Cruttwell, Geoffrey; **Lemay, Jean-Simon Pacaud**; **Lucyshyn-Wright, Rory B. B.**
Integral and differential structure on the free C^∞-ring modality. (English. French summary)
Zbl 07340839
Cah. Topol. Géom. Différ. Catég. 62, No. 2, 116-176 (2021).

The first notion of integration in a differential category was introduced in [*T. Ehrhard*, Math. Struct. Comput. Sci. 28, No. 7, 995–1060 (2018; Zbl 1456.03097)] with the introduction of differential categories with antiderivatives. *J. R. B. Cockett* and the second author [Math. Struct. Comput. Sci. 29, No. 2, 243–308 (2019; Zbl 1408.18012)] have provided the full story of integral categories, calculus categories and differential categories with antiderivatives, presenting an integral category of polynomial functions where it was not at all clear from its definition that the formula for its deriving transformation could be generalized to yield an integral category of arbitrary smooth functions. The principal objective in this paper is to present an integral category structure on the free C^∞-ring monad that is compatible with the known differential structure.

The following results in the paper are also to be noticed.

- *R. Blute* et al. [Cah. Topol. Géom. Différ. Catég. 57, No. 4, 243–279 (2016; Zbl 1364.13026)] defined derivations for codiferential categories. It is shown in this paper that derivations in this general sense, when applied to the C^∞-ring considered here, correspond precisely to derivations of the Fermat theory of smooth functions in [*E. J. Dubuc* and *A. Kock*, Commun. Algebra 12, 1471–1531 (1984; Zbl 1254.51005)], which provides additional evidence that the Blute/Lucyshyn-Wright/O'Nei definition is the appropriate generalization of derivations in the context of codifferential categories.
- Although this key example of integral categories does not possess a *codereliction* [*R. F. Blute* et al., Appl. Categ. Struct. 28, No. 2, 171–235 (2020; Zbl 07181514); Math. Struct. Comput. Sci. 16, No. 6, 1049–1083 (2006; Zbl 1115.03092)], it does possess many significant features of codereliction.
- An integral category obeys certain Rota-Baxter axiom [*G. Baxter*, Pac. J. Math. 10, 731–742 (1960; Zbl 0095.12705); *G. C. Rota*, Bull. Am. Math. Soc. 75, 325–329 (1969; Zbl 0192.33801); Bull. Am. Math. Soc. 75, 330–334 (1969; Zbl 0319.05008)], so that free C^∞-rings as examples of integral categories are Rota-Baxter algebras.

Reviewer: Hirokazu Nishimura (Tsukuba)

MSC:

18F40	Synthetic differential geometry, tangent categories, differential categories
18M05	Monoidal categories, symmetric monoidal categories
18C15	Monads (= standard construction, triple or triad), algebras for monads, homology and derived functors for monads
26B12	Calculus of vector functions
13N15	Derivations and commutative rings
13N05	Modules of differentials
26B20	Integral formulas of real functions of several variables (Stokes, Gauss, Green, etc.)
03F52	Proof-theoretic aspects of linear logic and other substructural logics

Keywords:

differential categories; C-infinity rings; Rota-Baxter algebras; monads; algebra modalities; monoidal categories; derivations; Kähler differentials

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