多変量判別関数値の分布を手がかりとした身体的発育発達の検討

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A Study on Physical Growth and Development through investigating in the Distributions of Multivariate Discriminant Function

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I. Introduction

Most of studies on physical growth and development have been carried out on such way that the changing tendencies of means are investigated in various physical growth and developmental variables with the longitudinal or the cross-sectional data. The mean is one of the statistical values representing the group investigated, so the investigation in the changing tendency of mean refers to the investigation in the changing tendency of group position on a certain scale due to the age or time change. In order to understand the group from the statistical point of view, the two parameters or constants are necessary if the distribution of data or attributes can be assumed as normal. They are mean and variance. Mean is, as mentioned precedingly, the representative value of group, which determines the location or position of the given group on a certain scale, and the variance is the one which shows the degree of individual difference within the given group. Thus, it may be inferred that the investigation in the physical growth and development only with the changes of means can give only some limited informations about them. More informations may be remained, because the individual differences are missed.

For statistical treatment of physical growth and developmental data, more informations can be obtained through investigating in the changing tendency of means due to age changes with taking the variance as the within group dispersion into consideration. For this, the standard deviations have been plotted very often on the growth and developmental curves as corresponding to each mean. This is one of the ways to compensate the shortage of informations that only means can give, but it is still difficult to guess what the standard deviation as the within group dispersion may suggest. That is, even if the age elapsed is one it can be reasonably assumed and actually observed that there are many subjects whose physical fitness level is not different but who belong to the different age or grade group. The degree of individual difference within group can give some sort of characteristics of group. For example, the following may be reasonably assumed; the degree of the within group individual difference may increase when the growth and development of a given physical attribute become vigorous and decrease at the reverse situation.[*3] Such information can not be obtained by only mean values. Thus, it is one of the purposes of this study to investigate in the physical growth and development with

taking the mean and the variance as the within group dispersion into consideration simultaneously.

The informations induced from investigating in the growth and developmental change of each measure instead of physical ability per se are also limited for assessing the actual physical growth and development, because the human growth and development should be evaluated as a whole and a test item can not always measure the chosen abilty. So it is no appropriate way to investigate in the changing trend of the test results due to the age increase for the study on the growth and development of such ability as physical fitness, motor ability and so on.[*4] Therefore, in this study, physical attributes and physical abilities were estimated with some reasonable functions of the measured items, and then the growth and developmental trends of the estimated physical attributes and abilities were investigated through investigating in the changing trends of the derived function values as the ability score due to the age increase or grade increase.

Among several ways to estimate the ability with the test items, factor analysis, component analysis, regression analysis, and discriminant analysis are major ones. The first two methods; factor analysis and component analysis, emphasize the ability domains hypothesized to be measured by the used test variables, but the last one; discriminant analysis emphasizes how effectively the different groups can be classified with the given test variables, but it does not deal with what ability or attribute the resulted function can represent. For the practical purpose, it is much better that the derived discriminant function can be interpreted as a certain score representing the actual ability domain. Thus, if the discriminant analysis procedure is applied to the test variables which can be reasonably assumed to measure the identical ability domain, the derived discriminant function may represent the chosen ability domain. Such derived discriminant function may be comparable to a first centroid factor which the chosen test variables involve,[*2] so this discriminant function value may be comparable to the first centroid factor score which represents the ability that can be measured by the chosen test variables.

Based upon such a logical development, the discriminant analysis will be applied to the one group of test variables, which can be reasonably assumed to measure the identical ability, in order that the ability may be represented by a linear function of the test variables as a discriminant function and also the different groups may be more strictly classified in terms of the chosen ability domain. And such group classification may cause the difference in the chosen ability between the groups more accentuated

II. Methods

The children of two elementary schools have been tested with the following 25 physical fitness test items since 1977, and the students of one junior high school were tested with the same items 1978; vital capacity, grip strength (left and right hand grip), back strength, chinning (flexed arm hang), vertical jump, push-and pull-arm strength, leg muscular endurance (leg raise), 50 m dash, softball throw for distance, trunk flexion, trunk extension, shuttle run, side step, foot balance on beam, zigzag run, stature, sitting height, lower limb length, body weight, skinfold fat (upper arm and back), chest girth, and upper arm girth. These items were administered by the skilled testers of graduate students and author himself. The physical fitness test has been scheduled in May once a year as Tsukuba Growth and Development Study since 1976. Thus, the sample of elementary school consists of the semilongitudinal data but junior high school sample is provided by the cross sectional. The data of this mixed type are used as the sample for this study. The sample sizes are shown in table 1.

Table 1 Sample size for use

	grade	1	2	3	4	5	6	7	8	9
1977	boy	113	79	73	58	47				
	girl	77	70	67	60	57				
1978	boy		92	79	80	50	61	67	46	60
	girl		86	76	68	60	60	47	54	42
1979	boy			93	95	94	67			
	girl			105	82	78	69			
Total	boy	113	171	245	233	191	128	67	46	60
	girl	77	156	248	210	195	129	47	54	42

It can be reasonably assumed that all the variables but skinfold fat distribute as normal, so these 23 variables were classified into the following five ability domains; (1). physique domain: stature, sitting height, lower limb length, body weight, chest girth and upper arm girth, (2). muscular strength domain: grip strength (left and right hand grip), back strength, push-and pull-arm strength, (3). fundamental motor skill domain: vertical jump, 50 m dash, and softball throw for distance, (4). flexibility domain: trunk flexion and trunk extension, (5). coordination domain: shuttle run, side step, foot balance on beam, and zigzag run. These five ability domains are validated as the important ability areas to estimate physical fitness in general. However, physique, muscular strength and fundamental motor skill areas will be discussed in this paper.

Then, in order that the scores of these ability domains may be determined for each individual, the discriminant function was determined for each ability domain, respectively. For this, it may be appropriate to apply such idea as making the within group dispersion less and the among group dispersion larger as much as possible, because the difference in physical fitness between different groups can be more accentuated. Therefore, let A be the among group variance covariance matrix, and W be the within group variance covariance matrix with q variables. They are as follows;

$$A = (a_{km}), \qquad a_{km} = \sum_{j=1}^{p} n_j (\bar{x}_k^j - \bar{x}_k)(\bar{x}_m^j - \bar{x}_m),$$

$$W = (w_{km}), \qquad w_{km} = \sum_{j=1}^{p} \sum_{i=1}^{n_j} (x_{ik}^j - \bar{x}_k^j)(x_{im}^j - \bar{x}_m^j),$$

$$k, m = 1, 2, 3, \ldots\ldots\ldots q$$

where a_{km} and w_{km} are (k, m) element of A and W matrix, x_{ik}^j the i th subject's value of k th variable in the j th group, $\bar{x}_k^j$ the mean value of the k th variable of the j th group, $\bar{x}_k$ the grand mean of the k th variable, p the number of groups, and n_j the number of cases in the j th group.

Then, suppose that $Y = B' X$ satisfies the preceding assumption; making the within group variance less and the among group variance larger as much as possible, where $B' = (b_1, b_2, \ldots\ldots\ldots b_q)$; the coefficient vector for X, and $X' = (x_1, x_2, \ldots\ldots\ldots x_q)$; the independent variable vector; that is, the variables used to estimate the score of the chosen ability domain with the discriminant function. Thus,

the problem is to determine B' such that Y may satisfy the assumption. Then, the among group variance covariance matrix and the within group variance covariance matrix are expressed as $B'AB$ and $B'WB$ respectively. Then, the question is to determine B' so that $B'AB$ may be maximized and $B'WB$ minimized. Thus, let t be $(B'AB) / (B'WB)$. This question can be solved by maximizing $(B'AB - tB'WB)$. Therefore, the solution is as follows;

$$\partial t/\partial B = 2A - 2tWB = 0,$$
$$(A - tW)B = 0,$$
$$(W^{-1}A - tI)B = 0,$$

In order to determine B, B must not be null matrix, so $(W^{-1}A - tI)$ must be equal to null matrix. In other words, $|W^{-1}A - tI| = 0$ must be satisfied. This is a latent equation of $W^{-1}A$. Then, t is equal to $(B'AB) / (B'WB)$, so the eigen vector corresponding to the maximum t; maximum eigen value, can give the coefficient for each independent variable of appropriate function. This rationale is just straightforwardly derived from the theory of multivariate discriminant function. Therefore, as mentioned in the preceding, the multivariate discriminant function was used to estimate the individual score of the chosen ability domains.

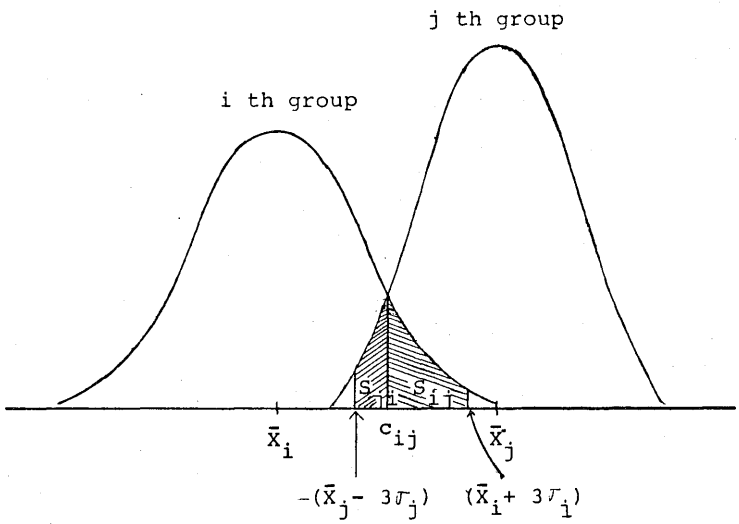
The discriminant function values are obtained for each individual in each ability area respectively. It can be reasonably assumed that these discriminant function values distribute as normal, because the discriminant function is a linear combination of the measures which are assumed to distribute as normal. The two parameters for normal distribution are mean and variance, and they are estimated with the unbiased statistic; that is, sample mean $\bar{x}_j$ and the unbiased variance $\hat{\sigma}_j^2 = n_j/(n_j - 1) S_j^2$, where S_j^2 stands for the sample variance of the j th group and n_j for the sample size. Then, under this assumption, the distribution of the chosen ability can be expressed by the following probability density function of normal;

$$f(x) = 1\Big/(\sqrt{2\pi}\ \sigma_j)\ \exp[-(x - \bar{x}_j)^2/2\hat{\sigma}_j^2]$$
$$j = 1, 2, \ldots\ldots\ldots p,$$

Then, these normal distribution curves are plotted in the identical coordinate for each grade group respectively. The configuration of distribution curves may show the growth and developmental tendency of the chosen ability, and these distributions are overlapped between different grade groups. This overlapping areas may be considered as the percentage of those who may be considered to belong to other group in reference with the chosen ability domain.

The criterion value for judging which group the one may belong to is the horizontal coordinate corresponding to the intersecting point of the two distribution curves of the given discriminant functions. Thus, suppose that there are two groups; i th group and j th group, and the mean $\bar{x}_i$ of i th group is larger than $\bar{x}_j$ of the j th group, and the criterion value is c_{ij}. If the one who actually belong to the i th group and show the larger value of the discriminant function than c_{ij}, he is judged to belong to the j th group in terms of some ability or attribute that the discriminant function may represent. Therefore, the area corresponding to the horizontal coordinates being greater than c_{ij} in the i th group may be assumed to show the percentage of those who actually belong to the i th group but may be judged

to belong to the j th group in terms of the used variables and/or ability that a linear combination of these used variables can represent. And the area corresponding to the horizontal coordinates being less than c_{ij} in the j th group may show the percentage of those who actually belong to the j the group but may be judged to belong to the i th group. Let this area be named the belonging percentage from i th to j th; shortly S_{ij}, and S_{ji} means the belonging percentage from j th to i th. S_{ij} was computed with such integral from c_{ij} to $(\bar{x}_i + 3\sigma_i)$ on the abscissa of distribution curve of i th group, and S_{ji} with such integral as from c_{ij} to $(\bar{x}_j - 3\sigma_j)$. Theoretically, the definition interval of normal distribution is from the negative infinitive to the positive infinitive, so the integral must be taken on t in this infinitive interval, but the area out of the interval from $(\bar{x} - 3\sigma)$ to $(\bar{x} + 3\sigma)$ is very small; 0.0027, so such small percentage can be neglected practically. Then, the actual integral is shown in the following figure;



$$S_{ij} = \frac{1}{\sqrt{2\pi}\,\sigma_i} \int_{c_{ij}}^{\bar{x}_i + 3\sigma_i} \exp\left[-\frac{(x - \bar{x}_i)^2}{2\sigma_i^2}\right] dx,$$

$$S_{ji} = \frac{1}{\sqrt{2\pi}\,\sigma_j} \int_{\bar{x}_j - 3\sigma_j}^{c_{ij}} \exp\left[-\frac{(x - \bar{x}_j)^2}{2\sigma_j^2}\right] dx,$$

These integrals are taken practically with the following integral of the unit normal probability density function through the transformation of x to $t = (x - \bar{x})/\sigma$.

$$S_{ij} = \frac{1}{\sqrt{2\pi}} \int_{\frac{c_{ij} - \bar{x}_i}{\sigma_i}}^{3} \exp\left(-\frac{t^2}{2}\right) dt,$$

$$S_{ji} = \frac{1}{\sqrt{2\pi}} \int_{-3}^{\frac{c_{ij} - \bar{x}_j}{\sigma_j}} \exp\left(-\frac{t^2}{2}\right) dt,$$

III. Results and Discussion

3.1. Physique domain

The independent variables used for estimating physique are as follows;

X_1 = stature,

X_2 = sitting height,

X_3 = lower limb length,

X_4 = body weight,

X_5 = chest girth, and

X_6 = upper arm girth (relaxed and bent rectangularly at elbow)

The coefficients for the independent variables of the derived discriminant functions are shown in table 2. And the means and standard deviations of discriminant function values as the unbiased statistic are shown in table 3.

Table 2. Coefficients for Variables of Discriminant Function; Physique

	Max. Eig. Value		X_1	X_2	X_3	X_4	X_5	X_6
Boy	6.84617	I	.12939	.26238	.54196	−.40533	.60324	−.30411
		II	.00803	.03535	.05241	−.03613	.06566	−.10947
		c	−8.50956					
Girl	7.28867	I	.82496	−.17623	.19335	−.01368	.27902	−.43233
		II	.05421	−.02411	.01634	−.00124	.02823	−.15379
		c	−5.39213					

I; Eigen vector corresponding to the maximum eigen value of $W^{-1}A$: Coefficients for the independent variables in the standard score form.
II; Coefficients for the independent variables in the raw score form.
c; Constant term in the discriminant function of the raw score form.

Table 3. Mean & SD of discriminant function value in each grade group (physique)

sex \ grade		1	2	3	4	5	6	7	8	9
BOY	MEAN	−1.08026	−0.79722	−0.44130	−0.21463	0.09456	0.36849	0.89442	1.18399	1.37456
	SD	0.25683	0.26007	0.28352	0.29765	0.37081	0.35003	0.38730	0.42270	0.40559
GIRL	MEAN	−0.92671	−0.64309	−0.33044	−0.09147	0.31530	0.47624	0.70973	0.95560	1.00885
	SD	0.23323	0.28625	0.29693	0.30922	0.30788	0.31368	0.32038	0.36234	0.33281

These means and variances as the unbiased statistic are put into the normal probability density function in each grade group respectively, and the normal distribution curves were plotted by a computer in each grade group respectively. These curves are shown in Fig. 1 for boys and in Fig. 2 for girls.

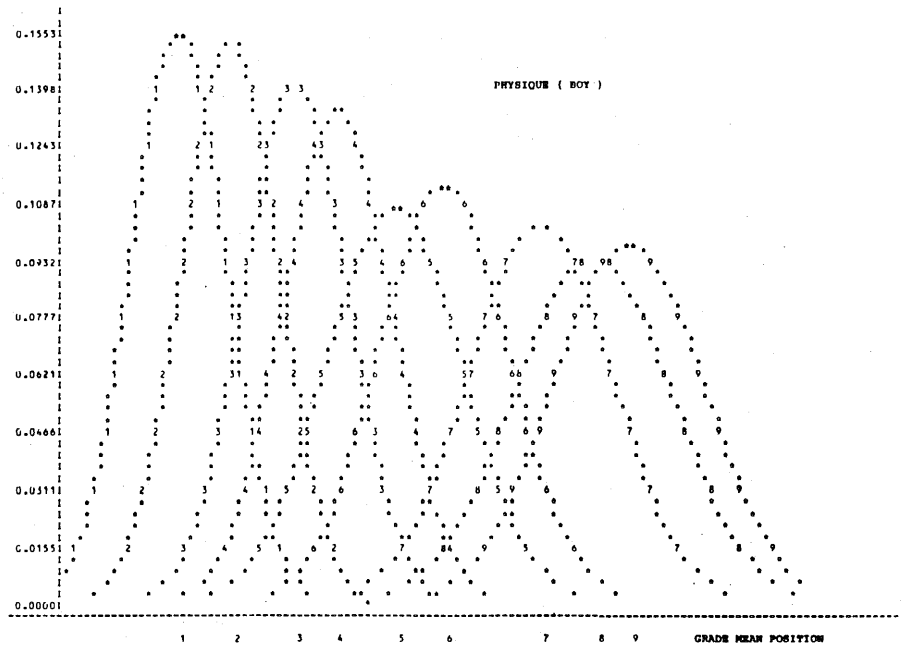


Fig. 1 Distribution of Discriminant Function Value in each Grade Group; Physique: Boy

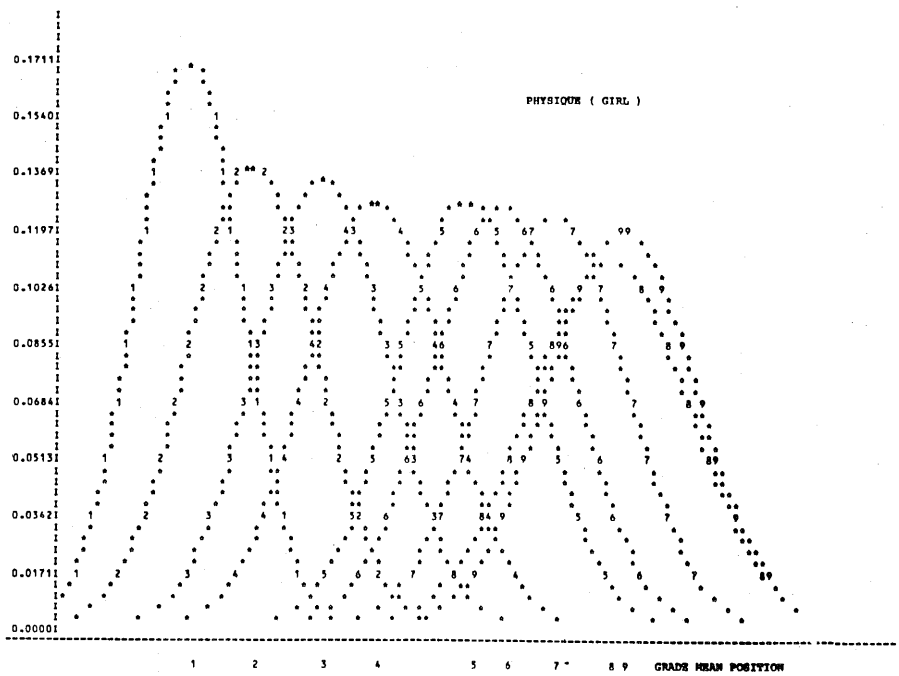


Fig. 2 Distribution of Discriminant Function Value in each Grade Group; Physique: Girl

The numbers described on the abscissa show the grade mean position; e. g., the means of the discriminant function values for each grade group. In boys the maximum ordinates of distribution curves; the ordinate corresponding to the grade mean, gradually declines and the range at the foot of distribution gradually is getting larger as the grade advances. This means that the individual difference within a grade group is getting larger, and also it may be inferred that the growth rate of physique is getting bigger, taking the larger distance in grade mean position between the adjoining grades. The largest distance in grade mean position is found between the 6th and the 7th, and this may mean the acceleration of physique growth, and the smaller distances between the 3rd and the 4th, the 5th and the 6th, and the 8th and 9th mean that a certain homogeneity in physique between them may be inferred, taking the larger amount of the belonging percentages; S_{34}, S_{56}, S_{89} and S_{43}, S_{65}, S_{98}, shown in table 4 into consideration. These belonging percentages are shown in table 4 for boys, and in table 5 for girls.

Table 4. Theoretical percentage of who judged to belong to the different grade group by the discriminant function of physique; boy

Grade	1	2	3	4	5	6	7	8	9
1		28.81	11.00	5.35	2.35	0.67	0.00	0.00	0.00
2	29.59		23.48	13.28	5.95	2.28	0.20	0.00	0.00
3	12.66	27.69		32.50	15.87	8.56	1.87	0.80	0.18
4	6.46	16.24	37.09		23.80	15.94	4.36	2.03	0.96
5	3.65	9.48	24.81	39.68		37.95	14.09	7.73	4.68
6	0.98	3.25	11.41	20.66	32.36		21.55	12.46	8.22
7	0.00	0.50	2.71	6.07	15.04	25.90		31.44	25.96
8	0.00	0.00	1.28	3.09	9.20	16.48	40.43		44.32
9	0.00	0.00	0.45	1.38	5.23	10.02	28.49	37.41	

Note: These figures show the percentage of those who actually belong to the grade described in the extreme left column but judged to belong to the different grade groups described in the upper most raw in terms of the discriminant function of physique.

Table 5. Theoretical percentage of who judged to belong to the different grade group by the discriminant function of physique; girl

Grade	1	2	3	4	5	6	7	8	9
1		22.79	10.68	5.02	0.90	0.35	0.00	0.00	0.00
2	35.08		28.40	16.61	5.09	2.93	1.20	0.56	0.21
3	15.12	30.76		32.76	13.90	8.97	4.38	2.24	1.56
4	7.19	18.77	36.54		25.60	17.90	9.91	5.33	4.13
5	1.24	5.58	14.66	25.38		38.39	25.46	14.76	13.14
6	0.57	3.27	9.66	18.31	41.17		34.56	20.70	19.47
7	0.00	1.37	4.81	10.40	27.53	36.70		29.58	30.85
8	0.00	0.76	2.84	6.52	19.01	26.91	41.91		59.34
9	0.00	0.33	1.78	4.52	14.73	21.50	33.83	33.83	

Note: These figures show the percentage of those who actually belong to the grade described in the extreme left column but judged to belong to the different grade groups described in the upper most raw in terms of the discriminant function of physique.

The belonging percentages are considerably larger between the adjoining grades each other, and they are decreasing as the grade difference increases. For instance, the belonging percentage of the 1st grade boys to the 2nd is 28.81%; that is, 28.81% of the 1st grade boys may be assumed to belong to the 2nd grade group in terms of physique, and 29.59% of the 2nd grade boys to the 1st grade group inversely and 11.00% to the 3rd, 5.3% to the 4th, 2.35% to the 5th, and 0.67% to the 6th. If the significance level of statistical test could be applied, the percentage of the 1st grade to the 5th grade; 2.35%, can be neglected, so the 1st grade boys' distribution of discriminant function values of physique can be evaluated to be quite different from the 5th grade boys' one. In other words, as long as physique is concerned, in order that the 1st grade boys may grow into a quite different group, it may take 4 years. Conversely, S_{ji} ($j > i$) may show the percentage of those who may be judged to belong to the younger grade group but actually belong to the older grade. For instance, S_{87} = 40.43% means that 40.43% of the 8th grade boys may be assumed to still remain in the 7th grade in terms of physique in spite of one year advancement of grade. Thus, from taking an investigation in S_{ji} ($j > i$) in table 4, 40% at most and 25% at least of the one-year-older grade boys may be assumed to remain in the one-year-younger grade in spite of one year advancement of grade. Of course, the belonging percentage of a fixed grade group to other younger grade is naturally decreasing as the grade difference increases.

In girls the ordinates corresponding to the grade means gradually decline up to the 4th grade, and thereafter, do not appear to change, but the distribution of the 8th seems to be overlapped to a great extent with the one of the 9th, so the individual differences are increasing gradually up to the 4th grade, and thereafter, the group heterogeneity may not change in terms of physique. Then, the largest distance in grade mean position between the 4th and 5th may show the growth acceleration in physique, taking the smaller belonging percentage shown in table 5 into consideration, and the smaller distance in grade mean position between the 5th and 6th, and the 8th and 9th, may show the greater degree of homogeneity in physique, taking the larger belonging percentage shown in table 5 into consideration. Then, the belonging percentage of the 1st grade to the 2nd is 22.79% and it is getting smaller according to grade advancement, and that to the 5th grade is only 0.90% , that can be neglected. This is the case for the 2nd grade, so 4 years are long enough to cause the girls of lower grade of elementary level grow into the quite different group in terms of physique. Contrary, the grade difference less than 4 years may not be influential to cause a grade group to grow into a quite different group in terms of physique for girls.

3.2. Muscular Strength

The independent variables used for estimating muscular strength are as follows;

X_1 = grip strength (left hand grip),

X_2 = grip strength (right hand grip),

X_3 = back strength,

X_4 = push arm strength, and

X_5 = pull arm strength.

Then, the coefficients for the independent variables of the derived discriminant functions are shown in table 6. As shown in the case of physique, the eigen value of $W^{-1}A$ corresponding to the derived discriminant function is very large, compared with other eigen values, so this discriminant function can be recognized as the most effective function for classifying the grade groups in terms of the

Table 6. Coefficients for Variables of Discriminant Function; Muscular Strength

	Max. Eig. Value		X_1	X_2	X_3	X_4	X_5
Boy	5.47279	I	.12949	.69765	.27963	.63647	.11504
		II	.01251	.07124	.00863	.06190	.01703
		c	−3.40912				
Girl	6.25846	I	.67767	.52679	.23282	.40585	−.21057
		II	.08351	.07034	.00924	.56180	−.04003
		c	−3.10797				

I; Eigen vector corresponding to the maximum eigen value of $W^{-1}A$: Coefficients for the independent variables in the standard score form.
II; Coefficients for the independent variables in the raw score form.
c; Constant term in the discriminant function of the raw score form.

used variables. The unbiased statistics for estimating the parameters are shown in table 7.

The distributions of discriminant function values in each grade group are shown in Fig. 3 for boys and in Fig. 4 for girls. In boys, the maximum ordinate of each grade group is decreasing following the grade increase, but the range of distribution is increasing. This means that the individual difference within grade group is increasing as the grade advances and the grade means increase. The considerable overlapping areas in the distribution between the adjoining grade groups are observed and they seem to be more than 45% , which is determined by ($S_{ij} + S_{ji}$), as shown in table 8. The largest belonging

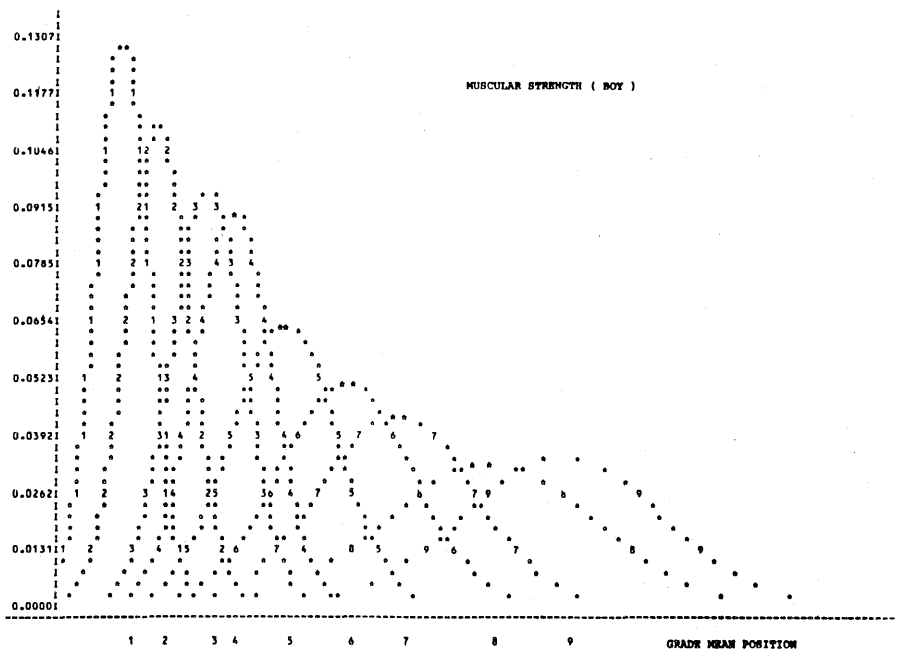


Fig. 3 Distribution of Discriminant Function Value in each Grade Group; Muscular Strength: Boy

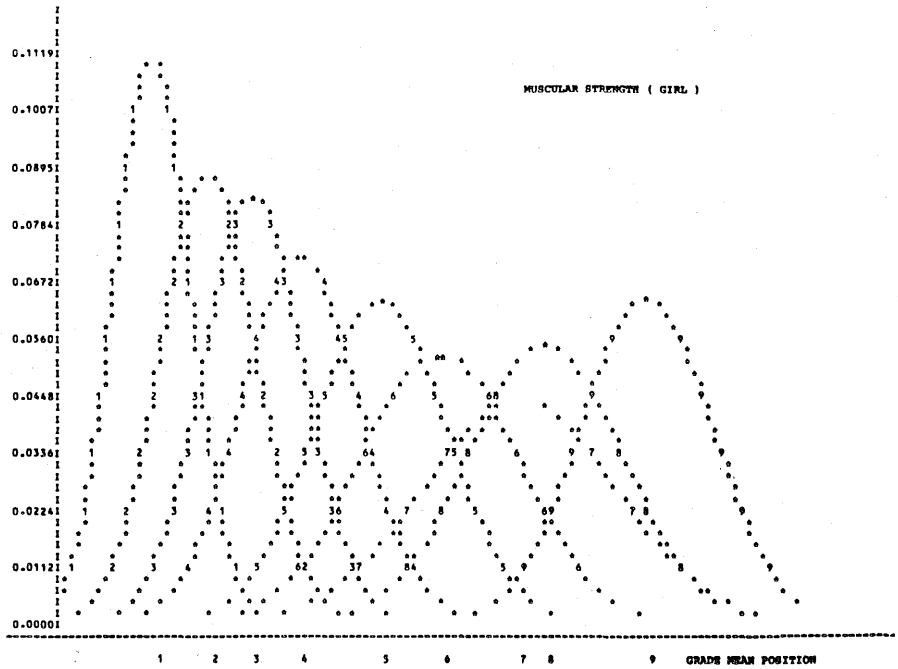


Fig. 4 Distribution of Discriminant Function Value in each Grade Group; Muscular Strength: Girl

Table 7. Mean & SD of discriminant function value in each grade group (muscular strength)

sex \ grade		1	2	3	4	5	6	7	8	9
BOY	MEAN	−1.95281	−1.57414	−0.97239	−0.68288	−0.08017	0.64296	1.23309	2.33618	3.17330
	SD	0.30515	0.36188	0.41738	0.43962	0.61229	0.77843	0.89810	1.20408	1.15400
GIRL	MEAN	−1.84420	−1.31520	−0.89072	−0.44367	0.31945	0.87650	1.63274	1.88473	2.84010
	SD	0.35645	0.44926	0.47542	0.55555	0.62825	0.74442	0.91350	0.72872	0.62039

percentage; S_{ij} and S_{ji}, is found between the 3rd and the 4th; 33.91% and 39.56%, respectively. Thus the 3rd grade boys may show a relatively similar distribution of muscular strength with the 4th grade boys. In other words, the 3rd grade boys show the relatively same group characteristics in muscular strength as the 4th grade boys. And such similarity may be found between the 6th and the 7th and between the 8th and the 9th, too.

As long as the changing trend of the grade mean position on the abscissa investigated, the distances between the adjoining grades are increasing as the grade advances. This shows the development of muscular strength due to the grade advancement, but there are 19% at least to 43.2% of boys who show the similar level of muscular strength but actually belong to the different adjoining grade group. The belonging percentage of one fixed grade group to others is decreasing naturally as the grade difference increases. For instance, the first grade boys show 22.44% of the belonging percentage to the 2nd, but only 6.83% to the 3rd, 3.36% to the 4th and 1.18% to the 5th. Therefore, it may be inferred that the

Table 8. Theoretical percentage of who judged to belong to the different grade group by the discriminant function of muscular strength; boy

Grade	1	2	3	4	5	6	7	8	9
1		22.44	6.83	3.36	1.18	0.28	0.00	0.00	0.00
2	31.48		20.04	11.97	4.28	1.45	0.62	0.00	0.00
3	10.37	25.83		33.91	13.18	5.08	2.42	0.80	0.00
4	5.27	15.91	39.56		18.77	8.00	4.01	1.34	0.21
5	2.68	8.38	24.35	36.37		22.18	13.14	4.96	1.97
6	1.10	3.60	11.56	18.13	37.20		28.73	12.58	6.88
7	0.50	1.82	6.20	10.07	24.24	43.23		20.52	13.74
8	0.09	0.87	2.78	4.46	12.13	25.30	38.13		38.34
9	0.00	0.00	0.52	1.03	4.22	11.69	20.29	33.87	

Note: These figures show the percentage of those who actually belong to the grade described in the extreme left column but judged to belong to the different grade groups described in the upper most raw in terms of the discriminant function of muscular strength.

1st grade boys develop into a quite different group in 3 years in terms of muscular strength. The development of muscular strength is considered to be accelerated in puberty.[*3,6,7] This is shown by the fact that the distances in grade mean position between the adjoining grade are larger in the 6th grade and up than in the younger grades. But the 6th grade boys show 28.73% of belonging percentage to the 7th grade, 12.58% to the 8th, and 6.88% to the 9th. Therefore, the significant increase of muscular strength can be observed in puberty but it should not be ignored that there are still fairly percentage of boys who show the similar level of muscular strength but actually belong to the younger grade or older. The belonging percentage derived from the distributions of muscular strength between the different grade groups are shown in table 8 for boys. As this figure shows, one year discrepancy may not be so influential to develop one group into a quite different group in terms of muscular strength, but two or three years may be considerably influential to do so, but still there are found around 10% of boys who are assumed to remain in the younger grade groups in terms of muscular strength.

In girls, as Fig. 4 shows, the configuration for distributions is quite different from the boys' case. As long as the changing trend of the grade mean position investigated, the development can exactly be observed. The largest distance between the adjoining grade mean positions, however, is found at the interval of the 8th to the 9th, although the individual difference within grade group is getting smaller. Although such within group individual difference gets larger in boys when the development is accelerated during puberty, this may not be observed in girls. The maximum ordinate of distribution curves for each grade group is getting smaller up to the 7th grade, and thereafter it tends to increase. In other words, the within group individual difference is the largest at the 7th grade and the distance in grade mean position between the corresponding grades; e. g., the 7th and the 8th, is the smallest, but that between the 6th and 7th is rather larger, so the acceleration of development of muscular strength can not always result in the increasing of the within group individual difference in girls. Table 9 shows the belonging percentages derived from the distribution between the different grade groups. They are 51.10% at most and 19.84% at least between the adjoining grade groups. Particularly, those between the 7th and the 8th are 38.29% (= S_{78}) and 51.10% (= S_{87}), so it may be inferred that the 7th grade

Table 9. Theoretical percentage of who judged to belong to the different grade group by the discriminant function of muscular strength; girl

Grade	1	2	3	4	5	6	7	8	9
1		19.87	9.89	4.38	0.91	0.27	0.00	0.00	0.00
2	30.67		30.08	15.85	5.00	2.25	0.86	0.00	0.00
3	14.91	34.48		26.72	10.77	5.12	2.04	0.77	0.00
4	7.71	22.38	39.24		22.81	11.95	5.20	2.89	0.00
5	1.76	7.69	16.14	28.91		26.65	13.55	11.08	2.19
6	0.87	4.14	9.16	18.54	41.21		24.44	25.17	8.36
7	0.32	1.98	4.48	9.95	24.81	39.59		38.29	27.46
8	0.00	0.31	1.28	4.02	13.69	24.19	51.10		27.26
9	0.00	0.00	0.00	0.02	2.16	6.58	14.36	20.34	

Note: These figures show the percentage of those who actually belong to the grade described in the extreme left column but judged to belong to the different grade groups described in the upper most raw in terms of the discriminant function of muscular strength.

and the 8th grade are very similar in terms of muscular strength. The 1st grade group shows the 19.84% of belonging percentage to the 2nd, 9.8% to the 3rd, 4.38% to the 4th, and only 0.91% to the 5th, so the 1st grade girls are quite different from the 4th grade girls in terms of muscular strength, and the 2nd grade shows 30.08% to the 3rd, 15.85% to the 4th, 5.0% to the 5th, and only 2.25% to the 6th, and 0.86% to the 7th grade group. As for the 3rd grade group, it is 51.2% to the 6th, and 5.20% to the 7th. Then, it may be inferred that such time discrepancy as three years are considerably influential to cause one grade of the lower elementary level to develop into a quite different group in terms of muscular strength, although there are still found less than 10% of the girls who may be judged to belong to the younger grade group in terms of muscular strength than the real grade group which they belong to.

3.3. Fundamental Motor Skill

The independent variables used for estimating fundamental motor skill are as follows; vertical jump (jumping ability), 50 m dash (running ability), and softball throw for distance (throwing ability). The coefficients for the independent variables of the derived discriminant function are shown in table 10. As shown in the preceding investigations, the eigen value corresponding to the derived discriminant function is very large, compared with others, so it can be assumed that this function is very effective to classify the different grade groups in terms of the used variables. And the used variables are concerning to the jumping skill, running skill and throwing skill, so a linear combination of these test variables may be reasonably assumed as a function representing the fundamental motor skill score in a sense of factor score.

Then, the distributions of this discriminant function values are shown for each grade group in Fig. 5 for boys and in Fig. 6 for girls. The unbiased statistics used for determining these distributions are shown in table 11. The features of these distributions seem to be quite different from the preceding two. In boys, from the investigation in the distances in grade mean position between the different grade groups, the development may be accelerated during the earlier stage of elementary level and the

Table 10. Coefficients for Variables of Discriminant Function; Fundamental Motor Skill.

	Max. Eig. Value		X_1	X_2	X_3
Boy	4.60857	I	.30030	−.87287	.38459
		II	.02525	−.65896	.02634
		c	4.60173		
Girl	4.19487	I	.18486	−.96001	.21026
		II	.01943	−.67333	.02713
		c	5.70314		

I; Eigen vector corresponding to the maximum eigen value of $W^{-1}A$: Coefficients for the independent variables in the standard score form.

II; Coefficients for the independent variables in the raw score form.

c; Constant term in the discriminant function of the raw score form.

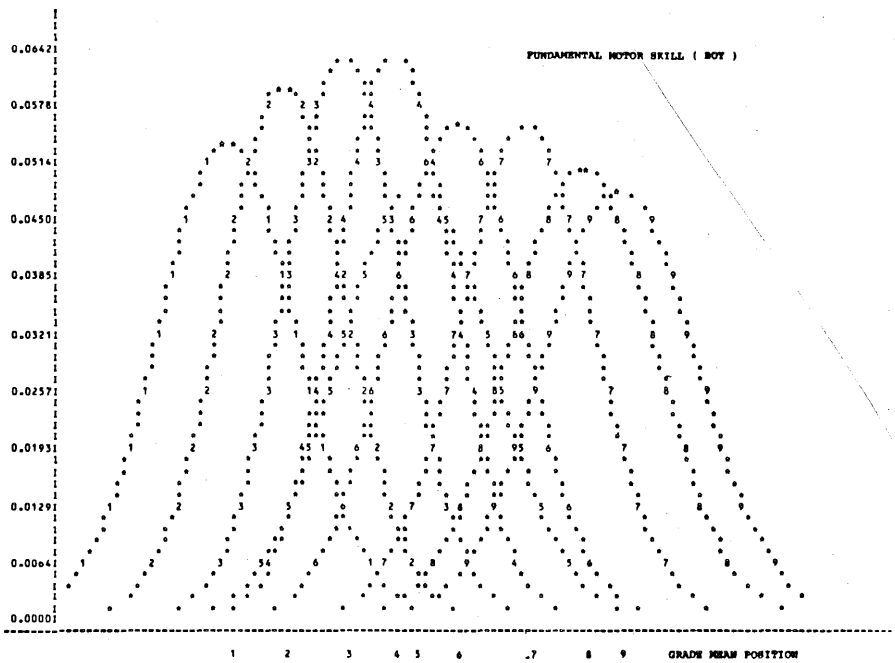


Fig.5, Distribution of Discriminant Function Value in each Grade Group; Fundamental Motor Skill: Boy

Fig. 5 Distribution of Discriminant Function Value in each Grade Group; Fundamental Motor Skill: Boy

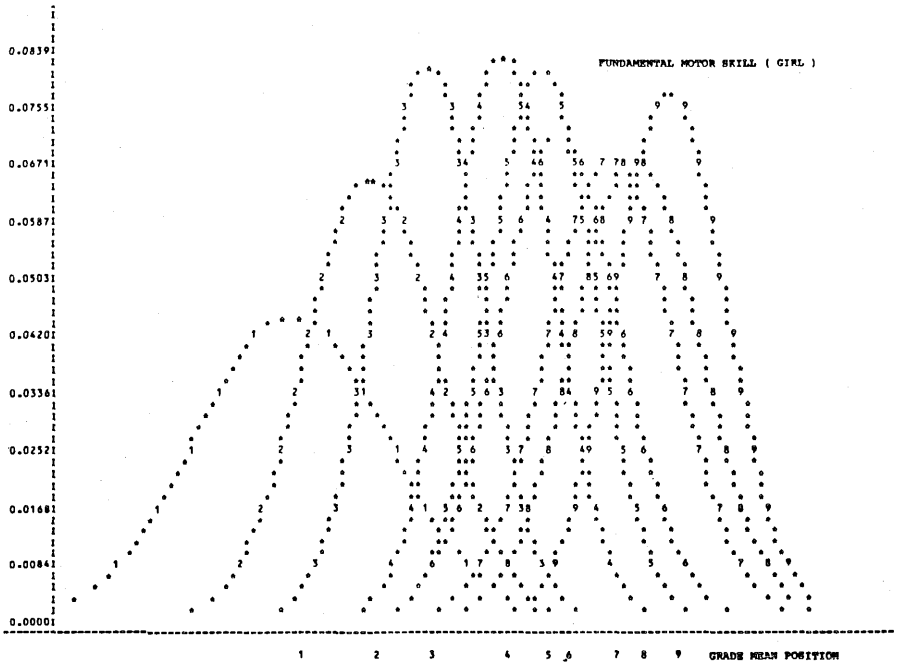


Fig.6, Distribution of Discriminant Function Value in each Grade Group;Fundamental Motor Skill:Girl

Fig. 6 Distribution of Discriminant Function Value in each Grade Group; Fundamental Motor Skill: Girl

Table 11. Mean & SD of discriminant function value in each grade group (Fundamental motor skill)

sex \ grade		1	2	3	4	5	6	7	8	9
BOY	MEAN	−1.91490	−1.30176	−0.65291	−0.10651	0.12073	0.55759	1.31769	1.92338	2.32566
	SD	0.73938	0.66163	0.63161	0.62111	0.81946	0.70858	0.71852	0.78421	0.82210
GIRL	MEAN	−1.77980	−1.03864	−0.51218	0.18098	0.54113	0.70567	1.18369	1.42061	1.73619
	SD	0.88247	0.61200	0.48741	0.47556	0.48946	0.56086	0.58894	0.58647	0.51598

earlier stage of junior high level. As for the within group individual difference, however, it tends to be decreasing up to the 4th grade, but it suddenly increases and becomes the largest at the 5th grade, and it decreases again at the 6th grade, and thereafter it tends to be increasing. The distance in grade mean position between the 4th and the 5th is the smallest and the within group individual difference is the largest at the 5th, so the 4th grade and the 5th grade may be very similar in terms of fundamental motor skill. But, as table 12 shows, this similarity seems to be dependent on the greater within group individual difference; that is, the belonging percentage of the 4th to the 5th is only 10% but that of 5th to the 4th is 75.52% . It is impossible to explain the reason for this fact, but probably some biasness of sample may influence it. Up to the 4th grade, taking the fact that the distances between the adjoining grade mean positions are fairly large but the within group individual difference is decreasing, it may be inferred that such degree of development that the fairly large distance in grade mean position between the adjoining grade groups suggests may be generated in the fairly large amount of individuals in

Table 12. Theoretical percentage of who judged to belong to the different grade group by the discriminant function of fundamental motor skill; boy

Grade	1	2	3	4	5	6	7	8	9
1		37.46	19.98	10.21	8.90	4.50	1.35	0.54	0.12
2	28.45		32.40	18.43	13.96	8.33	2.74	1.15	0.61
3	15.57	29.15		33.83	21.35	16.60	6.60	2.97	1.69
4	8.07	16.68	32.44		10.07	26.18	12.79	6.25	3.76
5	10.23	19.43	37.00	75.52		47.46	24.20	13.46	8.94
6	4.27	9.13	19.95	35.26	29.36		29.26	16.53	11.05
7	1.31	3.03	7.80	15.87	19.28	30.17		30.33	22.27
8	0.58	1.40	3.86	8.48	12.63	19.42	38.19		36.47
9	0.21	0.80	2.31	5.32	8.98	13.66	28.80	43.67	

Note: These figures show the percentage of those who actually belong to the grade described in the extreme left column but judged to belong to the different grade groups described in the upper most raw in terms of the discriminant function of fundamental motor skill.

terms of fundamental motor skill. In the 1st grade group, the within group individual difference is fairly large in spite of the low location of grade mean. This may suggest that the individual difference of this nature may refer to the degree of how well or safely the individual may perform the motor behavior.[*1,6] After the 5th grade, particularly from the 6th to the 7th, the development of fundamental motor skill may be accelerated but the within grade individual difference is relatively small, so it may be inferred that many boys may develop their fundamental motor skill with relatively small individual difference during these days. After the 7th grade, the developmental rate tends to decrease, as the decreasing trend of the distances in grade mean position between the adjoining grades show, but the within group individual difference tends to increase. Therefore, it can be inferred that the mean rate of development may decrease but the individual difference may increase in terms of fundamental motor skill. Such an increase of the individual difference may refer to how fast, powerful and long the individual can perform the motor activity.[*3,6] Thus, the individual difference at this stage seem to be quite different from that at the earlier stage. The belonging percentage derived from the distributions between the adjoining grades are rather large, except S_{45}; e. g., they are almost more than 30%. The 1st grade shows 37.46% of belonging percentage to the 2nd, 19.98% to the 3rd, 10.21% to the 4th, 8.90% to the 5th, 4.5% to the 6th and 1.35% to the 7th. Therefore, the 6th grade group may be quite different statistically from the 1st grade. The similarity of the 5th grade with the 4th is very high; that is, the belonging percentage is 75.52% , and the 5th shows 47.46% to the 6th grade, so it may be inferred that these three grade groups; 4th, 5th and 6th, are relatively similar in terms of fundamental motor skill.

In girls, as shown in Fig. 6, the distances in grade mean position between the adjoining grades are fairly large during the earlier stage of elementary level, but they get smaller after the 4th grade, although that between the 6th and 7th is a little larger. And the within group individual difference is the largest at the 1st grade of elementary level, and then it tends to decrease gradually up to the 4th, and it tends to increase up to the 8th, and thereafter, it tends to decrease again. Therefore, the develop-

ment may be accelerated during the earlier stage of elementary level, and such development may be generated commonly in many girls during this stage, for the within group individual difference tends to decrease. Then, the nature of this development may refer to how well and safely the individual can perform the motor behavior but not how strong, fast, and/or powerfull.[*6] Then, most of girls come to be able to perform the motor behaviors just as safely and/or well as the adult can do at the 4th and 5th grade, and thereafter, the nature of development of fundamental motor skill may refer to how strongly, fast, and powerfully the individual can perform them. Then, the development of this nature may be generated after the 6th grade, but the developmental rate tends to decrease after the 7th, and the within group individual difference may decrease as well. So the slow development may be generated for most of girls during these years. This is one of the differences between the boys and the girls; e. g., it is the same for both sexes that the developmental rate may decrease but it is just reverse that the within group individual difference tends to increase in boys but decrease in girls. As for the belonging percentage derived from the distributions between the adjoining grade groups, they are 18.81% at least and 66.18% , as shown in table 13. The 1st grade shows 41.05% of the belonging percentage to

Table 13. Theoretical percentage of who judged to belong to the different grade group by the discriminant function of fundamental motor skill; girl

Grade	1	2	3	4	5	6	7	8	9
1		41.05	23.69	9.68	5.81	5.25	2.65	1.77	0.73
2	18.81		39.02	15.29	8;61	7.23	3.29	2.06	0.75
3	9.61	23.27		24.10	14.00	11.01	5.04	3.17	1.21
4	4.36	10.63	23.05		34.01	24.88	14.32	10.28	5.53
5	2.81	6.42	14.09	36.87		20.77	22.32	17.45	11.29
6	3.00	6.45	13.44	35.92	66.18		31.71	25.47	18.00
7	1.64	3.14	6.41	19.99	32.34	35.99		42.42	35.25
8	1.10	1.96	3.98	13.84	23.82	27.82	41.60		46.56
9	0.32	0.60	1.29	6.13	12.16	15.81	26.15	30.38	

Note: These figures show the percentage of those who actually belong to the grade described in the extreme left column but judged to belong to the different grade groups described in the upper most raw in terms of the discriminant function of fundamental motor skill.

the 2nd, but the 2nd only 18.1% to the 1st, and the 1st 23.69% to the 3rd, 9.69% to the 4th, 5.81% to the 5th, 5.25% to the 6th. So it may be inferred that it may take nearly 5 years for the 1st graders to develop into a quite different group in terms of fundamental motor skill. As for the 2nd grade, the belonging percentage to the grade older than the 6th is less than 4% , and this is the case for the 3rd and 4th. Therefore, it may take 5 years for the lower graders to develop into a quite different groups in terms of fundamental motor skill in girls. Then, the 4th grade shows 34.01% to the 5th, 24.77% to the 6th, and the 5th 20.77% to the 6th, and moreover the 6th 66.18% to the 5th, so it may be inferred that these three grade groups are relatively similar in terms of fundamental motor skill. Furthermore, the 6th shows 31.71% to the 7th, and 25.47% to the 8th, and the 7th 35.99% to the 6th, so the 6th grade may be considered to be very similar as a group to the 7th and the 8th; 1st and 2nd of junior high level. And the inter-belonging percentage between the three grade groups of junior high level are

26% at least and 46% at most, so it may be inferred that these three grade groups are fairly similar in terms of fundamental motor skill in girls.

IV. Conclusions

In this paper, in order to satisfy the following multipurposes; to investigate in the growth and development of ability but not of item, to take the means and the within group individual differences into consideration simultaneously, and to emphasize the classification of one grade group from another to make the group difference more accentuated, the multivariate discriminant analysis was applied to nine grade groups with the 3 variable groups; physique domain, muscular strength domain and fundamental motor skill domain. Then, the following inferences were induced.

1. The hypothesis that the within group individual differences become larger when the development is accelerated is approved for physique in boys and girls, and for muscular strength in boys, but rejected for others. Therefore, the development of physique and muscular strength are generally accelerated in boys during puberty and this is shown by larger distance in grade mean position between the different grades at puberty, but there are found fairly percentage of those who can be judged to belong to the older or younger grade group than the real grade in terms of physique or muscular strength.
2. For the earlier grade of elementary level, 4 years for physique, 3 years for muscular strength may be long enough to cause them to develop into a quite different group in terms of physique and muscular strength, respectively. This may be recongnized in both boys and girls.
3. As for fundamental motor skill, the development may be accelerated twice up to the 9th grade; at earlier stage of elementary level and at the beginning of junior high level. At these stages, the relative larger amount of boys may develop together, compared with the rest of developmental stages in boys and girls.
4. The fairly large amount of the 4th, 5th, and 6th grade boys are at the relatively similar stage of development in terms of fundamental motor skill, because of relatively large belonging percentage among these grade groups each other. This may be considered applicable to the girls, too.
5. For physique, the largest vertical coordinate in the distribution of discriminant function values is decreasing more sharply in boys than in girls as the grade advances. This may show that the speed of the increasing within-group individual difference is larger in boys than in girls.
6. As for muscular strength, the configuration of boys' distributions is very different from the girls'; that is, the largest ordinate is rapidly decreasing as the grade advances in boys, but it decreases rapidly up to the 6th and 7th, and thereafter increases in girls. Contrary to this changing trend of the largest ordinate, the within group individual difference increases consistently in boys, but it increases up to the 6th and 7th, and then it decreases in girls.
7. As for fundamental motor skill, the largest ordinate increases up to the 4th grade and thereafter, it decreases in boys, and the within group individual difference decreases up to the 4th and then increases. Taking the nature of motor ability at the earlier stage of development into consideration, the grade group may change into a homogeneous group up to the 4th, and then the homogeneity of group may decrease in terms of fundamental motor skill. Such decreasing tendency of group homegeneity may suggest the developmental acceleration. The same tendency as the boys may be investigated up to the 8th grade in girls, too, but the largest ordinate becomes larger than the younger grade in junior high level and the within group individual difference tends to decrease again at 9th grade in girls.

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能力の一次モデルの立場から能力を測定変量の一次式であらわし，その能力からみた集団の区別を出来るだけ厳格にするため，同一能力領域を測定していると適切に仮定できる測定変量群による判別関数を求め，この関数値を能力スコアーとした。かつ，この関数値の各年齢集団分布を正規型と仮定し，この正規分布の加齢にともなう変化から身体的発育発達を検討した。

身体的発育発達属性としては正規性の仮定が妥当と考えられる体格，筋力，基礎運動技能の3体力領域をとりあげ，1977年，1978年，1979年の3年間にわたる同一小学校児童標本に対して行なわれた体力測定資料及び1978年に行なわれた中学校生徒標本の資料から半縦断的資料を上記3体力領域について構成し，これを分析対象とした。

判別関数値によってあらわされた体力の，その発達が急速になる時期における，集団内個人差の増大は体格，筋力については考察されたが，基礎運動能力については必ずしも認められなかった。同時に，体格，筋力については実際に属している年齢より低い年齢群に，又は高い年齢群に属すると判別関数値より判別されるものの割合は発育発達の急速な時期には増大すると推測された。かかるものの割合を能力の分布の重なりの程度として積分から求め，この積分値の評価から，体格で5年，筋力で4年，基礎運動技能では5～6年の差異が集団を全く分布上の重なりのない集団に変化させるのに要すると考えられた。これは，発育発達の盛んな時期における一年間の差異，平均値の有意な差異も必ずしも集団として大きく異った集団をあらわすとは限らない事を示すものと考えられた。