

Lambert, Michael

Discrete double fibrations. (English) Zbl 1464.18020
Theory Appl. Categ. 37, 671-708 (2021).

Ordinary fibrations have their analogues in 2-fibrations over a fixed base 2-category [*M. Buckley*, *J. Pure Appl. Algebra* 218, No. 6, 1034–1074 (2014; [Zbl 1296.18006](#))]. Discrete fibrations over a fixed base 2-category are of their 2-dimensional version in discrete 2-fibrations [*M. Lambert*, “Discrete 2-fibrations”, Preprint, [arXiv:2001.11477](#)]. They are justified by the existence of a representation theorem taking the form of an equivalence with representations of the base structure via a category of elements construction.

Within the double-categorical realm, *R. Paré* [*Theory Appl. Categ.* 25, 436–489 (2011; [Zbl 1251.18004](#))] proposed that certain span-valued, lax functors on a double category \mathbb{B} are presheaves on \mathbb{B} . The principal objective in this paper is to isolate the notion of *discrete double fibration* corresponding to this notion of presheaf. Paré’s double category of elements is exploited to exhibit a representation theorem as above. What is technically interesting is that an equivalence of virtual double categories is ultimately achieved, for which the language of monoids and modules in virtual double categories [*T. Leinster*, *Higher operads, higher categories*. Cambridge: Cambridge University Press (2004; [Zbl 1160.18001](#))] is in use.

Reviewer: [Hirokazu Nishimura \(Tsukuba\)](#)

MSC:

[18N10](#) 2-categories, bicategories, double categories
[18N25](#) Categorification

Keywords:

[double categories](#); [lax functors](#); [discrete fibrations](#); [virtual equipments](#); [monoids and modules](#)

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References:

- [1] M. Buckley. Fibred 2-categories and bicategories. *Journal of Pure and Applied Algebra*, 218(6):1034-1074, 2014. · [Zbl 1296.18006](#)
- [2] G.S.H. Crutwell and M.A. Shulman. A unified framework for generalized multicategories. *Theory and Applications of Categories*, 24(21):580-655, 2010. · [Zbl 1220.18003](#)
- [3] R. Diaconescu. Change of base for some toposes. PhD Thesis, Dalhousie University, 1973. · [Zbl 0353.18002](#)
- [4] R. Diaconescu. Change of base for toposes with generators. *Journal of Pure and Applied Algebra*, 6(3):191-218. · [Zbl 0353.18002](#)
- [5] C. Ehresmann. *Cat´egories et structures*. Dunod, Paris, 1963. DISCRETE DOUBLE FIBRATIONS707
- [6] M. Grandis and R. Par´e. Limits in Double Categories. *Cahiers de Topologie et G´eom´etrie Diff´erentielle Cat´egoriques*, 40(3):162-220.
- [7] M. Grandis and R. Par´e. Adjoints for Double Categories. *Cahiers de Topologie et G´eom´etrie Diff´erentielle Cat´egoriques*, 45(3):193-240.
- [8] J. Gray. *Formal Category Theory: Adjointness for 2-Categories*. Volume 391 of *Lecture Notes in Mathematics*, Springer, Berlin, 1974. · [Zbl 0285.18006](#)
- [9] P.T. Johnstone. *Topos Theory*. Dover, Minneapolis, 2014. · [Zbl 0368.18001](#)
- [10] M. Lambert. Discrete 2-fibrations. Preprint: <https://arxiv.org/abs/2001.11477>, 2020.
- [11] T. Leinster. *Higher Operads, Higher Categories*. Volume 298 of *London Mathematical Society Lecture Notes Series*, Cambridge University Press, Cambridge, 2004. · [Zbl 1160.18001](#)
- [12] S. MacLane. *Category Theory for the Working Mathematician*. Volume 5 of *Graduate Texts in Mathematics*, Springer, Berlin, 1998.
- [13] R. Par´e. Yoneda theory for double categories. *Theory and Applications of Categories*, 25(17):436-489, 2011. · [Zbl 1251.18004](#)
- [14] R. Par´e. Composition of modules for lax functors. *Theory and Applications of Categories*, 27(16):393-444, 2013.
- [15] M. Shulman. Framed bicategories and monoidal fibrations. *Theory and Applications of Categories*, 20(18):650-738, 2008. · [Zbl 1192.18005](#)

- [16] R. Street. Fibrations and Yoneda's lemma in a 2-category. In G.M. Kelly ed. *Category Seminar: Proceedings Sydney Category Theory Seminar 1972/1973*, Volume 420 of *Lecture Notes in Mathematics*, Springer, Berlin, pp. 104-133, 1974.
- [17] R. Street. Limits indexed by category-valued 2-functors. *Journal of Pure and Applied Algebra*, 8(2):149-181, 1976. · [Zbl 0335.18005](#)
- [18] R. Street and D. Verity. The comprehensive factorization and torsors. *Theory and Applications of Categories*, 23(3):42-75, 2010. · [Zbl 1315.18014](#)
- [19] R. Street and R.F.C. Walters. The comprehensive factorization of a functor. *Bulletin of the American Mathematical Society*, 79(5):936-941, 1973. · [Zbl 0274.18001](#)
- [20] R. Wood. Abstract proarrows I. *Cahiers de Topologie et Géométrie Différentielle Catégoriques*, 23(3):279-290, 1982.
- [21] R. Wood. Proarrows II. *Cahiers de Topologie et Géométrie Différentielle Catégoriques*, 26(2): 135-168, 1985.
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