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Lo Monaco, Giulio

Dependent products and 1-inaccessible universes. (English) Zbl 07323980

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The interaction between intuitionistic type theory and chains of universes has already been investigated, e.g., in [*E. Palmgren*, in: Twenty-five years of constructive type theory. Proceedings of a congress, Venice, Italy, Ocober 19–21, 1995. Oxford: Clarendon Press. 191–204 (1998; Zbl 0930.03090); *M. Rathjen* et al., Ann. Pure Appl. Logic 94, No. 1–3, 181–200 (1998; Zbl 0926.03074)], leading to a proof-theoretic study of type theory with universes in [*M. Rathjen*, Arch. Math. Logic 39, No. 1, 1–39 (2000; Zbl 0953.03065); Arch. Math. Logic 40, No. 3, 207–233 (2001; Zbl 0990.03048)]. This paper is to be considered part of the same stream of investigations in a somewhat complementary fashion, i.e., category-theoretic rather than proof-theoretic and semantic rather than syntactic.

The principal objective in this paper is to find the precise relationship between the notion of geometric or Grothendieck ∞-topos on the one hand and that of elementary ∞-topos as proposed by Mike Shulman in terms of the assumptions on our set-theoretical foundations. The author would like to add extra axioms positing the existence of internal universes closed under suitable operations. One possibility is to require that every family of objects live in a *weak* universe closed under finite limits, finite colimits and dependent sums, while a stronger possibility is to require that every family of objects live in a *strong* universe closed also under dependent products. The main results of the paper are

- Every geometric ∞-topos is an elementary ∞-topos with weak universes (Theorem 3.7);
- The statement that every geometric ∞-topos is an elementary ∞-topos with strong universes is equivalent to a large cardinal axiom (Theorem 3.13).

Reviewer: Hirokazu Nishimura (Tsukuba)

MSC:

03E55 Large cardinals
18N60 $(\infty, 1)$-categories (quasi-categories, Segal spaces, etc.); ∞-topoi, stable ∞-categories

Keywords:

higher categories; higher toposes; elementary higher toposes; Grothendieck universes; large cardinals; dependent products; dependent sums; classifiers; generic morphisms

Full Text: Link

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