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A closed model structure on the category of weakly unital dg categories. II. (English)

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Theory Appl. Categ. 37, 388-417 (2021).

This paper provides a cofibrantly generated Quillen model structure on the category $\mathbb{C}_{dgwu}(k)$ of *M. Kontsevich* and *Y. Soibelman* weakly unital dg categories over a field k [Lect. Notes Phys. 757, 153–219 (2009; Zbl 1202.81120)]. The Quillen equivalence between the model category $\mathbb{C}_{dgwu}(k)$ and the the category $\mathbb{C}_{dg}(k)$ of unital dg categories over k with *G. Tabuada*'s [C. R., Math., Acad. Sci. Paris 340, No. 1, 15–19 (2005; Zbl 1060.18010)] model sturucture is also established. All of them were studied by the authors in the previous paper ["A Quillen model structure on the category of Kontsevich-Soibelman weakly unital dg categories", Preprint, arXiv:1907.07970] for $\mathbb{C}^0_{dgwu}(k)$ of small Kontsevich-Soibelman weakly unital dg categories over a field k abiding by an extra condtion

$$\mathrm{id}_X \circ \mathrm{id}_X = \mathrm{id}_X$$

for any object X in place of $\mathbb{C}_{dgwu}(k)$.

Reviewer: Hirokazu Nishimura (Tsukuba)

MSC:

18N40 Homotopical algebra, Quillen model categories, derivators
18G35 Chain complexes (category-theoretic aspects), dg categories

Keywords:

dg-category; closed model category; weak units

Full Text: Link

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