Chen, Bang-Yen

g-Steiner, co-Steiner and normal points of bounded Euclidean submanifolds. (English)

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The Steiner point, aka the Steiner curvature centroid, was originally defined to be the geometric centroid of the system obtained by placing a mass equal to the magnitude of the exterior angle at each vertex of a triangle, being introduced by Jakob Steiner (1976–1863), a Swiss mathematician [ERAM 021.0662cj]. *H. Flanders* [Mathematika 13, 181–188 (1966; Zbl 0146.17502)] established that, for an even-dimensional convex closed hypersurface M^n in a Euclidean $(n+1)$-space $\mathbb{E}^{n+1}$, the Steiner point $s(M^n)$ is to be defined as

$$s(M^n) = \frac{1}{c_n} \int_{p \in M^n} \boldsymbol{x} K(p)\, dv$$

where $\boldsymbol{x}$ denotes the position vector field of M^n in $\mathbb{E}^{n+1}$, dv is the volume element of M^n, and $K(p)$ denotes the Gauss-Kronecker curvature of M^n at a point $p \in M^n$.

It is well known that the Steiner point by the hand of Flanders abides by the following properties [*G. C. Shephard*, Can. J. Math. 18, 1294–1300 (1966; Zbl 0145.42801); J. Lond. Math. Soc. 43, 439–444 (1968; Zbl 0162.25801)].

- For any similar transformation a, we have
$$s(aM^n) = as(M^n).$$
- For any constant vector $c \in \mathbb{E}^{n+1}$, we have
$$s(M^n + c) = s(M^n) + c.$$
- $s(M^n)$ is a continuous function of M^n.
- If $\dim M^n$ is positive, then $s(M^n)$ is a relative interior point of M^n.

This paper aims

- to extend the notion of Steiner points to the notion of g-Steiner points for bounded Euclidean submanifolds with arbitrary codimension,
- to introduce the notions of co-Steiner and normal points for bounded Euclidean submanifolds via the notion of G-total curvature [*B.-Y. Chen*, J. Differ. Geom. 7, 371–391 (1972; Zbl 0274.53060)],
- to establish several fundamental properties for such points, and
- to establish some links between g-Steiner, co-Steiner and normal points.

Reviewer: Hirokazu Nishimura (Tsukuba)

MSC:

52A20 Convex sets in n dimensions (including convex hypersurfaces)
53C40 Global submanifolds
52A39 Mixed volumes and related topics in convex geometry

Keywords:

Steiner point; g-Steiner point; co-Steiner point; normal point; g-total curvature; Gauss-Kronecker curvature

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