TSUKUBA J. MATH.
Vol. 24 No. 2 (2000), 249–255

ZERO-DIMENSIONAL SUBSETS OF HYPERSPACES

By

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Abstract. Let X be a metric continuum, let 2^X be the hyperspace of all the nonempty closed subsets of X and let $C(X)$ be the hyperspace of subcontinua of X. In this paper we prove:

THEOREM 1. *If $\mathscr{H}$ is a 0-dimensional subset of 2^X, then $2^X - \mathscr{H}$ is connected.*

THEOREM 2. *If $\mathscr{H}$ is a closed 0-dimensional subset of $C(X)$ such that $C(X) - \{A\}$ is arcwise connected for each $A \in \mathscr{H}$, then $C(X) - \mathscr{H}$ is arcwise connected.*

Theorem 2 answers a question by Sam B. Nadler, Jr.

Introduction

Throughout this paper X denotes a nondegenerate continuum, i.e., a compact connected metric space, with metric d. Let 2^X be the hyperspace of nonempty closed subsets of X, with the Hausdorff metric H, and let $C(X)$ be the hyperspace of subcontinua of X.

J. Krazinkiewicz proved in [5] that if $\mathscr{H}$ is a 0-dimensional subset of $C(X)$, then $C(X) - \mathscr{H}$ is connected. In this paper we use Krasinkiewicz' result to prove the following theorem:

THEOREM 1. *If $\mathscr{H}$ is a 0-dimensional subset of 2^X, then $2^X - \mathscr{H}$ is connected.*

On the other hand, in Krasinkiewicz' Theorem the word "connected" can not be replaced by "arcwise connected". Even if X is the $\sin(1/x)$-continuum and A is the limit segment, then $C(X) - \{A\}$ is not arcwise connected. In [7, Question 11.17], Nadler asked the following question: if $\mathscr{H}$ is a compact 0-dimensional

1991 Mathematics Subject Classification. Primary: 54B20
Key Words and Phrases: Arcwise connected, Hyperspace, 0-dimensional
Received August 3, 1999

subset of $C(X)$ and if $C(X) - \{A\}$ is arcwise connected for each $A \in \mathscr{H}$, does it follow that $C(X) - \mathscr{H}$ is arcwise connected? This question has been affirmatively answered for the following particular cases:

- if $\mathscr{H}$ has two elements (Nadler and Quinn, [8, Lemma 2.4]),
- if $\mathscr{H}$ is finite (Ward, [9])
- if $\mathscr{H}$ is numerable (Illanes, [3], this result was rediscovered by Hosokawa in [1]).

Furthermore, in [3], the author showed that any two elements of $C(X) - \mathscr{H}$ can be joined by an arc which intersects $\mathscr{H}$ only a finite number of times.

In this paper we finally solve the general question by proving the following theorem.

THEOREM 2. *If $\mathscr{H}$ is a closed 0-dimensional subset of $C(X)$ such that $C(X) - \{A\}$ is arcwise connected for each $A \in \mathscr{H}$, then $C(X) - \mathscr{H}$ is arcwise connected.*

Proof of Theorem 1

Throughout this section $\mathscr{H}$ will denote a 0-dimensional subset of 2^X. By Krasinkiewicz' result in [5], $C(X) - \mathscr{H}$ is connected. Let $\mathscr{L}$ be the component of $2^X - \mathscr{H}$ which contains $C(X) - \mathscr{H}$.

In order to prove that $2^X - \mathscr{H}$ is connected, it is enough to prove that $\mathscr{L}$ is dense in 2^X. Since the subset of 2^X which consists of all the nonempty finite subsets of X is dense in 2^X, we only need to prove the following claim:

Claim. For each finite subset $F = \{p_1, \ldots, p_m\}$ of X and for each $\varepsilon > 0$, there exists an element $L \in \mathscr{L}$ such that $H(F, L) < \varepsilon$.

Let $F = \{p_1, \ldots, p_m\}$ and $\varepsilon > 0$.

Take an order arc γ from a fixed one-point set $\{p_0\}$ to X (see [7, 1.2] for the definition of order arc). Since $\mathscr{H}$ is 0-dimensional, there exists an element $M \in \gamma - \mathscr{H} \subset C(X) - \mathscr{H}$ such that $H(M, X) < \varepsilon/2$ and M is nondegenerate. Choose points $q_1, \ldots, q_m \in M$ such that $d(p_i, q_i) < \varepsilon/2$ for each $i \in \{1, \ldots, m\}$. Let $\{U_n\}_{n=1}^{\infty}$ be a sequence of proper open subsets of M such that $q_1 \in U_n$ for every $n \geq 1$, $U_1 \supset \mathrm{cl}(U_2) \supset U_2 \supset \mathrm{cl}(U_3) \supset U_3 \supset \ldots, \mathrm{cl}(U_n) \to \{q_1\}$ (convergence in 2^X) and $M \neq \mathrm{cl}(U_1) \subset \{q \in X : d(q, q_1) < \varepsilon/2\}$.

Let $L_0 = \{q_1, \ldots, q_m\} \cup (\mathrm{Bd}_M(U_1) \cup \mathrm{Bd}_M(U_2) \cup \mathrm{Bd}_M(U_3) \cup \ldots)$. Clearly, $L_0 \in 2^X$. Fix a nondegenerate subcontinuum D of $U_1 - \mathrm{cl}(U_2)$. Then the set $\{L_0 \cup \{x\} \in 2^X : x \in D\}$ is a nondegenerate subcontinuum of 2^X. Since $\mathscr{H}$ is 0-dimensional, there exists a point $x_0 \in D$ such that $L_0 \cup \{x_0\} \notin \mathscr{H}$.

Define $L = L_0 \cup \{x_0\}$. Then $L \in 2^X - \mathscr{H}$ and $H(F, L) < \varepsilon$.

We will show that $L \in \mathscr{L}$.

For each $n \geq 1$, let $A_n = M - U_n \subset M - \mathrm{cl}(U_{n+1})$. Take an order arc γ_n from A_n to M. Since $M - \mathrm{cl}(U_{n+1})$ is an open subset of M, there exists a (nondegenerate) subarc σ_n of γ_n such that each of its elements is contained in $M - \mathrm{cl}(U_{n+1})$ and $A_n \in \sigma_n$. Consider the set $\theta_n = \{L \cup K : K \in \sigma_n\}$. It is easy to show that θ_n is a (nondegenerate) order arc from $L \cup A_n$ to some element in 2^X. Since $\mathscr{H}$ is 0-dimensional, we can choose an element $B_n = L \cup K_n \in \theta_n - \mathscr{H}$, where $K_n \in \sigma_n$. Notice that $A_n \subset K_n \subset A_{n+1}$.

Next, we will check that every component of B_n intersects L. Let C be a component of B_n. Since the subarc of θ_n which joins $L \cup A_n$ and B_n is an order arc, then (see [7, 1.8]), $C \cap (L \cup A_n) \neq \varnothing$. If $C \cap L = \varnothing$, we can take an element $x \in C \cap A_n$. Let C_1 be the component of A_n which contains x. Thus $C_1 \subset C$, and by ([7, 20.2]), $\varnothing \neq C_1 \cap \mathrm{Bd}_M(U_n) \subset C \cap L$. This contradiction completes the proof that $C \cap L \neq \varnothing$.

As a consequence of the claim of the paragraph above, we obtain that every component of B_{n+1} intersects B_n.

Let $B_0 = L$. Notice that B_{n-1} is a proper subset of B_n for every $n \geq 1$. By [7, 1,8], there exists a map $\beta_n : [0,1] \to 2^M$ such that $\beta_n(0) = B_{n-1}$, $\beta_n(1) = B_n$, and if $0 \leq s < t \leq 1$, then $\beta_n(s)$ is a proper subset of $\beta_n(t)$.

For each $n \geq 1$, let $\alpha_n : [0,1] \to 2^X$ be a map such that $\alpha_n(0) = \mathrm{Bd}_M(U_{n+2})$, $\alpha_n(1) = M$ and if $0 \leq s < t \leq 1$, then $\alpha_n(s)$ is a proper subset of $\alpha_n(t)$. Since $\mathrm{Bd}_M(U_{n+2}) \subset U_{n+1} - \mathrm{cl}(U_{n+3})$, there exists $t_n > 0$ such that $\alpha_n(t_n) \subset U_{n+1} - \mathrm{cl}(U_{n+3})$.

Let $\varphi_n : [0,1] \times [0,1] \to 2^M$ be given by $\varphi_n(s,t) = \alpha_n(st_n) \cup \beta_n(t)$. It is easy to check that φ_n is continuous, one-to-one, $\varphi_n(0,1) = B_n$ and $\varphi_n(0,0) = B_{n-1}$. Let $\mathscr{G}_n = \varphi_n([0,1] \times [0,1])$. Then $\mathscr{G}_n$ is a 2-cell. By [2, Theorem IV 4], $\mathscr{G}_n - \mathscr{H}$ is connected and contains B_{n-1} and B_n.

Let $\mathscr{G} = \bigcup\{\mathscr{G}_n : n \geq 1\}$. Then $\mathscr{G}$ is a connected subset of $2^X - \mathscr{H}$ and contains the element $B_0 = L$. On the other hand, since $A_n \to M$, and $A_n \subset B_n \subset M$ for each $n \geq 1$, we conclude that $B_n \to M$ and $M \in \mathrm{cl}_{2^X}(\mathscr{G})$. This implies that $\mathscr{G} \subset \mathscr{L}$. Therefore, $L \in \mathscr{L}$. This completes the proof of the claim and thus the proof of Theorem 1. ∎

Proof of Theorem 2

Throughout this section $\mathscr{H}$ will denote a closed 0-dimensional subset of $C(X)$ such that $C(X) - \{A\}$ is arcwise connected for each $A \in \mathscr{H}$.

LEMMA 1. *If $A, B \in C(X) - \mathscr{H}$, $A \cap B \neq \varnothing$, $A - B \neq \varnothing$ and $B - A \neq \varnothing$, then A and B can be joined by an arc in $C(X) - \mathscr{H}$.*

PROOF. Fix a component C of $A \cap B$. Then C is a proper subcontinuum of both A and B. Let $\alpha, \beta : [0,1] \to A \cup B$ be maps such that $\alpha(0) = C = \beta(0)$, $\alpha(1) = A$, $\beta(1) = B$ and $s < t$ implies that $\alpha(s)$ (resp., $\beta(s)$) is a proper subcontinuum of $\alpha(t)$ (resp., $\beta(t)$) (see [Nd78, 1.8]). Let $\mathscr{C} = [0,1] \times [0,1]$. Define $\varphi : \mathscr{C} \to C(A \cup B)$ by:

$$\varphi(s,t) = \alpha(s) \cup \beta(t).$$

Clearly, φ is continuous, $\varphi(1,0) = A$ and $\varphi(0,1) = B$. If D is a component of $\varphi^{-1}(\mathscr{H})$, then $\varphi(D)$ is a connected subset of $\mathscr{H}$. Thus $\varphi(D)$ has exactly one element. Therefore, D is a component of $\varphi^{-1}(E)$ for some $E \in \mathscr{H}$.

Since $\varphi(1,0)$ and $\varphi(0,1) \notin \mathscr{H}$ and $\mathscr{H}$ is compact, there exists $0 < r < 1/2$ such that $\{([1-r,1] \times [0,r]) \cup ([0,r] \times [1-r,1])\} \cap \varphi^{-1}(\mathscr{H}) = \varnothing$.

Let $G_1 = ([0,1-r] \times \{0\}) \cup (\{0\} \times [0,1-r])$ and $G_2 = (\{1\} \times [r,1]) \cup ([r,1] \times \{1\})$. Let $G = G_1 \cup G_2 \cup \varphi^{-1}(\mathscr{H})$. Then G is a compact subset of $\mathscr{C}$.

We will see that no component of $\varphi^{-1}(\mathscr{H})$ intersects both G_1 and G_2. Suppose, to the contrary, that there exists a component D of $\varphi^{-1}(\mathscr{H})$ such that $D \cap G_1 \neq \varnothing$ and $D \cap G_2 \neq \varnothing$. Then there exists an element $E \in \mathscr{H}$ such that D is a component of $\varphi^{-1}(E)$. Let $z = (s,t) \in D \cap G_1$ and $w = (u,v) \in D \cap G_2$. Then $\alpha(s) \cup \beta(t) = \varphi(z) = \varphi(w) = \alpha(u) \cup \beta(v)$. Notice that $s = 0$ or $t = 0$. If $s = 0$, then $\varphi(z) \subset B$. This implies that $\alpha(u) \subset A \cap B$. Hence $\alpha(u) = C$. Thus $u = 0$. This is a contradiction since $w \in G_2$. A similar contradiction can be obtained assuming that $t = 0$. Therefore, no component of $\varphi^{-1}(\mathscr{H})$ intersects both G_1 and G_2.

We are ready to apply the Cut Wire Theorem ([7, 20.6]) to the compact space $\varphi^{-1}(\mathscr{H})$ and the closed sets $\varphi^{-1}(\mathscr{H}) \cap G_1$ and $\varphi^{-1}(\mathscr{H}) \cap G_2$. Thus there exist two disjoint closed sets H_1, H_2 in $\mathscr{C}$ such that $\varphi^{-1}(\mathscr{H}) = H_1 \cup H_2$, $\varphi^{-1}(\mathscr{H}) \cap G_1 \subset H_1$ and $\varphi^{-1}(\mathscr{H}) \cap G_2 \subset H_2$. Define $L_1 = G_1 \cup H_1$ and $L_2 = G_2 \cup H_2$. Then L_1 and L_2 are disjoint closed subsets of $\mathscr{C}$. Thus there exist two disjoint open subsets U_1 and U_2 of $\mathscr{C}$ such that $L_1 \subset U_1$ and $L_2 \subset U_2$.

Let U be the component of U_1 which contains G_1 and let M be the component of $\mathscr{C} - U$ which contains G_2. It is easy to prove that $\mathscr{C} - M$ is connected. Since $\mathscr{C}$ is locally connected M is closed in $\mathscr{C}$ and $\mathrm{Bd}_{\mathscr{C}}(M) \subset \mathrm{Bd}_{\mathscr{C}}(U) \subset \mathrm{Bd}_{\mathscr{C}}(U_1)$. Let $L = \mathrm{Bd}_{\mathscr{C}}(M)$. Then $L \cap (L_1 \cup L_2) = \varnothing$. Since $G_1 \subset \mathscr{C} - M$, L separates G_1 and G_2 in $\mathscr{C}$. Since $\mathscr{C}$ is unicoherent ([6, Thm. 2 II, §57, Ch. VIII]), L is a subcontinuum of $\mathscr{C}$.

Since $[0,r] \times [1-r,1]$ is a connected subset of $\mathscr{C}$ that intersects both G_1

and G_2, we obtain this set intersects L. Similarly L intersects $[1-r,1]\times[0,r]$. Then the set $L_0 = L \cup ([1-r,1]\times[0,r]) \cup ([0,r]\times[1-r,1])$ is a subcontinuum of $\mathscr{C} - \varphi^{-1}(\mathscr{H})$. Since $\mathscr{C}$ is locally connected, there exists an open connected (and then arcwise connected) subset V of $\mathscr{C}$ such that $L_0 \subset V \subset \mathscr{C} - \varphi^{-1}(\mathscr{H})$. Let λ be an arc in V joining $(1,0)$ and $(0,1)$. Therefore, $\varphi(\lambda)$ is a path in $C(X) - \mathscr{H}$ joining A and B. ■

LEMMA 2. *If $A, B \in C(X) - \mathscr{H}$ and $A \subset B \neq A$, then A and B can be joined by an arc in $C(X) - \mathscr{H}$.*

PROOF. By [7, 1.8], there is an order arc from A to B. That is, there is a map $\alpha : [0,1] \to C(B)$ such that $\alpha(0) = A$, $\alpha(1) = B$ and if $s < t$, then $\alpha(s)$ is a proper subcontinuum of $\alpha(t)$. Let $\mathscr{G} = \alpha^{-1}(\mathscr{H})$.

First, we will show that for any $t \in \mathscr{G}$, there exists $\varepsilon_t > 0$ such that $(t-\varepsilon_t, t+\varepsilon_t) \subset (0,1)$ and for every $s \in (t-\varepsilon_t, t) - \mathscr{G}$ and every $r \in (t, t+\varepsilon_t) - \mathscr{G}$, $\alpha(s)$ and $\alpha(r)$ can be joined by an arc in $C(X) - \mathscr{H}$.

Since $\alpha(t) \in \mathscr{H}$, $C(X) - \{\alpha(t)\}$ is arcwise connected. Then there exists a one-to-one map $\beta : [0,1] \to C(X) - \{\alpha(t)\}$ such that $\beta(0) = A$ and $\beta(1) = B$. Let $u = \max\{v \in [0,1]; \beta(w) \subset \alpha(t)$ for each $w \in [0,v]\}$. Then $\beta(u)$ is a proper subcontinuum of $\alpha(t)$. Since β is continuous, there exists $z \in (u,1)$ such that the continuum $C = \cup\{\beta(w) : u \leq w \leq z\}$ does not contain $\alpha(t)$. Since $\mathscr{H}$ is 0-dimensional, we may assume that $C \notin \mathscr{H}$. By the definition of u, C is not contained in $\alpha(t)$.

We consider two cases:

CASE 1. $\alpha(t)$ is indecomposable.

By [7, 1.52.1 (2)], $\beta(u)$ is contained in the composant of $\alpha(t)$ which contains A. Then there exists a proper subcontinuum D of $\alpha(t)$ such that $D \cap A \neq \varnothing \neq D \cap \beta(u)$. Growing D by using an order arc from D to $\alpha(t)$, we may assume that D is not contained in C and $D \notin \mathscr{H}$. Let $\varepsilon_t > 0$ be such that $(t-\varepsilon_t, t+\varepsilon_t) \subset (0,1)$, $\alpha(t-\varepsilon_t)$ is not contained in D, $\alpha(t-\varepsilon_t)$ is not contained in C and $\alpha(t+\varepsilon_t)$ does not contain C.

In order to show that ε_t has the required properties, let $s \in (t-\varepsilon_t, t) - \mathscr{G}$ and $r \in (t, t+\varepsilon_t) - \mathscr{G}$. Then $\alpha(s) \cap D \neq \varnothing$ and $\alpha(s) - D \neq \varnothing$.

If $D - \alpha(s) \neq \varnothing$, then we may apply Lemma 1 to the pairs $\alpha(s)$ and D; D and C; C and $\alpha(r)$, and conclude that $\alpha(s)$ and $\alpha(r)$ can be joined by an arc in $C(X) - \mathscr{H}$.

If $D \subset \alpha(s)$, then we may apply Lemma 1 to the pairs $\alpha(s)$ and C; C and $\alpha(r)$, and conclude that $\alpha(s)$ and $\alpha(r)$ can be joined by an arc in $C(X) - \mathscr{H}$.

CASE 2. $\alpha(t)$ is decomposable.

In this case $\alpha(t) = E \cup F$, where E and F are proper subcontinua of $\alpha(t)$. We may assume that E, $F \notin \mathscr{H}$ and $E - C \neq \emptyset \neq F - C$.

Let $\varepsilon_t > 0$ be such that $(t - \varepsilon_t, t + \varepsilon_t) \subset (0, 1)$, $\alpha(t - \varepsilon_t)$ is not contained in any of the sets C, E and F, and C is not contained in $\alpha(t + \varepsilon_t)$.

Let $s \in (t - \varepsilon_t, t) - \mathscr{G}$ and $r \in (t, t + \varepsilon_t) - \mathscr{G}$. Then $\alpha(s)$ is not contained in any of the sets E, F and C. Since $\alpha(s)$ is a proper subcontinuum of $\alpha(t)$, $E - \alpha(s) \neq \emptyset$ or $F - \alpha(s) \neq \emptyset$. Suppose, for example, that E is not contained in $\alpha(s)$.

If $E \cap C \neq \emptyset$, then we may apply Lemma 1 to the pairs $\alpha(s)$ and E; E and C; C and $\alpha(r)$, and conclude that $\alpha(s)$ and $\alpha(r)$ can be joined by an arc in $C(X) - \mathscr{H}$.

If $F \cap C \neq \emptyset$, then we may apply Lemma 1 to the pairs $\alpha(s)$ and E; E and F; F and C; C and $\alpha(r)$, and conclude that $\alpha(s)$ and $\alpha(r)$ can be joined by an arc in $c(X) - \mathscr{H}$.

This completes the proof of the existence of ε_t.

Now we are ready to prove Lemma 2.

Let $t \in \mathscr{G}$ and let $\varepsilon_t > 0$ be as before. We claim that if $s, r \in (t - \varepsilon_t, t + \varepsilon_t) - \mathscr{G}$, then $\alpha(s)$ and $\alpha(r)$ can be joined by an arc in $C(X) - \mathscr{H}$. Indeed, if t is between s and r, this claim follows from the choice of ε_t, and if, for example, $s, r < t$, then fix $r_1 \in (t, t + \varepsilon_t) - \mathscr{G}$. By the choice of ε_t, both pairs $\alpha(s)$, $\alpha(r_1)$ and $\alpha(r)$, $\alpha(r_1)$ can be joined by an arc in $C(X) - \mathscr{H}$. Thus, $\alpha(r)$, $\alpha(s)$ can be joined by an arc in $C(X) - \mathscr{H}$.

Given a number $t \in [0, 1] - \mathscr{G}$, there exists $\varepsilon_t > 0$ such that $(t - \varepsilon_t, t + \varepsilon_t) \cap \mathscr{G} = \emptyset$. In this case, if $s, r \in (t - \varepsilon_t, t + \varepsilon_t) \cap [0, 1]$, then $\alpha(s)$ and $\alpha(r)$ can be joined by an arc in $C(X) - \mathscr{H}$.

For the open cover $\{(t - \varepsilon_t, t + \varepsilon_t) : t \in [0, 1]\}$, there exists $\delta > 0$ such that if $s, r \in [0, 1]$ and $|s - r| < \delta$, then $s, r \in (t - \varepsilon_t, t + \varepsilon_t)$ for some $t \in [0, 1]$.

Choose a partition $0 = t_0 < t_1 < \cdots < t_m = 1$ such that $t_i - t_{i-1} < \delta$ and $t_i \notin \mathscr{G}$ for each $i = 1, 2, \ldots, m$.

Thus, for each $i \in 1, 2, \ldots, m$, $\alpha(t_{i-1})$ and $\alpha(t_i)$ can be joined by an arc in $C(X) - \mathscr{H}$. Therefore, A and B can be joined by an arc in $C(X) - \mathscr{H}$. ■

PROOF OF THEOREM 2. We consider two cases:

CASE 1. X is indecomposable.

In this case $C(X)-\{X\}$ is not arcwise connected (see [7, 1.51]). Then $X \notin \mathscr{H}$. Given an element $A \in C(X)-(\mathscr{H} \cup \{X\})$, by Lemma 2, A and X can be connected by an arc in $C(X)-\mathscr{H}$.

CASE 2. X is decomposable.

Let $X = E \cup F$, where E and F are proper subcontinua of X. Since $\mathscr{H}$ is 0-dimensional, we may assume that $E, F \notin \mathscr{H}$. Given an element $A \in C(X)-(\mathscr{H} \cup \{X\})$, taking an order arc from A to X, we can find an element $B \in C(X)-\mathscr{H}$, such that A is a proper subcontinuum of B, $B \neq X$, $B-E \neq \varnothing$ and $B-F \neq \varnothing$. Notice that $E-B \neq \varnothing$ or $F-B \neq \varnothing$. Suppose, for example, that $E-B \neq \varnothing$. By Lemma 1, the pairs E, B and E, F can be joined by an arc in $C(X)-\mathscr{H}$, and by Lemma 2, A and B can be joined by an arc in $C(X)-\mathscr{H}$. Then A can be joined to both E and F in $C(X)-\mathscr{H}$. In the case that $X \notin \mathscr{H}$, by Lemma 2, X can be joined to both E and F in $C(X)-\mathscr{H}$. This completes the proof that $C(X)-\mathscr{H}$ is arcwise connected. ■

Acknowledgments

The author wishes to thank Jorge M. Martínez for the useful conversations they had.

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