Cockett, Robin; Cruttwell, Geoff; Gallagher, Jonathan; Pronk, Dorette

Latent fibrations: fibrations for categories of partial maps. (English) Zbl 07377656

Theory Appl. Categ. 36, 423-491 (2021)

Fibrations play a significant role in categorical logic, providing models of type theories [*B. Jacobs*, Categorical logic and type theory. Amsterdam: Elsevier (1999; Zbl 0911.03001)]. The principal objective in this paper is to develop a theory of fibrations (called *latent fibrations*) for *restriction categories* [*J. R. B. Cockett* and *S. Lack*, Theor. Comput. Sci. 270, No. 1–2, 223–259 (2002; Zbl 0988.18003); Theor. Comput. Sci. 294, No. 1–2, 61–102 (2003; Zbl 1023.18005); Math. Struct. Comput. Sci. 17, No. 4, 775–817 (2007; Zbl 1123.18003)]. Latent fibrations with various special properties are identified. It is shown that the notion of latent fibration corresponds precisely to the 2-categorical notion introduced in [*R. Street*, Lect. Notes Math. 420, 104–133 (1974; Zbl 0327.18006)] for the carefully chosen 2-category of restriction semi-functors and restriction transformations.

Reviewer: Hirokazu Nishimura (Tsukuba)

MSC:

18B99 Special categories
18D30 Fibered categories

Keywords:

latent fibrations; restriction categories; partial maps

Full Text: Link

References:

[1] Martin Abadi and Gordon D. Plotkin. A simple differentiable programming language.Proceedings of the ACM on Programming Languages, 4, 38:1-38:28, 2019. · Zbl 1315.68079

[2] J. B´enabou. Fibrations petites et localement petites.C. R. Acad. Sc. Paris, 281:831-834, 1975. · Zbl 0349.18006

[3] R. Blute, R. Cockett, and R. Seely. Cartesian differential categories.Theory and Applications of Categories, 22:622-672, 2009. · Zbl 1262.18004

[4] J.R.B. Cockett, G.S.H. Cruttwell, and J.D. Gallagher. Differential restriction categories.Theory and Applications of Categories, 25(21):537-613, 2011. · Zbl 1260.14004

[5] J.R.B. Cockett and Stephen Lack. Restriction categories I: categories of partial maps.Theoretical Computer Science, 270(1):223-259, 2002. · Zbl 0988.18003

[6] R. Cockett and G. Cruttwell.Differential structure, tangent structure, and SDG.Applied Categorical Structures, 22:331-417, 2014. · Zbl 1304.18031

[7] R. Cockett, X. Guo, and P. Hofstra. Range categories I: general theory.Theory and Applications of Categories, 26(17):412-452, 2012. · Zbl 1252.18003

[8] R. Cockett and S. Lack.Restriction categories III: colimits, partial limits and extensivity. Mathematical Structures in Computer Science, 17(4):775-817, 2007. · Zbl 1123.18003

[9] Robin Cockett, Geoffrey Cruttwell, Jonathan Gallagher, Jean-Simon Pacaud Lemay, Benjamin MacAdam, Gordon Plotkin, and Dorette Pronk. Reverse derivative categories. Proceedings of CSL 2020, 18:1-18:16, 2020.

[10] Robin Cockett and Richard Garner. Restriction categories as enriched categories.Theoretical Computer Science, 523:37-55, 2014. · Zbl 1288.18007

[11] Robin Cockett, Xiuzhan Guo, and Pieter Hofstra. Range categories II: towards regularity.Theory and Applications of Categories, 26(18):453-500, 2012. · Zbl 1252.18004

[12] G. Cruttwell, J. Gallagher, and D. Pronk. Categorical semantics of a simple differential programming language.Proceedings of the 3rd Annual International Applied Category Theory Conference 2020 inElectronic Proceedings in Theoretical Computer Science, 333:289-310, 2020.

[13] R. Garner and D. Lin. Cocompletion of restriction categories.Theory and Application of Categories, 35(22):809-844, 2020. · Zbl 1441.18005

[14] Jules Hedges.Coherence for lenses and open games.Technical Report arXiv:1704.02230v3 [math.CT], September 2017.

[15] B. Jacobs.Categorical Logic and Type Theory. Number 141 in Studies in logic and the foundations of mathematics. Elsevier, 1999.

[16] Michael Johnson, Robert Rosebrugh, and R. J. Wood. Lenses, fibrations and universal translations. Mathematical Structures in Computer Science, 22(1):25-42, 2012. · Zbl 1237.68075

[17] Andrej Karpathy.https://karpathy.medium.com/software-2-0-a64152b37c35.

[18] Anders Kock. The dual fibration in elementary terms. Technical Report arXiv:1501.01947, January 2015.

[19] Yann LeCun.https://www.facebook.com/yann.lecun/posts/10155003011462143.

[20] Fosco Loregian and Emily Riehl. Categorical notions of fibration.Expositiones Mathematicae, June 2019. · Zbl 1464.18010

[21] C. Nester. Turing categories and realizability. Master's thesis, University of Calgary, 2017.

[22] David I.Spivak.Generalized lens categories via functorsCop$-\!\!\longrightarrow$Cat.Technical Report arXiv:1908.02202v3 [math.CT], February 2020.

[23] Ross Street. Fibrations and Yoneda's lemma in a 2-category. In Gregory M. Kelly, editor,Category Seminar, pages 104-133, Berlin, Heidelberg, 1974. Springer Berlin Heidelberg. · Zbl 0327.18006

[24] P. Wilson and F. Zanasi. Reverse derivative ascent: a categorical approach to learning boolean circuits