Gindi, Harry

Coherent nerves for higher quasicategories. (English) Zbl 07377654

Theory Appl. Categ. 37, 709-817 (2021)

Oury [https://www.researchgate.net/publication/258453234_Duality_for_Joyal's_category_theta_and_homotopy_concepts_for_theta2-sets] has shown that his constructed model structure on Θ_2-sets is Cartesian closed. Around the same time, *C. Rezk* [Geom. Topol. 14, No. 1, 521–571 (2010; Zbl 1203.18015)] constructed a model structure on Θ_n-spaces, which, in the case of $n=2$, was expected to be Quillen equivalent to a model structure on the category of Θ_2-sets proposed by Joyal and Cisinski (later constructed by *D. Ara* [J. K-Theory 14, No. 3, 701–749 (2014; Zbl 1322.18002)] and by the author ["A homotopy theory of weak ω-categories", Preprint, arXiv:1207.0860]) that coincides with Oury's model structure.

J. E. Bergner and *C. Rezk* [Geom. Topol. 17, No. 4, 2163–2202 (2013; Zbl 1273.18031); "Comparison of models for (∞, n)-categories. II", Preprint, arXiv:1406.4182] have shown by means of a zig-zag of Quillen equivalences that the category of Θ_n-spaces with Rezk's model structure models the same homotopy theory as the model category of $\mathrm{Psh}_\Delta(\Theta_{n-1})$-enriched categories with the Bergner-Lurie model structure for categories enriched in Θ_{n-1}-spaces with Rezk's model structure. Since the equivalence is highly indirect, many of the ideas from *J. Lurie*'s work [Higher topos theory. Princeton, NJ: Princeton University Press (2009; Zbl 1175.18001)] on $(\infty, 1)$-categories could not be adapted straightforwardly, in particular, his construction of the Yoneda embedding and his proof of Yoneda's lemma.

The principal objective in this paper is to rectify this situation in two steps.

[1] The author exploits Oury's machinery to construct a model structure on $\Theta[\mathcal{C}]$-sets modelling weak enrichment in simplicial presheaves on $\mathcal{C}$, comparing it with an intermediate model structure of Rezk to demonstrate that they are Quillen equivalent. Therefore one can use results in [*C. Rezk*, Geom. Topol. 14, No. 1, 521–571 (2010; Zbl 1203.18015)] to localize this model structure hom-wise with respect to what Rezk calls a Cartesian presentation on , which is again Quillen equivalent to Rezk's localized model structure by merit of *D.-C. Cisinski*'s results on simplicial completion [Les préfaisceaux comme modèles des types d'homotopie. Paris: Société Mathématique de France (2006; Zbl 1111.18008)]. The author's model structure is Cartesian monoidal as a model category.

[2] The author constructs a version of the coherent realization and nerve adjunction between $\Theta[\mathcal{C}]$-sets and categories enriched in simplicial presheaves on $\mathcal{C}$, which turn out to be a Quillen equivalence between appropriate model structures by using an enhanced version of *D. Dugger* and *D. I. Spivak*'s calculus of necklaces [Algebr. Geom. Topol. 11, No. 1, 263–325 (2011; Zbl 1214.55013); Algebr. Geom. Topol. 11, No. 1, 225–261 (2011; Zbl 1213.55015)].

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MSC:

18A30	Limits and colimits (products, sums, directed limits, pushouts, fiber products, equalizers, kernels, ends and coends, etc.)
18E35	Localization of categories, calculus of fractions
55P10	Homotopy equivalences in algebraic topology
55U35	Abstract and axiomatic homotopy theory in algebraic topology

Keywords:

enriched categories; higher category theory; homotopy theory

Full Text: Link

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