zbMATH Open
THE FIRST RESOURCE FOR MATHEMATICS

Loregian, Fosco; Riehl, Emily
Categorical notions of fibration. (English) Zbl 1464.18010
Expo. Math. 38, No. 4, 496-514 (2020).

Fibrations in category theory, due to Grothendieck, were developed in [*J. W. Gray*, in: Proc. Conf. Categor. Algebra, La Jolla 1965, 21–83 (1966; Zbl 0192.10701)]. Definitions of fibrations internal to 2-categories were given in [*R. Street*, Lect. Notes Math. 420, 104–133 (1974; Zbl 0327.18006)], and those internal to bicategories were presented in [*R. Street*, Cah. Topologie Géom. Différ. Catégoriques 21, 111–159 (1980; Zbl 0436.18005)].

This expository paper tours the various categorical notions of fibration in order of increasing complexity. A synopsis of the paper goes as follows.

- §2 deals with the classical definitions of fibrations and discrete ones in ordinary 1-category theory.
- The internalization in a 2-category and generalization in a bicategory are given in §3 and §4.
- The real goal, pursued in parallel, is to define two-sided discrete fibrations in $\boldsymbol{Cat}$, where two-sided discrete fibrations encode functors
$$B^{\mathrm{op}} \times A \to \boldsymbol{Set}$$
known as *profunctors* from A to B, while in $\mathcal{V}$-$\boldsymbol{Cat}$ the dual two-sided codiscrete cofibrations encode $\mathcal{V}$-profunctors
$$B^{\mathrm{op}} \otimes A \to \mathcal{V}$$
- This paper concludes with a construction of a bicategory, defined internally to $\mathcal{V}$-$\boldsymbol{Cat}$, whose 1-cells are two-sided codiscrete cofibrations.

This theory has been extended to $(\infty, 1)$-categories modeled as quasi-categories by *J. Lurie* [Higher topos theory. Princeton, NJ: Princeton University Press (2009; Zbl 1175.18001)], where the equivalence between fibrations and pseudofunctors is implemented by *straightening* and *unstraightening* constructions.

Reviewer: Hirokazu Nishimura (Tsukuba)

MSC:

18D30 Fibered categories
18N10 2-categories, bicategories, double categories

Cited in **1** Document

Keywords:

Grothendieck fibration; two-sided fibration; profunctor

Full Text: DOI

References:

[1] Berger, C.; Kaufmann, R. M., Comprehensive factorisation systems, Tbilisi Math. J., 10, 3, 255-277 (2017) · Zbl 1397.18004

[2] Bourke, J.; Garner, R., Algebraic weak factorisation systems I: Accessible AWFS, J. Pure Appl. Algebra, 220, 1, 108-147 (2016) · Zbl 1327.18004

[3] Carboni, A.; Johnson, S.; Street, R.; Verity, D., Modulated bicategories, J. Pure Appl. Algebra, 94, 3, 229-282 (1994) · Zbl 0805.18002

[4] Fantechi, B.; Göttsche, L.; Illusie, L.; Kleiman, S. L.; Nitsure, N.; Vistoli, A., (Fundamental Algebraic Geometry. Fundamental Algebraic Geometry, Mathematical Surveys and Monographs, vol. 123 (2005), American Mathematical Society: American Mathematical Society Providence, RI), grothendieck's FGA explained · Zbl 1085.14001

[5] Gray, J. W., Fibred and cofibred categories, (Proc. Conf. Categorical Algebra (la Jolla, Calif., 1965) (1966), Springer: Springer New York), 21-83 · Zbl 0192.10701

[6] A. Grothendieck, Catégories fibrées et descente, seminaire de géométrie algébrique de l'Institute des Hautes Études Scientifiques (1961).

[7] Grothendieck, A., Technique de descente et théorèmes d'existence en géometrie algébrique. I. Généralités. Descente par

morphismes fidèlement plats, (Séminaire Bourbaki, Vol. 5 (1995), Soc. Math. France: Soc. Math. France Paris), pp. Exp. No. 190, 299-327

[8] Johnstone, P. T., (Sketches of an Elephant: A Topos Theory Compendium. Vol. 1. Sketches of an Elephant: A Topos Theory Compendium. Vol. 1, Oxford Logic Guides, vol. 43 (2002), The Clarendon Press, Oxford University Press: The Clarendon Press, Oxford University Press New York) · Zbl 1071.18001

[9] Johnstone, P. T.; Paré, R.; Wood, R.; Schumacher, D.; Wraith, G.; Rosebrugh, R., Indexed Categories and their Applications (1978), Springer

[10] Kelly, G. M., Elementary observations on 2-categorical limits, Bull. Aust. Math. Soc., 39, 2, 301-317 (1989) · Zbl 0657.18004

[11] Lawvere, F. W., Equality in hyperdoctrines and comprehension schema as an adjoint functor, Appl. Categ. Algebra, 17, 1-14 (1970) · Zbl 0234.18002

[12] Lurie, J., (Higher Topos Theory. Higher Topos Theory, Annals of Mathematics Studies, vol. 170 (2009), Princeton University Press: Princeton University Press Princeton, NJ) · Zbl 1175.18001

[13] Paré, R.; Schumacher, D., Abstract families and the adjoint functor theorems, (Indexed Categories and their Applications (1978), Springer), 1-125 · Zbl 0389.18002

[14] E. Riehl, D. Verity, Elements of \(\infty \)-category theory, draft textbook available from www.math.jhu.edu/ eriehl/elements.pdf (2019). · Zbl 1319.18005

[15] Street, R., Fibrations and Yoneda's lemma in a \(2\)-category, (Category Seminar (Proc. Sem., Sydney, 1972/1973). Category Seminar (Proc. Sem., Sydney, 1972/1973), Lecture Notes in Math., vol. 420 (1974), Springer: Springer Berlin), 104-133

[16] Street, R., Fibrations in bicategories, Cah. Topol. Géom. Différ., 21, 2, 111-160 (1980) · Zbl 0436.18005

[17] Street, R., Correction to: "Fibrations in bicategories" [Cahiers Topologie Géom. Différentielle 21 (1980), (2) 111-160; MR0574662 (81f:18028)], Cah. Topol. Géom. Différ. Catég., 28, 1, 53-56 (1987)

[18] Street, R.; Walters, R. F.C., The comprehensive factorization of a functor, Bull. Amer. Math. Soc., 79, 936-941 (1973) · Zbl 0274.18001

[19] Weber, M., Yoneda structures from 2-toposes, Appl. Categ. Struct., 15, 3, 259-323 (2007) · Zbl 1125.18001

This reference list is based on information provided by the publisher or from digital mathematics libraries. Its items are heuristically matched to zbMATH identifiers and may contain data conversion errors. It attempts to reflect the references listed in the original paper as accurately as possible without claiming the completeness or perfect precision of the matching.

© 2021 FIZ Karlsruhe GmbH