Heunen, Chris; **Lemay, Jean-Simon Pacaud**

Tensor-restriction categories. (English) Zbl 1464.18016

Theory Appl. Categ. 37, 635-670 (2021).

In a monoidal category such that subobjects of the tensor unit that are idempotents (called *subunits*) are closed under tensor products (such a monoidal category is called *firm*), we may think about the semilattice they form as a base space underlying the monoidal category, in which the tensor product provides methods to deal with partiality, restriction and support with respect to this base space. *Tensor topology* [*P. Enrique Moliner* et al., J. Pure Appl. Algebra 224, No. 10, Article ID 106378, 35 p. (2020; Zbl 1445.18010)] deals with objects and maps that are partially defined with respect to the tensor structure.

Restriction categories were introduced in [*J. R. B. Cockett* and *S. Lack*, Theor. Comput. Sci. 270, No. 1–2, 223–259 (2002; Zbl 0988.18003)] to deal with maps that are partially defined with respect to composition. For restriction categories, one can record the domain of definition of a morphism $f : A \to B$ in an endomorphism $\overline{f} : A \to A$ so that each object A has a space

$$\mathcal{O}(A) = \{e : A \to A \mid \overline{e} = e\}$$

underlying it, with respect to which there are methods to deal with partiality, restriction and support in terms of composition.

This paper aims to bring the two notions of partiality together. The authors introduce a construction turning a firm monoidal category $\boldsymbol{C}$ into a restriction category $\mathcal{S}[\boldsymbol{C}]$ in a functorial way and analyze which restriction categories arise this way to axiomatize them as *tensor-restriction categories*. It is shown that the known construction of taking the restriction-total maps turns a tensor-restriction category $\boldsymbol{X}$ into a firm monoidal category $\mathcal{T}[\boldsymbol{X}]$. We have

$$\mathcal{S}[\mathcal{T}[\boldsymbol{X}]] \simeq \boldsymbol{X}$$
$$\mathcal{T}[\mathcal{S}[\boldsymbol{C}]] \simeq \boldsymbol{C}$$

which gives an equivalence of categories.

Reviewer: Hirokazu Nishimura (Tsukuba)

MSC:

18M05 Monoidal categories, symmetric monoidal categories
18M30 String diagrams and graphical calculi

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