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Hjelmslev's model celebrating Juel's synthetic theory of surfaces. (English) Zbl 07334961

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This paper is concerned wth a unique surface in the model collection at the Department of Mathematical Sciences of the University of Copenhagen [https://www.math.ku.dk/bibliotek/arkivet/Models_at_the_Department_of_Mathematical_Sciences.pdf], which consists of nine plane polygonal figures soldered together. The paper explains the purpose of this unique model with a bit of digging. The clue to the enigma is an inscription on the surface reading "From J. Hjelmslev to C. Juel 25 Jan. 1925" in Danish. January 25th 1925 was the 70th birthday of the famous Danish geometer Christian Sophus Juel (1855–1935), the model being evidently a birthday gift from his collegue Johannes Hjelmslev (1873–1950), younger by one generation.

Juel graduated from the University of Copenhagen in 1879 as one of the first students of Hieronymus Georg Zeuthen (1839–1920), finishing his doctoral thesis "An introduction to the geometry of the imaginary line and the imaginary plane" in 1885. Hjelmslev also studied with Zeuthen, finishing his doctoral dissertation in 1897. The two mathematicians shared a synthetic and realist view of geometry. Juel's lifelong interest in the ideas of Karl Georg Christian von Staudt (1798–1867) [Geometrie der Lage. Korn, Nürnberg (1847)] began with his dissertation, culminating in his monograph [Zbl 0009.36704, JFM 60.0548.07]. Juel's research was centered around projective geometry, algebraic curves, polyhedra and surfaces of revolution from ovals. Hjelmslev's early papers on differential geometry of curves and surfaces [JFM 42.0595.04, JFM 45.0722.01] used purely synthetic definitions and methods inspired by Juel. He developed Juel's material view into a radical and systematic empirical approach to geometry, claiming finally that a systematic geometry of reality was superior to the Euclidean approach, not only didactically but also in practice and scientifically. He discovered the Hjelmslev transformation, a formale method for mapping an entire hyperbolic plane into a circle with a finite radius.

A *cubic surface*, an algebraic surface of the third degree, is defined by a polynomial in x, y and z in degree three. The theory of straight lines on such surfaces began in 1849 with Cayley and Salmon [Camb. Dublin Math. J. **4**, 118-138 (1849)], who showed that every nonsingular cubic surface contains exactly 27 straight lines, which holds only if one considers surfaces in *complex* projective space while a cubic surface typically contains less than 27 lines in *real* space. Ludwig Schlafli [Philos. Trans. Royal Soc. Lond. **6**, 201-241 (1863)] has shown that a cubic surface would contain 3, 7, 15 or 27 real lines, which Juel treated with purely geometric methods. Juel's main result [JFM 42.0576.02, p.1219] is that a simple and non-ruled surface of the third order without double points will always contain 3 or 7 or 15 or 21 straight lines. It was unfortunate that the paper contains a typographical error, which was that the number 21 should have been the magical number 27. The true nature of the model is only revealed in a paper written by Hjelmslev in a Festschrift to Juel on his 70th birthday in [Matermatisk Tidsskrift B **1925**, 1-107 (1925)]. Since polygons are in a sense simpler than general curves, Hjelmslev in the Festschrift developed a rather Hilbertian axiomatic theory of polygons in the projective plane, generalizing the ideas to polyhedra in projective space. He particularly studied lines on surfaces of order three remaking, claiming that polyhedra of the third order present on the properties similar to Juel's elementary surfaces of the third order and furnish in a sense the simplest types of such surfaces [JFM 51.0437.02, p.73].

The author concludes that the model was made by Hjelmslev to illustrate the strength of Juel's generalization of the theorem on the number of lines on third order surfaces.

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