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Orthomodular-valued models for quantum set theory. (English) Zbl 1421.03026

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Quantum logic was introduced by *G. Birkhoff* and *J. von Neumann* [Ann. Math. (2) 37, 823–843 (1936; Zbl 0015.14603)]. The study of quantum set theory with values in the lattice $\mathcal{Q}$ of closed linear subspaces of a Hilbert space was initiated by *G. Takeuti* ["Quantum set theory", in: Current issues in quantum logic. New York-London: Plenum Press (1981; Zbl 0537.03044), p. 303–322]. Takeuti adopted the Sasaki arrow [*U. Sasaki*, J. Sci. Hiroshima Univ., Ser. A 17, 293–302 (1954; Zbl 0055.25902)] as implication. The author [J. Symb. Log. 72, No. 2, 625–648 (2007; Zbl 1124.03045)] established the transfer principle for Takeuti's quantum set theory.

Transfer Principle. For any Δ_0-formula $\phi(x_1,\dots,x_n)$ in the language of set theory in ZFC and for any element $u_1,\dots,u_n$ in the universe $V^{(\mathcal{Q})}$ with the $\mathcal{Q}$-valued truth value $\|\phi(u_1,\dots,u_n)\|$, we have

$$\|\phi(u_1,\dots,u_n)\| \geq \underline{\vee}(u_1,\dots,u_n)$$

where $\underline{\vee}(u_1,\dots,u_n)$ stands for the commutator.

This paper considers orthomodular-valued set theory as a generalization of Boolean-valued set theory and quantum set theory, defining a general class of possible implications including non-polynomially definable ones, with respect to which it is shown that the general transfer principle holds. Furthermore, the author determines all polynomially definable operations for which the general transfer principle holds.

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MSC:

03E70 Nonclassical and second-order set theories
03E75 Applications of set theory
03G12 Quantum logic
06C15 Complemented lattices, orthocomplemented lattices and posets
46L60 Applications of selfadjoint operator algebras to physics
81P10 Logical foundations of quantum mechanics; quantum logic (quantum-theoretic aspects)

Keywords:

quantum logic; quantum set theory; Boolean-valued models; forcing; transfer principle; orthomodular lattices; commutator; implication; von Neumann algebras

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