**Lucatelli Nunes, Fernando**

**Descent data and absolute Kan extensions.** (English) Zbl 07361310

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The fundamental construction underlying descent theory, the lax descent category, comes with a functor forgetting the *descent data*. The main result of this paper is that, given any pseudofunctor $\mathcal{F}:\mathbb{C}^{\mathrm{op}}\to\mathrm{Cat}$, the forgetful functor

$$\text{lax-}\mathcal{D}\mathrm{esc}\left(\mathcal{F}\circ\mathrm{op}\left(a\right)\right)\to\mathcal{F}a\left(1\right)$$

between the $\mathcal{F}$-internal actions of a precategory $a:\Delta_3^{\mathrm{op}}\to\mathbb{C}$ and the category of internal actions of the underlying discrete precategory of $a$ creates absolute limits and colimits (Theorem 2.4).

A synopsis of the paper, consisting of four sections, goes as follows:

- §1 gives the basic definition of the *lax descent category*, presenting the corresponding definition for a general 2-category after [*F. Lucatelli Nunes*, "Semantic factorization and descent", Preprint, `arXiv:1902.01225`].
- §2 establishes the main theorems on the *morphisms forgetting descent data*. Based on Theorem 2.4, the monadicity theorem (Theorem 2.8) is esbalished. Theorem 2.11, claiming that a right adjoint functor is monadic iff it is a functor forgetting descent data composed with an equivalence, is established.
- §3 is concerned with the setting of *Grothendieck descent theory* [*G. Janelidze* and *W. Tholen*, Appl. Categ. Struct. 2, No. 3, 245–281 (1994; Zbl 0805.18005); *F. Lucatelli Nunes*, Theory Appl. Categ. 33, 390–444 (2018; Zbl 1405.18002)], aiming to establish Lemma 3.6 in order to recover the usual *descent factorization* [*G. Janelidze* and *W. Tholen*, Appl. Categ. Struct. 5, No. 3, 229–248 (1997; Zbl 0880.18007), §3] directly via the universal property of lax desecnt category.
- §4 is concerned with the relation monadicity and effective descent morphisms [*J. Benabou* and *J. Roubaud*, C. R. Acad. Sci., Paris, Sér. A 270, 96–98 (1970; Zbl 0287.18007); *G. Janelidze* and *W. Tholen*, Appl. Categ. Struct. 2, No. 3, 245–281 (1994; Zbl 0805.18005); *F. Lucatelli Nunes*, Theory Appl. Categ. 33, 390–444 (2018; Zbl 1405.18002)], establishing that, in bifibered categories, effective descent morphisms always induce monadic functors, even without satisfying the Beck-Chevalley condition.

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**MSC:**

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| 18N10 | 2-categories, bicategories, double categories |
| 18C15 | Monads (= standard construction, triple or triad), algebras for monads, homology and derived functors for monads |
| 18C20 | Eilenberg-Moore and Kleisli constructions for monads |
| 18F20 | Presheaves and sheaves, stacks, descent conditions (category-theoretic aspects) |
| 18A22 | Special properties of functors (faithful, full, etc.) |
| 18A30 | Limits and colimits (products, sums, directed limits, pushouts, fiber products, equalizers, kernels, ends and coends, etc.) |
| 18A40 | Adjoint functors (universal constructions, reflective subcategories, Kan extensions, etc.) |

**Keywords:**

descent theory; effective descent morphisms; internal actions; indexed categories; creation of absolute Kan extensions; Bénabou-Roubaud theorem; monadicity theorem

**Full Text:** Link

**References:**

[1] J.M. Beck. Triples, algebras and cohomology.Repr. Theory Appl. Categ., TAC(2):1- 59, 2003.

[2] J. B´enabou. Introduction to bicategories. InReports of the Midwest Category Seminar, pages 1-77. Springer, Berlin, 1967.

[3] J. B´enabou and J. Roubaud. Monades et descente.C. R. Acad. Sci. Paris S´er. A-B, 270:A96-A98, 1970. · Zbl 0287.18007

[4] D. Bourn and G. Janelidze. Protomodularity, descent, and semidirect products. Theory Appl. Categ., 4:No. 2, 37-46, 1998. · Zbl 0890.18003

[5] R. Brown and G. Janelidze. Van Kampen theorems for categories of covering morphisms in lextensive categories.J. Pure Appl. Algebra, 119(3):255-263, 1997. · Zbl 0882.18005

[6] E. Dubuc.Kan extensions in enriched category theory. Lecture Notes in Mathematics, Vol. 145. Springer-Verlag, Berlin-New York, 1970. · Zbl 0228.18002

[7] A. Grothendieck.Revˆetements ´etales et groupe fondamental. Lecture Notes in Mathematics, Vol. 224. Springer-Verlag, Berlin-New York, 1971. S´eminaire de G´eom´etrie Alg´ebrique du Bois Marie 1960-1961 (SGA 1), Dirig´e par Alexandre Grothendieck. Augment´e de deux expos´es de M. Raynaud.

[8] G. Janelidze. Categorical Galois theory: revision and some recent developments. InGalois connections and applications, volume 565 ofMath. Appl., pages 139-171. Kluwer Acad. Publ., Dordrecht, 2004. · Zbl 1072.18003

[9] G. Janelidze, M. Sobral, and W. Tholen. Beyond Barr exactness: effective descent morphisms. InCategorical foundations, volume 97 ofEncyclopedia Math. Appl., pages 359-405. Cambridge Univ. Press, Cambridge, 2004. · Zbl 1047.18010

[10] G. Janelidze and W. Tholen. Facets of descent. I.Appl. Categ. Structures, 2(3):245- 281, 1994. · Zbl 0805.18005

[11] G. Janelidze and W. Tholen. Facets of descent. II.Appl. Categ. Structures, 5(3):229- 248, 1997. · Zbl 0880.18007

[12] P.T. Johnstone.Topos theory. Academic Press [Harcourt Brace Jovanovich, Publishers], London-New York, 1977. London Mathematical Society Monographs, Vol. 10. · Zbl 0368.18001

[13] G.M. Kelly and R. Street. Review of the elements of 2-categories.Category Seminar (Proc. Sem., Sydney, 1972/1973), pages 75-103. Lecture Notes in Math., Vol. 420, 1974.

[14] S. Lack. Codescent objects and coherence.J. Pure Appl. Algebra, 175(1-3):223-241, 2002. Special volume celebrating the 70th birthday of Professor Max Kelly. · Zbl 1142.18301

[15] F. Lucatelli Nunes. On biadjoint triangles.Theory Appl. Categ., 31:No. 9, 217-256, 2016. · Zbl 1359.18003

[16] F. Lucatelli Nunes. On lifting of biadjoints and lax algebras.Categories and General Algebraic Structures with Applications, 9(1):29-58, 2018. · Zbl 1454.18018

[17] F. Lucatelli Nunes. Pseudo-Kan extensions and descent theory.Theory Appl. Categ., 33:No. 15, 390-444, 2018. · Zbl 1405.18002

[18] F. Lucatelli Nunes. Pseudoalgebras and non-canonical isomorphisms.Appl. Categ. Structures, 27(1):55-63, 2019. · Zbl 1411.18009

[19] F. Lucatelli Nunes. Semantic Factorization and Descent. 2019. arXiv: 1902.01225. · Zbl 1411.18009

[20] J. MacDonald and M. Sobral. Aspects of monads. InCategorical foundations, volume 97 ofEncyclopedia Math. Appl., pages 213-268. Cambridge Univ. Press, Cambridge, 2004. · Zbl 1043.18004

[21] M. Melo. Monadicidade e descida – da fibra¸c˜ao b´asica ‘a fibra¸c˜ao dos pontos. Master's thesis, University of Coimbra, Portugal, 2004.

[22] A.J. Power. A 2-categorical pasting theorem.J. Algebra, 129(2):439-445, 1990. DESCENT DATA AND ABSOLUTE KAN EXTENSIONS561 · Zbl 0698.18005

[23] J. Reiterman and W. Tholen. Effective descent maps of topological spaces.Topology Appl., 57(1):53-69, 1994. · Zbl 0829.54011

[24] M. Sobral. Descent for discrete (co)fibrations.Appl. Categ. Structures, 12(5-6):527- 535, 2004. · Zbl 1085.18002

[25] R. Street. The formal theory of monads.J. Pure Appl. Algebra, 2(2):149-168, 1972. · Zbl 0241.18003

[26] R. Street. Limits indexed by category-valued 2-functors.J. Pure Appl. Algebra, 8(2):149-181, 1976. · Zbl 0335.18005

[27] R. Street. Categorical and combinatorial aspects of descent theory.Appl. Categ. Structures, 12(5-6):537-576, 2004. · Zbl 1082.18011

[28] R. Street and R. Walters. Yoneda structures on 2-categories.J. Algebra, 50(2):350- 379, 1978 · Zbl 0401.18004