Tull, Sean

A categorical reconstruction of quantum theory. (English) Zbl 1456.18014

Log. Methods Comput. Sci. 16, No. 1, Paper No. 4, 39 p. (2020).

This paper lies in the long-standing tradition of *reconstruction theorems*. To mention a notable few, we have

- The so-called *Veblen-Young theorem* [*O. Veblen* and *J. W. Young*, Am. J. Math. 30, 347–380 (1908; JFM 39.0606.01); Bull. Sci. Math., II. Sér. 44, 105–112 (1920; JFM 47.0582.08); Projective Geometry. Vol. I. Boston and London: Ginn and Comp (1910; JFM 41.0606.06); Projective geometry. Vol. I, II. New York-Toronto-London: Blaisdell Publishing Company (1965; Zbl 0127.37604)] claims that a *projective space* of dimension at least 3 can be constructed as the projective space associated to a vector space over a division ring. Geometry of linear subspaces of a projective space or *projective geometry* in short was axiomatized lattice-theoretically as finite-dimensional complemented modular lattices. *G. Birkhoff* [Lattice theory. New York: American Mathematical Society (AMS) (1940; Zbl 0063.00402)] has shown that every complemented modular lattice of finite dimension is the direct product of lattices associated with projective geometries of finite dimension. *J. von Neumann* [Proc. Natl. Acad. Sci. USA 22, 92–100 (1936; Zbl 0014.22307); Continuous geometry. Princeton, N.J.: Princeton University Press (1960; Zbl 0171.28003)] generalized these considerations to continuous geometry under the name of *coordination theorems*.
- Quantum mechanics was formulated by orthomodular lattices. It was *C. Piron* [Helv. Phys. Acta 37, 439–468 (1964; Zbl 0141.23204)] who succeeded in establishing the coordination theorem. See Theorem 7.44 in [*V. S. Varadarajan*, Geometry of quantum theory. Vol. I. Princeton, N.J.-Toronto-London-Melbourne: D. van Nostrand Company, Inc. (1968; Zbl 0155.56802)] for its details.
- The category of modules was axiomatized categorically as *abelian categories*. The so-called *Freyd-Mitchell embedding theorem* [*P. Freyd*, Abelian categories. An introduction to the theory of functors. New York-Evanston-London: Harper and Row, Publishers (1964; Zbl 0121.02103); *B. Mitchell*, Theory of categories. New York and London: Academic Press (1965; Zbl 0136.00604)] claims that every abelian category is a full subcategory of a category of modules over some ring R and that the embedding is an exact functor.
- Grothendieck toposes were axiomatized categorically as *elementary toposes* by Lawvere and Tierney during the year 1969–1970. It was *J. Giraud* [Cohomologie non abélienne. Berlin-Heidelberg-New York: Springer-Verlag (1971; Zbl 0226.14011)] who succeeded in categorically characterizing elementary toposes for which one can reconstruct Grothendieck toposes.

This paper presents a genuinely categorical formalism of quantum mechanics based on dagger categories, establishing its coordination theorem to recover something like the standard Hilber-space formalism [*D. Hilbert* et al., Math. Ann. 98, 1–30 (1927; JFM 53.0849.03)].

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MSC:

18M40	Dagger categories, categorical quantum mechanics
81P10	Logical foundations of quantum mechanics; quantum logic (quantum-theoretic aspects)
03B70	Logic in computer science
81P05	General and philosophical questions in quantum theory
81P68	Quantum computation
03G12	Quantum logic

Keywords:

quantum reconstruction; dagger compact category; purification; dagger kernels; phased biproduct

Full Text: arXiv

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