Ghobadi, Aryan

Pivotal objects in monoidal categories and their Hopf monads. (English) Zbl 1458.18009

Theory Appl. Categ. 37, 287-325 (2021).

If $\mathcal{C}$ is a monoidal category and P an object in $\mathcal{C}$, one can construct a category of P-*intertwined* objects whose objects are pairs (A, σ) with A an object of $\mathcal{C}$ and

$$\sigma : A \otimes P \to P \otimes A$$

an invertible morphism. This category naturally lifts the monoidal structure of $\mathcal{C}$, in a similar fashion to the monoidal structure of the center of $\mathcal{C}$. If P is pivotal and a dual Q of P is chosen, then any invertible morphism σ induces two Q-*intertwinnings* on the object A as follows.

$$(\mathrm{ev} \otimes A \otimes Q)\,(Q \otimes \sigma \otimes Q)\,(Q \otimes A \otimes \mathrm{coev}) : Q \otimes A \to A \otimes Q$$
$$(Q \otimes A \otimes \underline{\mathrm{ev}})\left(Q \otimes \sigma^{-1} \otimes Q\right)(\underline{\mathrm{coev}} \otimes A \otimes Q) : A \otimes Q \to Q \otimes A$$

In order to obtain a closed monoidal category $\mathcal{C}\,(P, Q)$, whose monoidal structure is described in Theorem 4.1, one must restrict to the subcategory of pairs for which these induced Q-intertwinnings are inverses to each other. The principal objective in this paper is to show that $\mathcal{C}\,(P, Q)$ lifts left and right closed structures on $\mathcal{C}$, when they exist (Theorem 4.2 and Corollary 4.3). The construction is also discussed in the cases when $\mathcal{C}$ is rigid (Corollary 4.4), and when $\mathcal{C}$ has a pivotal structure compatible with P and Q (Theorem 4.5).

This review is closed with a list of significant remarks.

- Pivotal categories were introduced in [*D. N. Yetter*, Contemp. Math. 134, 325–349 (1992; Zbl 0812.18005)] under the name of sovereign categories, their study being vital for topological field theories [*V. Turaev* and *A. Virelizier*, Monoidal categories and topological field theory. Basel: Birkhäuser/Springer (2017; Zbl 1423.18001)]. However, the study of individual objects in a monoidal category having isomorphic left and right duals is indigenous to this paper. In [*K. Shimizu*, J. Algebra 428, 357–402 (2015; Zbl 1394.18004)] Shimizu introduced the pivotal cover of a rigid monoidal category in connection with Frobenius-Schur indicators discussed in [*S.-H. Ng* and *P. Schauenburg*, Adv. Math. 211, No. 1, 34–71 (2007; Zbl 1138.16017)]. This paper introduces the pivotal cover $\mathcal{C}^{piv}$ of an arbitrary monoidal category $\mathcal{C}$ in Definition 3.5 from a distinct viewpoint arising in [*A. Ghobadi*, "Hopf algebroids, bimodule connections and noncommutative geometry", Preprint, `arXiv:2001.08673`]. The pivotal cover of a monoidal category is of pivotal pairs as objects and suitable pivotal morphisms between them, so that any strong monoidal functor from a pivotal category to the original category factors through the pivotal cover (Theorem 3.7). The construction of Shimizu requires all objects to have left duals and a choice of distinguished left dual for each object, while the construction in this paper gets rid of these issues by taking pivotal pairs as objects of $\mathcal{C}^{piv}$.
- Duals of monoidal functors were introduced in [*S. Majid*, in: Category theory 1991. Proceedings of an international summer category theory meeting, held in Montréal, Québec, Canada, June 23-30, 1991. Providence, RI: American Mathematical Society. 329–343 (1992; Zbl 0784.18004)] as a generalization of the center of a monoidal category. Tannaka-Krein reconstruction for Hopf monads, as described by *K. Shimizu* ["Tannaka theory and the FRT construction over non-commutative algebras", Preprint, `arXiv:1912.13160`], takes the data of a strong monoidal functor, producing a Hopf monad whose module category recovers the dual of the monoidal functor. §6.3 shows that pivotal pairs in monoidal category correspond to strict monoidal functors from the smallest pivotal category Piv (1) into the category, from which one can recover the same Hopf monad structure from the approach of Tannaka-Krein reconstruction. An additional result concerning the pivotal structure of the dual monoidal category is also provided.
- The Hopf monads constructed in this paper should be the simplest examples generalizing the theory of Hopf algebroids with bijective antipodes [*G. Böhm* and *K. Szlachányi*, J. Algebra 274, No. 2,

708–750 (2004; Zbl 1080.16035)] to the monoidal setting. A categorical characterization of the antipode for Hopf monads is essential. While antipodes for Hopf monads have been discussed in both the rigid setting [*A. Bruguières* and *A. Virelizier*, Adv. Math. 215, No. 2, 679–733 (2007; Zbl 1168.18002)] and the general setting [*G. Böhm* and *S. Lack*, J. Pure Appl. Algebra 220, No. 6, 2177–2213 (2016; Zbl 1353.18002)], neither cover the case of Hopf algebroids over noncommutative bases, which admit bijective antipodes. When restricted to the category of bimodules over an arbitrary algebra, Hopf monads in this paper correspond to Hopf algebroids which in fact admit involutory antipodes (Example 5.9).

- It is shown in Theorem 5.4 that the Hopf monad constructed is augmented iff the pair P and Q is a pivotal pair in the center of the monoidal category, so that the braided Hopf algebra corresponding to every pivotal pair in the center of the monoidal category is constructed by putting the theory of augmented Hopf monads [*A. Bruguières* et al., Adv. Math. 227, No. 2, 745–800 (2011; Zbl 1233.18002)] to use.
- The applications of this work are scattered throughout the article. §3 observes that Frobenius bimodules [*L. Kadison*, New examples of Frobenius extensions. Providence, RI: American Mathematical Society (1999; Zbl 0929.16036)] and ambidextrous adjunctions are examples of pivotal objects in monoidal categories. The author ["Unravelling the complex behavior of Mrk 421 with simultaneous X-ray and VHE observations during an extreme flaring activity in April 2013", Preprint, `arXiv:2001.08678`, §4.2] has presented several other examples, in the format of differential calculi, where the space of 1-forms is a pivotal object in the monoidal category of bimodules over the algebra of noncommutative functions. This setting is briefly explained in Example 3.3. The Hopf monad constructed in this case becomes a Hopf algebroid (Example 5.9) which is a subalgebra of the Hopf algebroid of differential operators in the above setting. Additionally, to construct the sheaf of differential operators in this setting, the wedge product between the space of 1-forms and 2-forms is required to be a pivotal morphism. A direct consequence of Example 3.6 is that any bicovariant calculus over a Hopf algebra obeys this condition.

Reviewer: Hirokazu Nishimura (Tsukuba)

MSC:

18M05 Monoidal categories, symmetric monoidal categories
18D15 Closed categories (closed monoidal and Cartesian closed categories, etc.)
16T99 Hopf algebras, quantum groups and related topics
18C20 Eilenberg-Moore and Kleisli constructions for monads

Keywords:

monoidal category; closed category; pivotal category; Hopf monad; Hopf algebra; tensor category

Full Text: Link

References:

Beggs und Majid 2020Beggs, Edwin J. ;Majid, Shahn:Quantum Riemannian Geometry. Bd. 355. Springer Nature, 2020

B¨ohm 2018B¨ohm, Gabriella:Hopf Algebras and Their Generalizations from a Category Theoretical Point of View. Bd. 2226. Springer, 2018

B¨ohm und Lack 2016B¨ohm, Gabriella ;Lack, Stephen: Hopf comonads on naturally Frobenius map-monoidales. In:Journal of Pure and Applied Algebra220 (2016), Nr. 6, S. 2177-2213 · Zbl 1353.18002

B¨ohm und Szlach´anyi 2004B¨ohm, Gabriella ;Szlach´anyi, Korn´el: Hopf algebroids with bijective antipodes: axioms, integrals, and duals. In:Journal of Algebra274 (2004), Nr. 2, S. 708-750

Bruguieres u. a. 2011Bruguieres, Alain ;Lack, Steve ;Virelizier, Alexis: Hopf monads on monoidal categories. In:Advances in Mathematics227 (2011), Nr. 2, S. 745-800

Brugui'eres und Natale 2011Brugui'eres, Alain ;Natale, Sonia: Exact sequences of tensor categories. In:International Mathematics Research Notices2011 (2011), Nr. 24, S. 5644-5705 · Zbl 1250.18005

Bruguieres und Virelizier 2007Bruguieres, Alain ;Virelizier, Alexis: Hopf monads. In:Advances in Mathematics215 (2007), Nr. 2, S. 679-733

Brugui'eres und Virelizier 2012Brugui'eres, Alain ;Virelizier, Alexis: Quantum double of Hopf monads and categorical centers. In:Transactions of the American Mathematical Society364 (2012), Nr. 3, S. 1225-1279

Etingof u. a. 2016Etingof, Pavel ;Gelaki, Shlomo ;Nikshych, Dmitri ;Ostrik, Victor:Tensor categories. Bd. 205. American Mathematical Soc., 2016 · Zbl 1365.18001

Ghobadi 2020Ghobadi, Aryan: Hopf Algebroids, Bimodule Connections and Noncommutative Geometry. In:arXiv preprint arXiv:2001.08673(2020)

Kadison 1999Kadison, Lars:New examples of Frobenius extensions. Bd. 14. American Mathematical Soc., 1999

Lauda 2006Lauda, Aaron D.:Frobenius algebras and ambidextrous adjunctions. In:Theory and Applications of Categories16 (2006), Nr. 4, S. 84-122 · Zbl 1092.18003

Mac Lane 2013Mac Lane, Saunders:Categories for the working mathematician. Bd. 5. Springer Science, 2013

Majid 1992Majid, Shahn: Braided groups and duals of monoidal categories.In: Canad. Math. Soc. Conf. ProcBd. 13, 1992, S. 329-343 · Zbl 0784.18004

[1] 324ARYAN GHOBADI

Majid 1994Majid, Shahn: Algebras and Hopf algebras in braided categories, Advances in Hopf algebras (Chicago, IL, 1992), 55-105. In:Lecture Notes in Pure and Appl. Math158 (1994) · Zbl 0812.18004

Manin 1988Manin, Yu I.: Quantum groups and non-commutative geometry, Publ. In:Centre de Recherches Math., Univ. de Montreal(1988) · Zbl 0724.17006

Moerdijk 2002Moerdijk, Ieke: Monads on tensor categories. In:Journal of Pure and Applied Algebra168 (2002), Nr. 2-3, S. 189-208

Ng und Schauenburg 2007Ng, Siu-Hung ;Schauenburg, Peter: Frobenius-Schur indicators and exponents of spherical categories. In:Advances in Mathematics211 (2007), Nr. 1, S. 34-71 · Zbl 1138.16017

Schauenburg 2000Schauenburg, Peter: Duals and doubles of quantum groupoids (×R-Hopf algebras).In:New Trends in Hopf Algebra Theory: Proceedings of the Colloquium on Quantum Groups and Hopf Algebras, La Falda, Sierras de C´ordoba, Argentina, August 9-13, 1999Bd. 267 American Mathematical Soc. (Veranst.), 2000, S. 273

Schauenburg 2017Schauenburg, Peter: The dual and the double of a Hopf algebroid are Hopf algebroids. In:Applied Categorical Structures25 (2017), Nr. 1, S. 147-154 · Zbl 1384.16023

Shimizu 2015Shimizu, Kenichi: The pivotal cover and Frobenius-Schur indicators. In:Journal of Algebra428 (2015), S. 357-402 · Zbl 1394.18004

Shimizu 2019Shimizu, Kenichi:Tannaka theory and the FRT construction over non-commutative algebras. In:arXiv preprint arXiv:1912.13160(2019)

ˇSkoda 2003Skodaˇ, Zoran: Localizations for construction of quantum coset spaces. In:Banach Center Publications1 (2003), Nr. 61, S. 265-298 · Zbl 1070.14002

Szlach´anyi 2003Szlach´anyi, Korn´el: The monoidal Eilenberg-Moore construction and bialgebroids.In:Journal of Pure and Applied Algebra182 (2003), Nr. 2-3, S. 287-315 · Zbl 1025.18005

Turaev und Virelizier 2017Turaev, Vladimir G. ;Virelizier, Alexis:Monoidal categories and topological field theory. Bd. 322. Springer, 2017 · Zbl 1423.18001

Woronowicz 1989Woronowicz, Stanis law L: Differential calculus on compact matrix pseudogroups (quantum groups). In:Communications in Mathematical Physics 122 (1989), Nr. 1, S. 125-170

Yetter 1992Yetter, David N.: Framed tangles and a theorem of Deligne on braided deformations of Tannakian categories. In:Contemp. Math134 (1992), S. 325-350 · Zbl 0812.18005

[2] School of Mathematics, Queen Mary University of London

[3] Mile End Road, London, E1 4NS, United Kingdom PIVOTAL OBJECTS IN MONOIDAL CATEGORIES AND THEIR HOPF MONADS325

[4] Email:a.ghobadi@qmul.ac.uk

[22] Assistant TEX editor.Gavin Seal, Ecole Polytechnique F ´ed ´erale de Lausanne:
[23] gavin seal@fastmail.fm
[24] Transmitting editors.
[25] Clemens Berger, Universit ´e de Nice-Sophia Antipolis:cberger@math.unice.fr
[26] Julie Bergner, University of Virginia:jeb2md (at) virginia.edu
[27] Richard Blute, Universit ´e d' Ottawa:rblute@uottawa.ca
[28] Gabriella B¨ohm, Wigner Research Centre for Physics:bohm.gabriella (at) wigner.mta.hu
[29] Valeria de Paiva: Nuance Communications Inc:valeria.depaiva@gmail.com
[30] Richard Garner, Macquarie University:richard.garner@mq.edu.au
[31] Ezra Getzler, Northwestern University:getzler (at) northwestern(dot)edu
[32] Kathryn Hess, Ecole Polytechnique F ´ed ´erale de Lausanne:kathryn.hess@epfl.ch
[33] Dirk Hofmann, Universidade de Aveiro:dirk@ua.pt
[34] Pieter Hofstra, Universit ´e d' Ottawa:phofstra (at) uottawa.ca
[35] Anders Kock, University of Aarhus:kock@math.au.dk
[36] Joachim Kock, Universitat Aut'onoma de Barcelona:kock (at) mat.uab.cat
[37] Stephen Lack, Macquarie University:steve.lack@mq.edu.au
[38] Tom Leinster, University of Edinburgh:Tom.Leinster@ed.ac.uk
[39] Matias Menni, Conicet and Universidad Nacional de La Plata, Argentina:matias.menni@gmail.com
[40] Ieke Moerdijk, Utrecht University:i.moerdijk@uu.nl
[41] Susan Niefield, Union College:niefiels@union.edu
[42] Kate Ponto, University of Kentucky:kate.ponto (at) uky.edu
[43] Robert Rosebrugh, Mount Allison University:rrosebrugh@mta.ca
[44] Jiri Rosicky, Masaryk University:rosicky@math.muni.cz
[45] Giuseppe Rosolini, Universit'a di Genova:rosolini@disi.unige.it
[46] Alex Simpson, University of Ljubljana:Alex.Simpson@fmf.uni-lj.si
[47] James Stasheff, University of North Carolina:jds@math.upenn.edu
[48] Ross Street, Macquarie University:ross.street@mq.edu.au