an economy without aggregate risk adopted by Ellison and Sargent (2015) is the same as that of De Santis (2007). That is, aggregate consumption growth in that economy equals the expected growth of aggregate consumption in the economy with aggregate shocks. This is equivalent to considering a deterministic process only for aggregate consumption c_t in (9). Then, the welfare-gain measure is

$$\lambda = \frac{\beta}{1-\beta}\frac{\epsilon\tau_1^2}{2} - \frac{\beta}{1-\beta}\frac{1}{\sqrt{T}}\frac{\epsilon\tau_1^2}{\sqrt{\epsilon(\tau_1^2+\tau_2^2)}}\Phi^{-1}\left(\overline{p(\sigma)}\right). \tag{12}$$

Table 5: The Effect of Idiosyncratic Risk

	Detection error probability					
	50%	45%	40%	20%	10%	5%
A. Model with aggregate shocks only						
λ	0.80%	1.63%	2.48%	6.38%	9.30%	11.71%
λ_{nofear}	0.80	0.80	0.80	0.80	0.80	0.80
λ_{fear}	0	0.83	1.68	5.58	8.50	10.91
B. Model with both aggregate and idiosyncratic shocks						
λ	0.80%	1.03%	1.27%	2.35%	3.17%	3.84%
λ_{nofear}	0.80	0.80	0.80	0.80	0.80	0.80
λ_{fear}	0	0.23	0.47	1.55	2.37	3.04

Note: The results in panels A and B are based on equations (11) and (12), respectively. The welfare-gain estimates are multiplied by 100 to express them in percentage terms.

The derivation of (12) is given in Appendix C. We can establish the validity of this formula in two ways. First, the closed-form solution of the detection error probability $p(\sigma)$ used in this formula replicates the calibration results of the robustness parameter σ reported in Table 2 of Ellison and Sargent (2015) (see Appendix D for details). Second, as shown below, this formula replicates the results for the case of log preferences reported in Table 3 of Ellison and Sargent (2015).

Table 5 reports the values of welfare gains based on these two formulas. Here, we use the following parameter settings based on Ellison and Sargent (2015): $\sqrt{\epsilon}\tau_1 = 0.029$; $\sqrt{\epsilon}\tau_2 = 0.10$; $\beta = 0.95$; and $T = 69$. Panels A and B of the table show the results from (11) and (12), respectively. In addition, for ease of comparison with Table 3 in Ellison and Sargent (2015), we present results for detection error probabilities of 0.50, 0.45, and 0.40, as well as for the previous three cases. We report the results in percentage terms.[13] The welfare-gain measures in Panel B, λ_{nofear} and λ_{fear}, correspond to "Baseline cost" and "Contribution of fear of model misspecification," respectively, in Table 3 of Ellison and Sargent (2015).

The major findings are as follows. First, the result for the no idiosyncratic risk case in Panel

[13] The welfare-gain measure is multiplied by 100 to express it as a percentage because it is defined as the difference of log consumption.

A of Table 5 is comparable with that for the United States in column (1) of Table 2 based on the WDI data. Two factors account for the difference between them: one is the sample size used in calculating detection error probabilities; the other is the magnitude of the variance of aggregate shocks. In our case, where the sample size T increases from 48 to 69 and the estimate of σ_ϵ increases from 0.016 to 0.029, the latter effect dominates the former one. Hence, the estimates of λ_{fear} become higher than those based on the WDI data.

Second, the results for detection error probabilities of 0.50, 0.45, and 0.40 in Panel B of Table 5 are entirely consistent with those for log preferences in Table 3 of Ellison and Sargent (2015). For detection error probabilities of 0.20, 0.10, and 0.05 (which are not reported in Ellison and Sargent (2015)), as expected from our formula (12), smaller detection error probabilities (i.e., larger fears of model misspecification) lead to larger estimates of λ_{fear}. For example, λ_{fear} rises from 0.23% at $\overline{p(\sigma)} = 0.45$ to 2.37% at $\overline{p(\sigma)} = 0.10$.

Third, the comparison between Panels A and B of Table 5 indicates that the exclusion of idiosyncratic risk by assumption results in overestimating the contribution of model uncertainty to welfare gains. For example, for a detection error probability of 0.10, the value of λ_{fear} rises from 2.37% in Panel B (the idiosyncratic risk case) to 8.50% in Panel A (the no idiosyncratic risk case). However, this finding should not be taken as evidence against the claim of Ellison and Sargent (2015) because, as is clear from Panel B, λ_{fear} still raises the overall value of the welfare gain substantially under the idiosyncratic risk model.

The result that the value of λ is smaller for the idiosyncratic risk case (Panel B) than for the no idiosyncratic risk case (Panel A) depends on the assumption of log preferences. We can see from Table 3 of Ellison and Sargent (2015) that the value of λ increases by relaxing the assumption of log preferences. For example, the value of λ at $\overline{p(\sigma)} = 0.45$ is 1.03% in Panel B of our table (or equivalently, the first panel of Table 3 in Ellison and Sargent (2015)). By contrast, it rises to 1.428% under constant relative risk aversion preferences with a relative risk aversion coefficient of

1.5, owing to the contribution of idiosyncratic risk and the combined effect of idiosyncratic risk and fear of model misspecification (see the third panel of Table 3 in Ellison and Sargent (2015)). Thus, by relaxing the assumption of log preferences, the value of λ at $\overline{p(\sigma)} = 0.45$ becomes closer to the value of 1.63% shown in Panel A of our table.

To summarize, if we restrict our specification of consumption processes to the random walk, trend stationary, and idiosyncratic risk models used in the literature, the estimates of the welfare gain from the random walk model represent the upper bounds. Although the result here is limited to the United States, we can state with certainty that the welfare gain from eliminating model uncertainty is not trivial, irrespective of the presence or absence of idiosyncratic risk.

6 Conclusion

A broad range of countries tends to have large welfare gains from eliminating model uncertainty relative to the United States. In addition, there is a negative relationship between the welfare gains and the level of economic development. These findings indicate that the welfare gains from eliminating aggregate consumption fluctuations are much larger than indicated by the previous Lucas-style calculations, and that countries at higher stages of economic development have lower welfare gains because the gains from eliminating model uncertainty are smaller in those countries.

For developing countries, we agree with Pallage and Robe's (2003, p. 695) conclusion that the welfare cost in those countries is "a large multiple of that in the United States." However, the evidence presented in this paper shows that the source for it is model uncertainty, not only consumption risk drawn from a known probability distribution. Our finding concerning the cross-country difference in the welfare gains from eliminating model uncertainty suggests that policies that change the amount of model uncertainty may have significantly different effects among countries.

Appendix

A. List of Sample Countries and Classification

The sample countries and their PPP-converted GDP-based classifications are as follows. The classification by the World Bank is presented in Appendix Table A.

Poor Countries: Benin, Burkina Faso, Lesotho, Mali, Rwanda, Togo, Cameroon, Kenya, Madagascar, Bangladesh

Emerging Countries: Bolivia, Chile, (Republic of) Congo, Costa Rica, Dominican Republic, Ecuador, El Salvador, Gabon, Guatemala, Mauritania, Nicaragua, Paraguay, Uruguay, Algeria, Colombia, South Korea, Malaysia, Peru, South Africa, Sri Lanka, Thailand, Venezuela, Brazil, Egypt, India, Indonesia, Mexico, the Philippines

Rich Countries: Austria, Belgium, Denmark, Finland, Greece, Hong Kong, Ireland, Israel, Luxembourg, the Netherlands, New Zealand, Norway, Portugal, Puerto Rico, Singapore, Sweden, Switzerland, Australia, Canada, France, Italy, Spain, the United Kingdom, Germany, Japan, the United States

B. Data Description

The World Development Indicators (WDI) database is available from the World Bank's website (https://databank.worldbank.org/) as of March 2020. The definition of each variable used in the paper is as follows.

Real private consumption per capita is based on "Household final consumption expenditure (constant LCU)" (code: NE.CON.PRVT.KN) and "Population, total" (code: SP.POP.TOTL), where LCU means that the data are in local currency. Final consumption expenditure includes expenditure on durables as well as that on nondurables and services.

PPP-converted GDP per capita is "GDP per capita, PPP (constant 2011 international dollars)" (code: NY.GDP.PCAP.PP.KD). This series was available from 1990 to 2018 at the time of writing

(March 2020). Hence, its geometric average was calculated for the period 1990–2018, as described in the text.

Trade openness is defined as the ratio of exports plus imports to GDP. This variable was calculated as the sum of "Exports of goods and services (% of GDP)" (code: NE.EXP.GNFS.ZS) and "Imports of goods and services (% of GDP)" (code: NE.IMP.GNFS.ZS) divided by 100. These data on exports and imports were not available for Venezuela in 2018. Therefore, 2017 values were used for Venezuela instead.

C. Derivation of Equation (12)

Consider the consumption process (9) reproduced here as

$$\begin{aligned} c_t^i &= c_t + \delta_t^i, \\ \Delta c_t &= \sqrt{\epsilon} w_{1t}, \\ \Delta \delta_t^i &= \sqrt{\epsilon} w_{2t}, \end{aligned} \tag{C1}$$

where

$$\begin{bmatrix} w_{1t} \\ w_{2t} \end{bmatrix} \sim N\left(\begin{bmatrix} g - \tau_1^2/2 \\ -\tau_2^2/2 \end{bmatrix}, \begin{bmatrix} \tau_1^2 & 0 \\ 0 & \tau_2^2 \end{bmatrix} \right).$$

To simplify the notation below, let $\epsilon_{1t} \equiv \sqrt{\epsilon} w_{1t}$ and $\epsilon_{2t} \equiv \sqrt{\epsilon} w_{2t}$.

We first derive a deterministic process for aggregate consumption, for which consumption growth is equal to the expected growth of the random process $\{C_t\}$, where $C_t = \exp(c_t)$. Because $C_{t+1} = \exp(c_t + \epsilon_{1t+1}) = C_t \cdot \exp(\epsilon_{1t+1})$, the expected growth of C_t is $E[C_{t+1}/C_t] = E[\exp(\epsilon_{1t+1})] = \exp\left(E(\epsilon_{1t+1}) + \frac{1}{2}\mathrm{Var}(\epsilon_{1t+1})\right)$. Therefore, letting C_t^d be the level of aggregate consumption in the economy without aggregate shocks, it follows that

$$\frac{C_{t+1}^d}{C_t^d} = \exp\left(E(\epsilon_{1t+1}) + \frac{1}{2}\mathrm{Var}(\epsilon_{1t+1})\right) = \exp\left(\sqrt{\epsilon}\left(g - \frac{\tau_1^2}{2}\right) + \frac{1}{2}\epsilon\tau_1^2\right). \tag{C2}$$

Thus, taking logarithms of both sides of (C2), we have

$$c_{t+1}^d = c_t^d + \sqrt{\epsilon}\left(g - \frac{\tau_1^2}{2}\right) + \frac{1}{2}\epsilon\tau_1^2, \tag{C3}$$

where $c_t^d = \ln C_t^d$.

Next, we replace c_t in (C1) with c_t^d and derive the analytical solution to the corresponding value function recursion. More specifically, we consider the following consumption process:

$$\begin{aligned}
\bar{c}_t^i &= c_t^d + \delta_t^i, \\
\Delta c_t^d &= \sqrt{\epsilon}\left(g - \frac{\tau_1^2}{2}\right) + \frac{1}{2}\epsilon\tau_1^2, \\
\Delta \delta_t^i &= \epsilon_{2t}.
\end{aligned} \tag{C4}$$

Substituting the second and third equations of (C4) into the first one (after taking the first-difference), the consumption process can be rewritten as

$$\bar{c}_{t+1}^i = \bar{c}_t^i + \sqrt{\epsilon}\left(g - \frac{\tau_1^2}{2}\right) + \frac{1}{2}\epsilon\tau_1^2 + \epsilon_{2t+1}. \tag{C5}$$

The value function recursion defined by this $\bar{c}_t^i$ is

$$\bar{U}_t = (1-\beta)\bar{c}_t^i - \frac{1}{\sigma}\ln E_t\left[\exp\left(-\sigma\beta\bar{U}_{t+1}\right)\right]. \tag{C6}$$

To solve for $\bar{U}_t$, guess $\bar{U}_t = k_0 + k_1\bar{c}_t^i$. Then,

$$\begin{aligned}
\bar{U}_{t+1} &= k_0 + k_1\bar{c}_{t+1}^i, \\
&= k_0 + k_1\left[\bar{c}_t^i + \sqrt{\epsilon}\left(g - \frac{\tau_1^2}{2}\right) + \frac{1}{2}\epsilon\tau_1^2 + \epsilon_{2t+1}\right].
\end{aligned} \tag{C7}$$

Substituting (C7) into (C6) yields

$$\begin{aligned}
\bar{U}_t &= (1-\beta)\bar{c}_t^i - \frac{1}{\sigma}\ln E_t\left[\exp\left(-\sigma\beta\left\{k_0 + k_1\left[\bar{c}_t^i + \sqrt{\epsilon}\left(g - \frac{\tau_1^2}{2}\right) + \frac{1}{2}\epsilon\tau_1^2 + \epsilon_{2t+1}\right]\right\}\right)\right], \\
&= (1-\beta)\bar{c}_t^i + \beta\left\{k_0 + k_1\left[\bar{c}_t^i + \sqrt{\epsilon}\left(g - \frac{\tau_1^2}{2}\right) + \frac{1}{2}\epsilon\tau_1^2\right]\right\} - \frac{1}{\sigma}\ln E_t\left[\exp\left(-\sigma\beta k_1\epsilon_{2t+1}\right)\right].
\end{aligned} \tag{C8}$$

Noting that $\exp\left(-\sigma\beta k_1\epsilon_{2t+1}\right)$ is a log-normally distributed random variable with mean $-\sigma\beta k_1 E(\epsilon_{2t+1})$ and variance $\sigma^2\beta^2k_1^2\mathrm{Var}(\epsilon_{2t+1})$ and rewriting the last term of the right-hand side, (C8) can be written as

$$\begin{aligned}
\bar{U}_t &= (1-\beta)\bar{c}_t^i + \beta\left\{k_0 + k_1\left[\bar{c}_t^i + \sqrt{\epsilon}\left(g - \frac{\tau_1^2}{2}\right) + \frac{1}{2}\epsilon\tau_1^2\right]\right\} - \frac{1}{\sigma}\left(\sigma\beta k_1\sqrt{\epsilon}\frac{\tau_2^2}{2} + \frac{1}{2}\sigma^2\beta^2k_1^2\epsilon\tau_2^2\right), \\
&= \left[(1-\beta) + \beta k_1\right]\bar{c}_t^i + \beta k_0 + \beta k_1\left[\sqrt{\epsilon}\left(g - \frac{\tau_1^2 + \tau_2^2}{2}\right) + \frac{1}{2}\epsilon\tau_1^2 - \frac{1}{2}\sigma\beta k_1\epsilon\tau_2^2\right].
\end{aligned} \tag{C9}$$

Matching the coefficients in (C9) and $\bar{U}_t = k_0 + k_1 \bar{c}_t^i$, we have

$$\begin{aligned} k_1 &= 1, \\ k_0 &= \frac{\beta}{1-\beta}\left[\sqrt{\epsilon}\left(g - \frac{\tau_1^2+\tau_2^2}{2}\right) + \frac{1}{2}\epsilon\tau_1^2 - \frac{1}{2}\sigma\beta\epsilon\tau_2^2\right]. \end{aligned} \tag{C10}$$

Thus, we obtain

$$\bar{U}_t = \frac{\beta}{1-\beta}\left[\sqrt{\epsilon}\left(g - \frac{\tau_1^2+\tau_2^2}{2}\right) + \frac{1}{2}\epsilon\tau_1^2 - \frac{1}{2}\sigma\beta\epsilon\tau_2^2\right] + \bar{c}_t^i. \tag{C11}$$

Following the same steps, we can derive the following solution to the value function recursion (10) (i.e., equation (11) of Ellison and Sargent (2015, p. 47)):

$$\tilde{U}_t = \frac{\beta}{1-\beta}\left[\sqrt{\epsilon}\left(g - \frac{\tau_1^2+\tau_2^2}{2}\right) - \frac{1}{2}\sigma\beta\epsilon(\tau_1^2+\tau_2^2)\right] + c_t^i. \tag{C12}$$

Finally, we solve $\tilde{U}_0 = \bar{U}_0$ to obtain the welfare gain λ, defined as the difference of the initial consumption, $c_0^i - \bar{c}_0^i$. This gives

$$\lambda \equiv c_0^i - \bar{c}_0^i = \frac{\beta}{1-\beta}\frac{1}{2}\epsilon\tau_1^2 + \frac{\beta}{1-\beta}\sigma\beta\frac{1}{2}\epsilon\tau_1^2. \tag{C13}$$

One more step is required to obtain equation (12). As Okubo (2018a) showed, the overall detection probability under the consumption process (C1) is given by

$$p(\theta^{-1}) = \Phi\left(-\frac{\sqrt{T}}{2}\theta^{-1}\frac{\sqrt{\epsilon(\tau_1^2+\tau_2^2)}}{1-\beta}\right). \tag{C14}$$

A proof of (C14) is provided in Okubo (2018b) (in Appendix D, we present additional evidence that demonstrates the validity of (C14), rather than outlining the proof). Because $\theta^{-1} = \sigma\beta(1-\beta)$, (C14) can be rewritten as

$$p(\sigma) = \Phi\left(-\frac{\sqrt{T}}{2}\sigma\beta\sqrt{\epsilon(\tau_1^2+\tau_2^2)}\right). \tag{C15}$$

Therefore, given any value of the overall detection error probability (denoted by $\overline{p(\sigma)}$ in the text), we have

$$\sigma = -\frac{2}{\sqrt{T}}\frac{1}{\beta\sqrt{\epsilon(\tau_1^2+\tau_2^2)}}\Phi^{-1}\left(\overline{p(\sigma)}\right). \tag{C16}$$

Substituting (C16) into (C13) gives equation (12).

D. Replication of the Robustness Parameter σ

This appendix shows that our formula (C16) can replicate the calibration results of σ for the case of log preferences reported in Table 2 of Ellison and Sargent (2015). Here, we use the following parameter settings of Ellison and Sargent (2015): $\sqrt{\epsilon}\tau_1 = 0.029$; $\sqrt{\epsilon}\tau_2 = 0.10$; $\beta = 0.95$; and $T = 69$.

The calculation results of the robustness parameter σ based on (C16) are reported in Appendix Table F. The first row of the table presents the results from our formula, and the second row presents the results from Table 2 of Ellison and Sargent (2015). As $\Phi^{-1}(0.5) = 0$ by the definition of the standard normal cumulative distribution function, it is obvious from (C16) that $\sigma = 0$ for $\overline{p(\sigma)} = 0.5$. The other two cases are calculated as follows:

$$\sigma = -\frac{2}{\sqrt{69}} \cdot \frac{1}{0.95\sqrt{(0.029)^2 + (0.10)^2}} \cdot (-0.1256613) \approx 0.31 \quad \text{for } \overline{p(\sigma)} = 0.45, \tag{D1}$$

$$\sigma = -\frac{2}{\sqrt{69}} \cdot \frac{1}{0.95\sqrt{(0.029)^2 + (0.10)^2}} \cdot (-0.2533471) \approx 0.62 \quad \text{for } \overline{p(\sigma)} = 0.40. \tag{D2}$$

It turns out that the differences between Ellison and Sargent's results and ours are due to rounding errors.

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Appendix for "Model Uncertainty, Economic Development, and Welfare Costs of Business Cycles"

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This appendix has several tables.

- Table A lists the 64 countries in our sample and the country classification used in the paper, along with the World Bank classification.
- Table B reports parameter estimates of the random walk and trend stationary models for individual countries.
- Tables C.1 and C.2 report welfare-gain estimates for individual countries based on the random walk and trend stationary models, respectively.
- Tables D.1 and D.2 report regression results using the welfare-gain estimates from the random walk model under detection error probabilities of 0.05 and 0.20, respectively.
- Tables E.1 and E.2 report regression results using the welfare-gain estimates from the trend stationary model under detection error probabilities of 0.05 and 0.20, respectively.
- Table F reports the calculation results of the robustness parameter σ based on (C16) in Appendix D of the paper.

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Appendix Table A
List of Sample Countries and Classification

No.	Country	Classification: This paper	Classification: World Bank
1	Algeria [DZA]	Emerging	Upper middle income
2	Australia [AUS]	Rich	High income
3	Austria [AUT]	Rich	High income
4	Bangladesh [BGD]	Poor	Lower middle income
5	Belgium [BEL]	Rich	High income
6	Benin [BEN]	Poor	Low income
7	Bolivia [BOL]	Emerging	Lower middle income
8	Brazil [BRA]	Emerging	Upper middle income
9	Burkina Faso [BFA]	Poor	Low income
10	Cameroon [CMR]	Poor	Lower middle income
11	Canada [CAN]	Rich	High income
12	Chile [CHL]	Emerging	High income
13	Colombia [COL]	Emerging	Upper middle income
14	Congo [COG]	Emerging	Lower middle income
15	Costa Rica [CRI]	Emerging	Upper middle income
16	Denmark [DNK]	Rich	High income
17	Dominican Republic [DOM]	Emerging	Upper middle income
18	Ecuador [ECU]	Emerging	Upper middle income
19	Egypt [EGY]	Emerging	Lower middle income
20	El Salvador [SLV]	Emerging	Lower middle income
21	Finland [FIN]	Rich	High income
22	France [FRA]	Rich	High income
23	Gabon [GAB]	Emerging	Upper middle income
24	Germany [DEU]	Rich	High income
25	Greece [GRC]	Rich	High income
26	Guatemala [GTM]	Emerging	Upper middle income
27	Hong Kong [HKG]	Rich	High income
28	India [IND]	Emerging	Lower middle income
29	Indonesia [IDN]	Emerging	Lower middle income
30	Ireland [IRL]	Rich	High income
31	Israel [ISR]	Rich	High income
32	Italy [ITA]	Rich	High income
33	Japan [JPN]	Rich	High income
34	Kenya [KEN]	Poor	Lower middle income
35	Korea [KOR]	Emerging	High income
36	Lesotho [LSO]	Poor	Lower middle income
37	Luxembourg [LUX]	Rich	High income
38	Madagascar [MDG]	Poor	Low income
39	Malaysia [MYS]	Emerging	Upper middle income
40	Mali [MLI]	Poor	Low income
41	Mauritania [MRT]	Emerging	Lower middle income
42	Mexico [MEX]	Emerging	Upper middle income
43	Netherlands [NLD]	Rich	High income
44	New Zealand [NZL]	Rich	High income
45	Nicaragua [NIC]	Emerging	Lower middle income
46	Norway [NOR]	Rich	High income
47	Paraguay [PRY]	Emerging	Upper middle income
48	Peru [PER]	Emerging	Upper middle income
49	Philippines [PHL]	Emerging	Lower middle income
50	Portugal [PRT]	Rich	High income

Appendix Table A (continued)			
51	Puerto Rico [PRI]	Rich	High income
52	Rwanda [RWA]	Poor	Low income
53	Singapore [SGP]	Rich	High income
54	South Africa [ZAF]	Emerging	Upper middle income
55	Spain [ESP]	Rich	High income
56	Sri Lanka [LKA]	Emerging	Upper middle income
57	Sweden [SWE]	Rich	High income
58	Switzerland [CHE]	Rich	High income
59	Thailand [THA]	Emerging	Upper middle income
60	Togo [TGO]	Poor	Low income
61	United Kingdom [GBR]	Rich	High income
62	United States [USA]	Rich	High income
63	Uruguay [URY]	Emerging	High income
64	Venezuela [VEN]	Emerging	Upper middle income

Note : The country code in the WDI database is in square brackets.

Appendix Table B
Parameter Estimates of the Random Walk and Trend Stationary Models

		Model					
		Random walk		Trend stationary			
No.	Country	σ_ε		ρ		σ_ε	
1	Algeria [DZA]	0.053	(0.006)	0.919	(0.041)	0.051	(0.005)
2	Australia [AUS]	0.015	(0.002)	0.914	(0.062)	0.014	(0.001)
3	Austria [AUT]	0.019	(0.002)	0.921	(0.033)	0.014	(0.002)
4	Bangladesh [BGD]	0.054	(0.006)	0.919	(0.030)	0.039	(0.004)
5	Belgium [BEL]	0.017	(0.002)	0.873	(0.036)	0.012	(0.001)
6	Benin [BEN]	0.046	(0.005)	0.712	(0.102)	0.042	(0.004)
7	Bolivia [BOL]	0.025	(0.003)	1.015	(0.035)	0.023	(0.002)
8	Brazil [BRA]	0.046	(0.005)	0.803	(0.062)	0.040	(0.004)
9	Burkina Faso [BFA]	0.065	(0.007)	0.766	(0.093)	0.061	(0.006)
10	Cameroon [CMR]	0.075	(0.008)	0.811	(0.083)	0.071	(0.007)
11	Canada [CAN]	0.017	(0.002)	0.851	(0.064)	0.016	(0.002)
12	Chile [CHL]	0.068	(0.007)	0.873	(0.054)	0.063	(0.006)
13	Colombia [COL]	0.023	(0.002)	0.935	(0.051)	0.023	(0.002)
14	Congo [COG]	0.198	(0.020)	0.443	(0.144)	0.172	(0.018)
15	Costa Rica [CRI]	0.039	(0.004)	0.944	(0.053)	0.037	(0.004)
16	Denmark [DNK]	0.024	(0.002)	0.821	(0.086)	0.022	(0.002)
17	Dominican Republic [DOM]	0.043	(0.004)	0.817	(0.092)	0.041	(0.004)
18	Ecuador [ECU]	0.042	(0.004)	0.833	(0.073)	0.040	(0.004)
19	Egypt [EGY]	0.029	(0.003)	0.837	(0.063)	0.026	(0.003)
20	El Salvador [SLV]	0.057	(0.006)	0.925	(0.052)	0.056	(0.006)
21	Finland [FIN]	0.024	(0.003)	0.885	(0.066)	0.023	(0.002)
22	France [FRA]	0.014	(0.001)	0.899	(0.035)	0.010	(0.001)
23	Gabon [GAB]	0.104	(0.011)	0.738	(0.090)	0.095	(0.010)
24	Germany [DEU]	0.016	(0.002)	0.918	(0.033)	0.013	(0.001)
25	Greece [GRC]	0.035	(0.004)	0.974	(0.042)	0.031	(0.003)
26	Guatemala [GTM]	0.019	(0.002)	0.967	(0.046)	0.019	(0.002)
27	Hong Kong [HKG]	0.040	(0.004)	0.935	(0.038)	0.037	(0.004)
28	India [IND]	0.027	(0.003)	0.997	(0.032)	0.021	(0.002)
29	Indonesia [IDN]	0.034	(0.003)	0.888	(0.063)	0.032	(0.003)
30	Ireland [IRL]	0.032	(0.003)	0.954	(0.049)	0.032	(0.003)
31	Israel [ISR]	0.035	(0.004)	0.808	(0.088)	0.033	(0.003)
32	Italy [ITA]	0.022	(0.002)	0.989	(0.027)	0.018	(0.002)
33	Japan [JPN]	0.020	(0.002)	0.963	(0.028)	0.016	(0.002)
34	Kenya [KEN]	0.070	(0.007)	0.662	(0.103)	0.064	(0.007)
35	Korea [KOR]	0.039	(0.004)	0.999	(0.042)	0.036	(0.004)
36	Lesotho [LSO]	0.077	(0.008)	0.987	(0.031)	0.059	(0.006)
37	Luxembourg [LUX]	0.020	(0.002)	0.981	(0.034)	0.017	(0.002)
38	Madagascar [MDG]	0.042	(0.004)	0.934	(0.049)	0.039	(0.004)
39	Malaysia [MYS]	0.047	(0.005)	0.871	(0.080)	0.046	(0.005)
40	Mali [MLI]	0.053	(0.005)	0.921	(0.072)	0.052	(0.005)
41	Mauritania [MRT]	0.103	(0.011)	0.612	(0.116)	0.093	(0.010)
42	Mexico [MEX]	0.035	(0.004)	0.816	(0.081)	0.033	(0.003)
43	Netherlands [NLD]	0.019	(0.002)	0.965	(0.044)	0.018	(0.002)
44	New Zealand [NZL]	0.025	(0.003)	0.931	(0.051)	0.024	(0.002)
45	Nicaragua [NIC]	0.103	(0.011)	0.929	(0.052)	0.099	(0.010)
46	Norway [NOR]	0.021	(0.002)	0.849	(0.076)	0.020	(0.002)
47	Paraguay [PRY]	0.035	(0.004)	0.902	(0.051)	0.034	(0.004)
48	Peru [PER]	0.057	(0.006)	0.979	(0.043)	0.054	(0.006)
49	Philippines [PHL]	0.018	(0.002)	1.043	(0.028)	0.015	(0.002)

Appendix Table B (continued)							
50	Portugal [PRT]	0.035	(0.004)	0.883	(0.058)	0.033	(0.003)
51	Puerto Rico [PRI]	0.137	(0.014)	0.240	(0.140)	0.108	(0.011)
52	Rwanda [RWA]	0.077	(0.008)	0.866	(0.065)	0.071	(0.007)
53	Singapore [SGP]	0.031	(0.003)	0.915	(0.053)	0.028	(0.003)
54	South Africa [ZAF]	0.026	(0.003)	0.938	(0.050)	0.026	(0.003)
55	Spain [ESP]	0.025	(0.003)	0.938	(0.046)	0.024	(0.002)
56	Sri Lanka [LKA]	0.056	(0.006)	0.719	(0.095)	0.051	(0.005)
57	Sweden [SWE]	0.019	(0.002)	0.894	(0.065)	0.019	(0.002)
58	Switzerland [CHE]	0.013	(0.001)	0.797	(0.068)	0.011	(0.001)
59	Thailand [THA]	0.036	(0.004)	0.920	(0.064)	0.035	(0.004)
60	Togo [TGO]	0.146	(0.015)	0.561	(0.112)	0.125	(0.013)
61	United Kingdom [GBR]	0.022	(0.002)	0.972	(0.050)	0.022	(0.002)
62	United States [USA]	0.016	(0.002)	0.918	(0.060)	0.016	(0.002)
63	Uruguay [URY]	0.056	(0.006)	0.880	(0.063)	0.053	(0.005)
64	Venezuela [VEN]	0.088	(0.009)	0.960	(0.078)	0.085	(0.009)
Mean :							
Rich countries		0.027				0.024	
Emerging countries		0.054				0.050	
Poor countries		0.070				0.062	

Note : Standard errors are in parentheses. The sample period is from 1970 to 2018. The sample size is 48, which corresponds to the number of observations on Δc_t .

Appendix Table C.1
Welfare-Gain Estimates for Individual Countries: Random Walk Model

		Detection error probability						
		5%		10%		20%		
No.	Country	λ	λ_{fear}	λ	λ_{fear}	λ	λ_{fear}	λ_{nofear}
1	Algeria [DZA]	0.268	0.241	0.215	0.188	0.151	0.123	0.027
2	Australia [AUS]	0.067	0.065	0.053	0.051	0.035	0.033	0.002
3	Austria [AUT]	0.090	0.087	0.071	0.068	0.048	0.045	0.004
4	Bangladesh [BGD]	0.270	0.243	0.216	0.189	0.152	0.124	0.027
5	Belgium [BEL]	0.077	0.075	0.061	0.058	0.041	0.038	0.003
6	Benin [BEN]	0.225	0.206	0.180	0.160	0.125	0.105	0.020
7	Bolivia [BOL]	0.119	0.113	0.094	0.088	0.064	0.058	0.006
8	Brazil [BRA]	0.225	0.205	0.179	0.160	0.125	0.105	0.020
9	Burkina Faso [BFA]	0.335	0.294	0.270	0.229	0.191	0.151	0.040
10	Cameroon [CMR]	0.390	0.337	0.315	0.262	0.225	0.172	0.053
11	Canada [CAN]	0.078	0.075	0.061	0.059	0.041	0.039	0.003
12	Chile [CHL]	0.349	0.306	0.282	0.238	0.200	0.157	0.044
13	Colombia [COL]	0.110	0.105	0.087	0.082	0.059	0.054	0.005
14	Congo [COG]	1.265	0.893	1.067	0.696	0.829	0.457	0.372
15	Costa Rica [CRI]	0.188	0.174	0.149	0.135	0.103	0.089	0.014
16	Denmark [DNK]	0.112	0.107	0.088	0.083	0.060	0.055	0.005
17	Dominican Republic [DOM]	0.211	0.194	0.169	0.151	0.117	0.099	0.018
18	Ecuador [ECU]	0.208	0.191	0.166	0.149	0.115	0.098	0.017
19	Egypt [EGY]	0.137	0.130	0.109	0.101	0.074	0.066	0.008
20	El Salvador [SLV]	0.290	0.259	0.233	0.202	0.164	0.133	0.031
21	Finland [FIN]	0.114	0.109	0.090	0.085	0.061	0.056	0.006
22	France [FRA]	0.063	0.062	0.050	0.048	0.033	0.031	0.002
23	Gabon [GAB]	0.568	0.467	0.465	0.364	0.341	0.239	0.102
24	Germany [DEU]	0.074	0.071	0.058	0.056	0.039	0.037	0.002
25	Greece [GRC]	0.168	0.156	0.133	0.122	0.091	0.080	0.011
26	Guatemala [GTM]	0.091	0.088	0.072	0.068	0.048	0.045	0.004
27	Hong Kong [HKG]	0.198	0.182	0.157	0.142	0.109	0.093	0.016
28	India [IND]	0.128	0.121	0.101	0.094	0.069	0.062	0.007
29	Indonesia [IDN]	0.162	0.151	0.128	0.118	0.088	0.077	0.011
30	Ireland [IRL]	0.153	0.144	0.122	0.112	0.083	0.074	0.010
31	Israel [ISR]	0.170	0.159	0.135	0.124	0.093	0.081	0.012
32	Italy [ITA]	0.102	0.098	0.081	0.076	0.055	0.050	0.005
33	Japan [JPN]	0.095	0.091	0.075	0.071	0.050	0.047	0.004
34	Kenya [KEN]	0.364	0.317	0.294	0.247	0.209	0.162	0.047
35	Korea [KOR]	0.190	0.176	0.151	0.137	0.104	0.090	0.014
36	Lesotho [LSO]	0.404	0.347	0.327	0.271	0.234	0.178	0.056
37	Luxembourg [LUX]	0.094	0.090	0.074	0.070	0.050	0.046	0.004
38	Madagascar [MDG]	0.207	0.190	0.165	0.148	0.114	0.097	0.017
39	Malaysia [MYS]	0.235	0.214	0.188	0.167	0.131	0.109	0.021
40	Mali [MLI]	0.264	0.237	0.211	0.185	0.148	0.122	0.026
41	Mauritania [MRT]	0.567	0.466	0.464	0.363	0.339	0.238	0.101
42	Mexico [MEX]	0.167	0.155	0.132	0.121	0.091	0.080	0.011
43	Netherlands [NLD]	0.087	0.084	0.069	0.065	0.046	0.043	0.003
44	New Zealand [NZL]	0.117	0.111	0.092	0.087	0.063	0.057	0.006
45	Nicaragua [NIC]	0.566	0.465	0.463	0.362	0.339	0.238	0.101
46	Norway [NOR]	0.100	0.096	0.079	0.075	0.053	0.049	0.004
47	Paraguay [PRY]	0.171	0.159	0.136	0.124	0.093	0.081	0.012
48	Peru [PER]	0.286	0.256	0.230	0.199	0.162	0.131	0.031
49	Philippines [PHL]	0.084	0.081	0.066	0.063	0.045	0.042	0.003

Appendix Table C.1 (continued)								
50	Portugal [PRT]	0.170	0.159	0.135	0.124	0.093	0.081	0.012
51	Puerto Rico [PRI]	0.797	0.618	0.660	0.482	0.495	0.316	0.179
52	Rwanda [RWA]	0.403	0.346	0.326	0.270	0.233	0.177	0.056
53	Singapore [SGP]	0.147	0.138	0.116	0.107	0.079	0.070	0.009
54	South Africa [ZAF]	0.125	0.118	0.099	0.092	0.067	0.061	0.007
55	Spain [ESP]	0.120	0.114	0.095	0.089	0.064	0.058	0.006
56	Sri Lanka [LKA]	0.283	0.253	0.227	0.197	0.160	0.130	0.030
57	Sweden [SWE]	0.091	0.087	0.072	0.068	0.048	0.045	0.004
58	Switzerland [CHE]	0.059	0.058	0.047	0.045	0.031	0.030	0.002
59	Thailand [THA]	0.174	0.162	0.138	0.126	0.095	0.083	0.012
60	Togo [TGO]	0.858	0.656	0.713	0.511	0.537	0.336	0.201
61	United Kingdom [GBR]	0.106	0.101	0.084	0.079	0.057	0.052	0.005
62	United States [USA]	0.077	0.074	0.060	0.058	0.041	0.038	0.003
63	Uruguay [URY]	0.282	0.253	0.227	0.197	0.159	0.129	0.030
64	Venezuela [VEN]	0.473	0.399	0.385	0.311	0.278	0.204	0.074

Note : There are cases in which the sum of the estimates λ_{nofear} and λ_{fear} is not completely equal to the estimate of λ , owing to rounding errors.

Appendix Table C.2
Welfare-Gain Estimates for Individual Countries: Trend Stationary Model

		Detection error probability						
		5%		10%		20%		
No.	Country	λ	λ_{fear}	λ	λ_{fear}	λ	λ_{fear}	λ_{nofear}
1	Algeria [DZA]	0.0963	0.0901	0.0764	0.0702	0.0523	0.0461	0.0062
2	Australia [AUS]	0.0249	0.0244	0.0195	0.0190	0.0129	0.0125	0.0005
3	Austria [AUT]	0.0265	0.0260	0.0208	0.0203	0.0138	0.0133	0.0005
4	Bangladesh [BGD]	0.0727	0.0691	0.0574	0.0538	0.0390	0.0353	0.0036
5	Belgium [BEL]	0.0167	0.0165	0.0131	0.0128	0.0087	0.0084	0.0003
6	Benin [BEN]	0.0309	0.0293	0.0245	0.0228	0.0166	0.0150	0.0016
7	Bolivia [BOL]	0.1572	0.1454	0.1251	0.1133	0.0862	0.0744	0.0119
8	Brazil [BRA]	0.0399	0.0380	0.0316	0.0296	0.0214	0.0194	0.0020
9	Burkina Faso [BFA]	0.0546	0.0506	0.0435	0.0394	0.0299	0.0259	0.0040
10	Cameroon [CMR]	0.0760	0.0696	0.0606	0.0542	0.0420	0.0356	0.0064
11	Canada [CAN]	0.0188	0.0184	0.0147	0.0143	0.0098	0.0094	0.0004
12	Chile [CHL]	0.0897	0.0830	0.0714	0.0646	0.0492	0.0425	0.0068
13	Colombia [COL]	0.0476	0.0462	0.0374	0.0360	0.0251	0.0236	0.0015
14	Congo [COG]	0.0845	0.0671	0.0697	0.0523	0.0517	0.0344	0.0174
15	Costa Rica [CRI]	0.0851	0.0809	0.0673	0.0630	0.0456	0.0414	0.0042
16	Denmark [DNK]	0.0236	0.0229	0.0185	0.0179	0.0124	0.0117	0.0007
17	Dominican Republic [DOM]	0.0434	0.0412	0.0343	0.0321	0.0233	0.0211	0.0022
18	Ecuador [ECU]	0.0454	0.0432	0.0359	0.0337	0.0243	0.0221	0.0022
19	Egypt [EGY]	0.0298	0.0288	0.0234	0.0225	0.0157	0.0148	0.0010
20	El Salvador [SLV]	0.1116	0.1037	0.0887	0.0808	0.0609	0.0531	0.0079
21	Finland [FIN]	0.0331	0.0322	0.0260	0.0251	0.0174	0.0165	0.0010
22	France [FRA]	0.0162	0.0160	0.0127	0.0125	0.0084	0.0082	0.0002
23	Gabon [GAB]	0.0808	0.0718	0.0649	0.0560	0.0457	0.0368	0.0089
24	Germany [DEU]	0.0228	0.0224	0.0179	0.0175	0.0119	0.0115	0.0004
25	Greece [GRC]	0.0995	0.0948	0.0786	0.0739	0.0532	0.0485	0.0047
26	Guatemala [GTM]	0.0548	0.0533	0.0431	0.0415	0.0288	0.0273	0.0016
27	Hong Kong [HKG]	0.0789	0.0750	0.0623	0.0585	0.0423	0.0384	0.0039
28	India [IND]	0.0914	0.0878	0.0720	0.0684	0.0485	0.0449	0.0036
29	Indonesia [IDN]	0.0481	0.0462	0.0379	0.0360	0.0256	0.0236	0.0019
30	Ireland [IRL]	0.0789	0.0754	0.0622	0.0588	0.0420	0.0386	0.0035
31	Israel [ISR]	0.0335	0.0321	0.0264	0.0250	0.0178	0.0164	0.0014
32	Italy [ITA]	0.0690	0.0668	0.0542	0.0521	0.0363	0.0342	0.0022
33	Japan [JPN]	0.0438	0.0427	0.0343	0.0333	0.0229	0.0219	0.0010
34	Kenya [KEN]	0.0418	0.0386	0.0333	0.0300	0.0230	0.0197	0.0033
35	Korea [KOR]	0.1742	0.1619	0.1384	0.1262	0.0951	0.0829	0.0122
36	Lesotho [LSO]	0.2337	0.2118	0.1869	0.1650	0.1302	0.1084	0.0219
37	Luxembourg [LUX]	0.0572	0.0556	0.0449	0.0433	0.0300	0.0285	0.0016
38	Madagascar [MDG]	0.0829	0.0786	0.0655	0.0613	0.0445	0.0402	0.0043
39	Malaysia [MYS]	0.0636	0.0600	0.0503	0.0468	0.0343	0.0307	0.0036
40	Mali [MLI]	0.0996	0.0931	0.0791	0.0725	0.0542	0.0476	0.0065
41	Mauritania [MRT]	0.0564	0.0500	0.0454	0.0390	0.0320	0.0256	0.0064
42	Mexico [MEX]	0.0340	0.0327	0.0268	0.0254	0.0181	0.0167	0.0014
43	Netherlands [NLD]	0.0496	0.0483	0.0389	0.0376	0.0260	0.0247	0.0013
44	New Zealand [NZL]	0.0472	0.0457	0.0371	0.0356	0.0249	0.0234	0.0015
45	Nicaragua [NIC]	0.2144	0.1888	0.1727	0.1471	0.1222	0.0966	0.0256
46	Norway [NOR]	0.0241	0.0235	0.0189	0.0183	0.0126	0.0120	0.0006
47	Paraguay [PRY]	0.0555	0.0531	0.0438	0.0414	0.0296	0.0272	0.0024
48	Peru [PER]	0.1890	0.1736	0.1506	0.1352	0.1042	0.0888	0.0154
49	Philippines [PHL]	0.3588	0.3619	0.2789	0.2819	0.1821	0.1851	-0.0031

	Appendix Table C.2 (continued)							
50	Portugal [PRT]	0.0477	0.0457	0.0376	0.0356	0.0253	0.0234	0.0020
51	Puerto Rico [PRI]	0.0374	0.0315	0.0304	0.0246	0.0220	0.0161	0.0059
52	Rwanda [RWA]	0.0990	0.0906	0.0790	0.0706	0.0547	0.0464	0.0084
53	Singapore [SGP]	0.0500	0.0482	0.0393	0.0375	0.0265	0.0246	0.0018
54	South Africa [ZAF]	0.0554	0.0535	0.0436	0.0417	0.0293	0.0274	0.0019
55	Spain [ESP]	0.0500	0.0484	0.0393	0.0377	0.0264	0.0248	0.0016
56	Sri Lanka [LKA]	0.0390	0.0366	0.0310	0.0285	0.0212	0.0187	0.0025
57	Sweden [SWE]	0.0289	0.0282	0.0226	0.0219	0.0151	0.0144	0.0007
58	Switzerland [CHE]	0.0105	0.0104	0.0082	0.0081	0.0055	0.0053	0.0001
59	Thailand [THA]	0.0658	0.0628	0.0519	0.0489	0.0351	0.0321	0.0030
60	Togo [TGO]	0.0712	0.0605	0.0578	0.0472	0.0416	0.0310	0.0106
61	United Kingdom [GBR]	0.0663	0.0641	0.0521	0.0499	0.0350	0.0328	0.0022
62	United States [USA]	0.0284	0.0278	0.0222	0.0216	0.0148	0.0142	0.0006
63	Uruguay [URY]	0.0778	0.0728	0.0617	0.0567	0.0423	0.0372	0.0050
64	Venezuela [VEN]	0.2424	0.2153	0.1948	0.1678	0.1372	0.1102	0.0270

Note : There are cases in which the sum of the estimates λ_{nofear} and λ_{fear} is not completely equal to the estimate of λ, owing to rounding errors.

Appendix Table D.1

Cross-Sectional Regressions of Welfare Gains on Variables that Measure Economic Development, Country Size, and Openness: Random Walk Model (the 5% Detection Error Probability)

	Dependent variable											
	λ				λ_{fear}				λ_{nofear}			
Regressor	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
A. Results based on all 64 countries												
ln PPP_GDP	-0.082	-0.090	-0.096	-0.092	-0.066	-0.072	-0.076	-0.073	-0.017	-0.019	-0.020	-0.019
	(0.020)	(0.020)	(0.025)	(0.024)	(0.015)	(0.014)	(0.017)	(0.016)	(0.006)	(0.006)	(0.008)	(0.007)
ln pop		-0.056		-0.054		-0.042		-0.041		-0.014		-0.013
		(0.017)		(0.016)		(0.012)		(0.012)		(0.005)		(0.004)
openness			0.062	0.009			0.045	0.005			0.017	0.004
			(0.036)	(0.033)			(0.024)	(0.023)			(0.012)	(0.010)
SER	0.191	0.176	0.188	0.177	0.138	0.126	0.136	0.127	0.055	0.052	0.054	0.052
R^2	0.204	0.339	0.241	0.339	0.238	0.378	0.274	0.378	0.113	0.223	0.150	0.224
B. Results based on 61 countries excluding outliers												
ln PPP_GDP	-0.065	-0.071	-0.072	-0.070	-0.055	-0.059	-0.060	-0.059	-0.011	-0.012	-0.012	-0.011
	(0.011)	(0.010)	(0.012)	(0.011)	(0.008)	(0.008)	(0.010)	(0.009)	(0.002)	(0.002)	(0.003)	(0.002)
ln pop		-0.032		-0.033		-0.026		-0.026		-0.006		-0.007
		(0.009)		(0.011)		(0.007)		(0.009)		(0.002)		(0.002)
openness			0.026	-0.006			0.021	-0.004			0.004	-0.002
			(0.012)	(0.017)			(0.010)	(0.014)			(0.002)	(0.003)
SER	0.107	0.098	0.106	0.099	0.086	0.079	0.086	0.080	0.021	0.020	0.021	0.020
R^2	0.333	0.448	0.350	0.449	0.350	0.463	0.368	0.464	0.249	0.369	0.263	0.371

Note : Heteroskedasticity-robust standard errors are in parentheses. The welfare gains, λ and λ_{fear}, are those for a detection error probability of 0.05. PPP_GDP is the geometric average of PPP-converted GDP per capita over the period 1990--2018, a measure of economic development. pop is the total population in 2018. openness is the trade openness measured by the ratio of exports plus imports to GDP in 2018. See Appendix B for details of each variable. The coefficient on the constant term in the regression is not reported. *SER* is the standard error of the regression. The results for λ_{nofear} (columns (9)--(12)) are the same as those in Table 3.

Appendix Table D.2

Cross-Sectional Regressions of Welfare Gains on Variables that Measure Economic Development, Country Size, and Openness: Random Walk Model (the 20% Detection Error Probability)

	Dependent variable											
	λ				λ_{fear}				λ_{nofear}			
Regressor	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
A. Results based on all 64 countries												
ln PPP_GDP	-0.050	-0.055	-0.059	-0.057	-0.034	-0.037	-0.039	-0.037	-0.017	-0.019	-0.020	-0.019
	(0.013)	(0.013)	(0.016)	(0.016)	(0.007)	(0.007)	(0.009)	(0.008)	(0.006)	(0.006)	(0.008)	(0.007)
ln pop		-0.035		-0.034		-0.022		-0.021		-0.014		-0.013
		(0.011)		(0.010)		(0.006)		(0.006)		(0.005)		(0.004)
openness			0.040	0.007			0.023	0.002			0.017	0.004
			(0.024)	(0.022)			(0.012)	(0.012)			(0.012)	(0.010)
SER	0.124	0.115	0.122	0.115	0.071	0.064	0.070	0.065	0.055	0.052	0.054	0.052
R^2	0.185	0.316	0.222	0.316	0.238	0.378	0.274	0.378	0.113	0.223	0.150	0.224
B. Results based on 61 countries excluding outliers												
ln PPP_GDP	-0.039	-0.042	-0.042	-0.041	-0.028	-0.030	-0.031	-0.030	-0.011	-0.012	-0.012	-0.011
	(0.006)	(0.006)	(0.007)	(0.007)	(0.004)	(0.004)	(0.005)	(0.004)	(0.002)	(0.002)	(0.003)	(0.002)
ln pop		-0.019		-0.020		-0.013		-0.014		-0.006		-0.007
		(0.006)		(0.007)		(0.004)		(0.004)		(0.002)		(0.002)
openness			0.015	-0.004			0.011	-0.002			0.004	-0.002
			(0.007)	(0.010)			(0.005)	(0.007)			(0.002)	(0.003)
SER	0.065	0.060	0.065	0.060	0.044	0.040	0.044	0.041	0.021	0.020	0.021	0.020
R^2	0.321	0.438	0.338	0.439	0.350	0.463	0.368	0.464	0.249	0.369	0.263	0.371

Note : Heteroskedasticity-robust standard errors are in parentheses. The welfare gains, λ and λ_{fear}, are those for a detection error probability of 0.20. PPP_GDP is the geometric average of PPP-converted GDP per capita over the period 1990--2018, a measure of economic development. pop is the total population in 2018. openness is the trade openness measured by the ratio of exports plus imports to GDP in 2018. See Appendix B for details of each variable. The coefficient on the constant term in the regression is not reported. *SER* is the standard error of the regression. The results for λ_{nofear} (columns (9)--(12)) are the same as those in Table 3.

Appendix Table E.1

Cross-Sectional Regressions of Welfare Gains on Variables that Measure Economic Development, County Size, and Openness: Trend Stationary Model (the 5% Detection Error Probability)

	Dependent variable											
	λ				λ_{fear}				λ_{nofear}			
Regressor	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
A. Results based on all 64 countries												
ln PPP_GDP	-0.019	-0.019	-0.021	-0.021	-0.017	-0.017	-0.019	-0.019	-0.002	-0.002	-0.002	-0.002
	(0.006)	(0.006)	(0.007)	(0.007)	(0.006)	(0.006)	(0.006)	(0.006)	(0.0005)	(0.0005)	(0.0006)	(0.0006)
ln pop		-0.002		0.0004		-0.001		0.002		-0.001		-0.001
		(0.005)		(0.006)		(0.005)		(0.006)		(0.0004)		(0.0004)
openness			0.011	0.011			0.010	0.011			0.001	0.0001
			(0.005)	(0.007)			(0.005)	(0.007)			(0.0007)	(0.0006)
SER	0.059	0.059	0.059	0.059	0.056	0.056	0.056	0.056	0.005	0.005	0.005	0.005
R^2	0.126	0.128	0.140	0.140	0.114	0.114	0.126	0.127	0.148	0.229	0.168	0.229
B. Results based on 62 countries excluding outliers												
ln PPP_GDP	-0.015	-0.016	-0.017	-0.017	-0.013	-0.014	-0.015	-0.015	-0.002	-0.002	-0.002	-0.002
	(0.005)	(0.005)	(0.006)	(0.006)	(0.004)	(0.005)	(0.005)	(0.005)	(0.0006)	(0.0006)	(0.0006)	(0.0006)
ln pop		-0.005		-0.004		-0.004		-0.003		-0.001		-0.001
		(0.004)		(0.004)		(0.003)		(0.003)		(0.0004)		(0.0004)
openness			0.011	0.007			0.009	0.006			0.001	0.0003
			(0.006)	(0.004)			(0.005)	(0.004)			(0.0007)	(0.0006)
SER	0.047	0.047	0.047	0.047	0.042	0.042	0.042	0.042	0.005	0.005	0.005	0.005
R^2	0.126	0.149	0.146	0.155	0.118	0.137	0.138	0.144	0.160	0.227	0.183	0.228

Note : Heteroskedasticity-robust standard errors are in parentheses. The welfare gains, λ and λ_{fear}, are those for a detection error probability of 0.05. PPP_GDP is the geometric average of PPP-converted GDP per capita over the period 1990--2018, a measure of economic development. pop is the total population in 2018. openness is the trade openness measured by the ratio of exports plus imports to GDP in 2018. See Appendix B for details of each variable. The coefficient on the constant term in the regression is not reported. *SER* is the standard error of the regression. The results for λ_{nofear} (columns (9)--(12)) are the same as those in Table 4.

Appendix Table E.2
Cross-Sectional Regressions of Welfare Gains on Variables that Measure Economic Development, County Size, and Openness: Trend Stationary Model (the 20% Detection Error Probability)

	Dependent variable											
	λ				λ_{fear}				λ_{nofear}			
Regressor	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
A. Results based on all 64 countries												
ln PPP_GDP	-0.011	-0.011	-0.012	-0.012	-0.009	-0.009	-0.010	-0.010	-0.002	-0.002	-0.002	-0.002
	(0.003)	(0.003)	(0.004)	(0.004)	(0.003)	(0.003)	(0.003)	(0.003)	(0.0005)	(0.0005)	(0.0006)	(0.0006)
ln pop		-0.002		-0.0004		-0.0005		0.001		-0.001		-0.001
		(0.003)		(0.003)		(0.003)		(0.003)		(0.0004)		(0.0004)
openness			0.006	0.006			0.005	0.006			0.001	0.0001
			(0.003)	(0.004)			(0.003)	(0.003)			(0.0007)	(0.0006)
SER	0.032	0.032	0.032	0.032	0.028	0.029	0.028	0.029	0.005	0.005	0.005	0.005
R^2	0.135	0.140	0.150	0.150	0.114	0.114	0.126	0.127	0.148	0.229	0.168	0.229
B. Results based on 62 countries excluding outliers												
ln PPP_GDP	-0.009	-0.009	-0.010	-0.010	-0.007	-0.007	-0.008	-0.008	-0.002	-0.002	-0.002	-0.002
	(0.003)	(0.003)	(0.003)	(0.003)	(0.002)	(0.002)	(0.003)	(0.003)	(0.0006)	(0.0006)	(0.0006)	(0.0006)
ln pop		-0.003		-0.002		-0.002		-0.001		-0.001		-0.001
		(0.002)		(0.002)		(0.002)		(0.002)		(0.0004)		(0.0004)
openness			0.006	0.004			0.005	0.003			0.001	0.0003
			(0.003)	(0.002)			(0.003)	(0.002)			(0.0007)	(0.0006)
SER	0.026	0.026	0.026	0.026	0.022	0.022	0.022	0.022	0.005	0.005	0.005	0.005
R^2	0.131	0.158	0.152	0.164	0.118	0.137	0.138	0.144	0.160	0.227	0.183	0.228

Note : Heteroskedasticity-robust standard errors are in parentheses. The welfare gains, λ and λ_{fear}, are those for a detection error probability of 0.20. PPP_GDP is the geometric average of PPP-converted GDP per capita over the period 1990--2018, a measure of economic development. pop is the total population in 2018. openness is the trade openness measured by the ratio of exports plus imports to GDP in 2018. See Appendix B for details of each variable. The coefficient on the constant term in the regression is not reported. *SER* is the standard error of the regression. The results for λ_{nofear} (columns (9)--(12)) are the same as those in Table 4.

Appendix Table F
Replication of the Robustness Parameter

	Detection error probability		
	50%	45%	40%
Formula (C16)	0	0.31	0.62
Ellison and Sargent (2015)	0	0.3	0.6

Note : The results in the second row are from Ellison and Sargent (2015, Table 2)