García-Martínez, Xabier; Gray, James R. A.
Algebraic exponentiation for Lie algebras. (English) Zbl 07377649
Theory Appl. Categ. 36, 288-305 (2021)

Motivated by [*J. R. A. Gray*, J. Pure Appl. Algebra 216, No. 8–9, 1964–1967 (2012; Zbl 1275.18021)] where it was shown that the category of LIe algebras over a commutative ring is *locally algebraically cartesian closed* (LACC), this paper aims to show that, under certain conditions, the category of LIe algebras in a monoidal category is (LACC), adding the categories of Lie superalgebras, $\mathbb{Z}$-graded Lie algebras and differential graded Lie algebras amongst others with the category of Lie algebras in the Loday-Pirashvili category [*J. L. Loday* and *T. Pirashvili*, Georgian Math. J. 5, No. 3, 263–276 (1998; Zbl 0909.18003)] as another example.

Reviewer: Hirokazu Nishimura (Tsukuba)

MSC:

18E13 Protomodular categories, semi-abelian categories, Mal'tsev categories
16W25 Derivations, actions of Lie algebras
17A32 Leibniz algebras
18M05 Monoidal categories, symmetric monoidal categories

Keywords:

locally algebraically cartesian closed; semi-abelian category; algebraic exponentiation; Lie algebra

Full Text: Link

References:

[1] F. Borceux, G. Janelidze, and G. M. Kelly,On the representability of actions in a semi-abelian category, Theory Appl. Categ.14(2005), no. 11, 244-286. · Zbl 1103.18006

[2] D. Bourn,Normalization equivalence, kernel equivalence and affine categories, Category Theory, Proceedings Como 1990 (A. Carboni, M. C. Pedicchio, and G. Rosolini, eds.), Lecture Notes in Math., vol. 1488, Springer, 1991, pp. 43-62. · Zbl 0756.18007

[3] D. Bourn,Normal functors and strong protomodularity, Theory Appl. Categ.7 (2000), no. 9, 206-218. · Zbl 0947.18004

[4] D. Bourn,The cohomological comparison arising from the associated abelian object, preprintarXiv:1001.0905, 2010.

[5] D. Bourn and J. R. A. Gray,Aspects of algebraic exponentiation, Bull. Belg. Math. Soc. Simon Stevin19(2012), 823-846. · Zbl 1264.18003

[6] S. Caenepeel and I. Goyvaerts,Monoidal Hom-Hopf algebras, Comm. Algebra39 (2011), no. 6, 2216-2240. · Zbl 1255.16032

[7] J. M. Casas and X. Garc´ıa-Mart´ınez,Abelian extensions and crossed modules of Hom-Lie algebras, J. Pure Appl. Algebra224(2020), no. 3, 987-1008. · Zbl 1436.17028

[8] A. S. Cigoli, J. R. A. Gray, and T. Van der Linden,On the normality of Higgins commutators, J. Pure Appl. Algebra219(2015), 897-912. · Zbl 1306.18006

[9] R. Fern´andez-Casado, X. Garc´ıa-Mart´ınez, and M. Ladra,A natural extension of the universal enveloping algebra functor to crossed modules of Leibniz algebras, Appl. Categ. Structures25(2017), no. 6, 1059-1076.

[10] X. Garc´ıa-Mart´ınez and M. Ladra,Universal central extensions of Leibniz superalgebras over superdialgebras, Mediterr. J. Math.14(2017), 14: 63.

[11] X. Garc´ıa-Mart´ınez, M. Tsishyn, T. Van der Linden, and C. Vienne,Algebras with representable representations, To appear in Proc. Edinburgh Math. Soc., 2021.

[12] X. Garc´ıa-Mart´ınez and T. Van der Linden,A characterisation of Lie algebras amongst anti-commutative algebras, J. Pure Appl. Algebra223(2019), no. 11, 4857- 4870. · Zbl 07076020

[13] X. Garc´ıa-Mart´ınez and T. Van der Linden,A characterisation of Lie algebras via algebraic exponentiation, Adv. Math.341(2019), 92-117.

[14] I. Goyvaerts and J. Vercruysse,A note on the categorification of Lie algebras, Lie theory and its applications in physics, Springer Proc. Math. Stat., vol. 36, Springer, Tokyo, 2013, pp. 541-550. · Zbl 1280.17027

[15] I. Goyvaerts and J. Vercruysse,Lie monads and dualities, J. Algebra414(2014), 120-158. · Zbl 1312.17016

[16] I. Goyvaerts and J. Vercruysse,On the duality of generalized Lie and Hopf algebras, Adv. Math.258(2014), 154-190. · Zbl

1295.16021

[17] J. R. A. Gray,Algebraic exponentiation in general categories, Ph.D. thesis, University of Cape Town, 2010. · Zbl 1276.18002

[18] J. R. A. Gray,Algebraic exponentiation for categories of Lie algebras, J. Pure Appl. Algebra216(2012), 1964-1967. · Zbl 1275.18021

[19] J. R. A. Gray,Algebraic exponentiation in general categories, Appl. Categ. Structures 20(2012), 543-567. · Zbl 1276.18002

[20] G. Janelidze, L. M´arki, and W. Tholen,Semi-abelian categories, J. Pure Appl. Algebra168(2002), no. 2-3, 367-386. · Zbl 0993.18008

[21] G. M. Kelly,Doctrinal adjunction, Category Seminar (Proc. Sem., Sydney, 1972/1973), 1974, pp. 257-280. Lecture Notes in Math., Vol. 420.

[22] Stephen Lack,Note on the construction of free monoids, Applied Categorical Structures18(2010), no. 1, 17-29. · Zbl 1226.18010

[23] J. L. Loday and T. Pirashvili,The tensor category of linear maps and Leibniz algebras, Georgian Math. J.5(1998), no. 3, 263-276. · Zbl 0909.18003

[24] S. Mac Lane,Categories for the working mathematician, second ed., Grad. Texts in Math., vol. 5, Springer, 1998. · Zbl 0906.18001

[25] N. Martins-Ferreira and T. Van der Linden,A note on the "Smith is Huq" condition, Appl. Categ. Structures20(2012), no. 2, 175-187. · Zbl 1255.18008

[26] A. Rovi,Lie algebroids in the Loday-Pirashvili category, SIGMA Symmetry Integrability Geom. Methods Appl.11(2015), Paper 079, 16. · Zbl 1395.17038