Alonderis, R.; **Pliuškevičius, R.**; **Pliuškevičienė, A.**; **Giedra, H.**
Loop-type sequent calculi for temporal logic. (English) Zbl 07311961
J. Autom. Reasoning 64, No. 8, 1663-1684 (2020).

Loop-type sequent calculi were first considered in [*P. Wolper*, Log. Anal., Nouv. Sér. 28, 119–136 (1985; Zbl 0585.03008)]. observing that some global constraints (*loops*) must be detected on branches to identify a tree as a proof. This approach uses loops not only for closing failed proof trees as in standard cases but also for playing the role of axioms. The loop-type sequent calculi for **BDI** (Beliefs, Desires, Intentions) logic based on **CTL** were presened in [*N. Naoyuki* et al., Electron. Notes Theor. Comput. Sci. 70, No. 5, 140–152 (2002; Zbl 1270.68286)]. Loop-type sequent calculi for temporal, mutual belief and dynamic logics were considered in [*A. Pliuškevičienė*, Liet. Mat. Rink. 41, 413–420 (2001; Zbl 1043.03516); *R. Pliuškevičius*, Lect. Notes Comput. Sci. 711, 640–649 (1993; Zbl 0925.03107); Lect. Notes Comput. Sci. 713, 289–300 (1993; Zbl 0793.03014); J. Math. Sci., New York 87, No. 1, 1 (1995; Zbl 0928.03021); translation from Zap. Nauchn. Semin. POMI 220, 123–144 (1995); Lect. Notes Comput. Sci. 2083, 107–120 (2001; Zbl 0988.03018); *R. Pliuškevičius* and *A. Pliuškevičienė*, Lect. Notes Comput. Sci. 3900, 112–128 (2006; Zbl 1235.03050); Lect. Notes Comput. Sci. 735, 299–311 (1993); J. Autom. Reason. 13, 391–407 (1994)].

The principal objective in this paper is to give a loop-type sequent calculus $\mathbf{G}_{\mathrm{L}}\mathbf{T}$ for propositional linear temporal logic ($PLTL$) with temporal operators *next* $\bigcirc$ and *henceforth always* $\Box$, denoted by $PLTL^{n,a}$, which is shown to be sound and complete. The sequent calculus $\mathbf{G}_{\mathrm{L}}\mathbf{T}^{\mathcal{U}}$ for $PLTL^{n,u}$ obtained from $PLTL^{n,a}$ by adding the temporal operator *until* is also considered.

Construction of sound and complete cut-free sequent calculi for $PLTL$ is utmost problematic, which has to do with the induction principle

$$(\phi \wedge \Box(\phi \supset \bigcirc\phi)) \supset \Box\phi$$

Some considerations on cut-free sequent calculi for $PLTL$ in the literature go as follows:

- Infinitary sequent calculi with the ω-type induction rule

$$\frac{\Gamma \to \Delta, \phi \quad \Gamma \to \Delta, \bigcirc\phi \quad \ldots \quad \Gamma \to \Delta, \overbrace{\bigcirc\ldots\bigcirc}^{n}\phi \quad \ldots}{\Gamma \to \Delta, \Box\phi} \ (\to \Box_\omega)$$

  were considered in [*G. Sundholm*, Theoria 43, 47–51 (1977; Zbl 0364.02011); Bull. Sect. Logic, Pol. Acad. Sci. 6, 70–73 (1977; Zbl 0403.03011)].

- Calculi with an invariant-like rule

$$\frac{\Gamma \to \Delta, I \quad I \to \bigcirc I \quad I \to \phi}{\Gamma \to \Delta, \Box\phi} \ (\to \Box_{\mathrm{I}})$$

  were considered in [*B. Paech*, Lect. Notes Comput. Sci. 385, 240–253 (1989; Zbl 0712.03015)].

996; Zbl 0872.68171) gave a cut-free calculus **LB** even for the first-order temporal logic with the weak induction principle

$$(\phi \wedge \bigcirc\Box\phi) \supset \Box\phi$$

in place of the above full one.

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MSC:

03F03 Proof theory, general (including proof-theoretic semantics)
03B44 Temporal logic

Keywords:

temporal logic; sequent calculus; derivation loops

Software:

TTM

Full Text: DOI

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