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ON FOUR-MANIFOLDS FIBERING OVER SURFACES

By

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Abstract. We study closed connected topological or smooth 4-manifolds fibering over a surface in terms of classifying spaces, characteristic classes, and intersection forms.

1. Introduction.

Let F and X be closed connected oriented surfaces of genus h and g, respectively. We are going to study closed connected 4-manifolds M which admit a fibration

$$F \longrightarrow M \xrightarrow{\pi} X \tag{1}$$

with base X and fiber F.

It was shown by Meyer [11] that for a fixed $h \geq 3$ any integer $4m \in \boldsymbol{Z}$ may appear as signature of such a manifold M. So these manifolds provide an interesting class of 4-manifolds (see also [1] and [2] for related examples).

More recently Hillman ([5] and [6]) has proved that the necessary conditions:

$$\chi(M') = \chi(X)\chi(F)$$

and

$$\Pi_1(M') \quad \text{is an extension of} \quad \Pi_1(F) \quad \text{by} \quad \Pi_1(X)$$

are sufficient for the closed 4-manifold M' is homotopy equivalent to a 4-manifold M with fibration structure (1). Here χ denotes the Euler characteristic as

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usual. Moreover, if the homotopy equivalence is simple, then M' and M are topologically s-cobordant.

In this paper, we are going to study the problem of uniqueness of the fibration structure (1). If for example $M = X \times F$, then there are at least two such structures. But even if we fix the base X and the fiber F, the following question arises:

Does M admit non-isomorphic fibration

$$F \xrightarrow{i} M \xrightarrow{\pi} X \quad \text{and} \quad F \xrightarrow{i'} M \xrightarrow{\pi'} X?$$

We are trying to construct invariants of such fibrations which are not invariants of the manifold M. A complete set of such invariants will give a classification of closed 4-manifolds fibering as in (1) in terms of homotopy classes of maps from X to certain classifying spaces described below. As a general reference on the algebraic theory of closed 4-manifolds see for example [6]. For a standard text on differential topology of fiber bundles we refer to [8].

2. The description in terms of classifying spaces.

Let X and F be given as above. We fix an orientation on $F = F_h$, where h is the genus of F.

Let

$$G = \begin{cases} \mathrm{Aut}^+(F) & \text{with the compact-open topology} \\ \mathrm{Diff}^+(F) & \text{with the } C^\infty\text{-topology,} \end{cases}$$

where $\mathrm{Aut}^+(F)$ (resp. $\mathrm{Diff}^+(F)$) is the group of orientation preserving homeomorphisms (resp. diffeomorphisms) of F.

Any fiber bundle (1), $F \to M \to X$, is classified by a map $f : X \to BG$. Isomorphism classes of bundles (1) correspond bijectively to homotopy classes of maps $X \to BG$, i.e. to elements of $[X, BG]$.

The component $G_0 \subset G$ of the identity is (weakly) contractible if $h \geq 2$ (see [3] and [4]). Classical results, due to Nielsen, Dehn, and Birman, imply the following canonical group isomorphisms:

$$G/G_0 \cong \mathrm{Aut}^+(\Pi_1(F_h))/\mathrm{Inn}(\Pi_1(F_h)) \cong E^+(F_h),$$

where

$$E^+(F_h) = \{[\varphi] \in [F_h, F_h] : \varphi \text{ orientation preserving homotopy equivalence of } F_h\}.$$

Note that $\Gamma_h := E^+(F_h)$ is just the *Teichmüller group* of F_h.

It follows that G/G_0 is a discrete group. Since $G_0 \simeq \{*\}$ (at least weakly), the fibration

$$G/G_0 \longrightarrow BG_0 \longrightarrow BG$$

implies that $BG \simeq K(\Gamma_h, 1) = B\Gamma_h$.

Assuming that the genus of X is ≥ 2, i.e. $X = K(\Pi_1(X), 1) = B\Pi_1(X)$, we obtain

$$[X, BG] \xrightarrow{\cong} [B\Pi_1(X), B\Gamma_h] \cong \mathrm{Hom}(\Pi_1(X), \Gamma_h)/\Gamma_h,$$

where Γ_h acts on $\mathrm{Hom}(\Pi_1(X), \Gamma_h)$ by setting $(\gamma\alpha)(c) = \gamma\alpha(c)\gamma^{-1}$ for any $\gamma \in \Gamma_h$, $\alpha \in \mathrm{Hom}(\Pi_1(X), \Gamma_h)$, and $c \in \Pi_1(X)$. The last isomorphism holds for discrete groups $\Pi = \Pi_1(X)$ and $\Gamma = \Gamma_h$. Under this isomorphism the classifying map $f : X \to BG$ of the fibration (1) goes to the induced homomorphism

$$f_* : \Pi_1(X) \to \Pi_1(BG) \cong \Pi_0(G/G_0) \cong \Gamma_h.$$

In particular, we note that the obvious map

$$\mathrm{Diff}^+(F) \to E^+(F) \tag{2}$$

is a homotopy equivalence (see [4]).

On the other hand, any fibration $F \to M \to X$ defines an element in

$$\mathrm{Ext}(\Pi_1(F), \Pi_1(X))$$

by the sequence:

$$1 \longrightarrow \Pi_1(F) \longrightarrow \Pi_1(M) \longrightarrow \Pi_1(X) \longrightarrow 1.$$

Conversely, given

$$[1 \to \Pi_1(F) \to \Pi \to \Pi_1(X) \to 1] \in \mathrm{Ext}(\Pi_1(F), \Pi_1(X)),$$

where $h =$ genus $F \geq 2$ and $g =$ genus $X \geq 2$, we obtain a homotopy fibration $F \to B\Pi \to X$ and a classifying map $f_1 : X \to BE^+(F)$. Using (2), we find a unique DIFF fiber bundle (up to bundle isomorphism) $F \to E \to X$ inducing the sequence:

$$1 \longrightarrow \Pi_1(F) \longrightarrow \Pi_1(E) = \Pi \longrightarrow \Pi_1(X) \longrightarrow 1.$$

Let $f : X \to BG$ be its classifying map. Now it is well-known that the set of extensions

$$[1 \to \Pi_1(F) \to \Pi \to \Pi_1(X) \to 1] \in \mathrm{Ext}(\Pi_1(F), \Pi_1(X)),$$

inducing the same homomorphism $f_* : \Pi_1(X) \to \Pi_1(BG) \cong \Gamma_h$, is isomorphic to $H^2(X; \zeta\Pi_1(F))$, where $\zeta\Pi_1(F)$ denotes the center of $\Pi_1(F)$ (see [10], p. 128). But $\zeta\Pi_1(F) = \{1\}$ if $h \geq 3$. Hence we conclude that the fibration structure $F \to M \to X$ is uniquely defined by the element

$$(3) \qquad [1 \to \Pi_1(F) \to \Pi \to \Pi_1(X) \to 1] \in \mathrm{Ext}(\Pi_1(F), \Pi_1(X))$$

if $h =$ genus $F \geq 3$.

In this case our question is equivalent to:

In how many elements of $\mathrm{Ext}(\Pi_1(F), \Pi_1(X))$ *can a given group* Π *occur* (*as in* (3))?

As pointed out by Hillman in [7], a closely related question was considered by Johnson in [9] from a purely algebraic point of view. He proved that if the homomorphism f_* is not injective but has infinite image, then the extension is unique; if f_* has finite image, there are at most two distinct extension structures, and that there are such groups Π with two extension structures. If Π has one extension structure with f_* injective, then all have this property, but he does not settle the question completely in this case. Note also that $\mathrm{Wh}(\Pi) = 0$ in all cases (see for example [6], V.1, p. 68).

3. Characteristic classes.

Let $F \to M \xrightarrow{\pi} X$ be given as in Section 1. Let $\xi \subset TM$ be the subbundle of vertical vectors, i.e. vectors tangent to the fibers. We assume that $\xi \to M$ is an orientable bundle. Such fibrations $F \to M \to X$ are called *orientable* in [12] and [13]. Let $e(\xi) \in H^2(M; \mathbf{Z})$ be the Euler class of ξ. Note that $e(M) = e(\xi)\pi^*(e(X))$, where $e(M) \in H^4(M; \mathbf{Z})$ and $e(X) \in H^2(X; \mathbf{Z})$ denote the Euler classes of M and X, respectively.

Following [12], we define $e_1(\xi) = \mathscr{G}_*(e(\xi)^2) \in H^2(X; \mathbf{Z})$, where $\mathscr{G}_*$ is the Gysin homomorphism

$$\begin{array}{ccc} H^4(M;\mathbf{Z}) & \xrightarrow{\mathscr{G}_*} & H^2(X;\mathbf{Z}) \\ \mathrm{PD}\downarrow\cong & & \cong\downarrow\mathrm{PD} \\ H_0(M;\mathbf{Z}) & \xrightarrow[\pi_*]{} & H_0(X;\mathbf{Z}). \end{array}$$

It is clear that the higher classes $e_j = \mathscr{G}_*(e^{j+1}) = 0$ in our case. However there is another characteristic class. The classifying map $f_* : \Pi_1(X) \to \Gamma_h$ composes with

$\sigma_* : \Gamma_h \to \mathrm{Sp}(2h; \boldsymbol{Z})$, which is induced by the action of

$$\Gamma_h = \mathrm{Aut}(\Pi_1(F))/\mathrm{Inn}(\Pi_1(F))$$

on $H^1(F; \boldsymbol{Z})$. The composition of the maps

$$\Pi_1(X) \to \Gamma_h \to \mathrm{Sp}(2h; \boldsymbol{Z}) \subset \mathrm{Sp}(2h; \boldsymbol{R}) \tag{4}$$

induces $X \to B\,\mathrm{Sp}(2h; \boldsymbol{R})$. Here we continue to assume that the genus of X is ≥ 2.

Let us consider the bundle η associated to the fiber bundle $F \to M \to X$. For every $x \in X$, the fiber of η over x is the real cohomology of the fiber F_x. Since the unitary group $\mathscr{U}(h)$ is a maximal torus in $\mathrm{Sp}(2h; \boldsymbol{R})$, the structure group of this bundle can be reduced to the unitary group, i.e. η can be considered as a complex vector bundle over X.

Then we have the first Chern class $c_1(\eta) \in H^2(X; \boldsymbol{Z})$.

Now from [12], p. 555, it follows that $e_1(\xi) = -12c_1(\eta)$.

One of the results proved by Meyer in [11], p. 246, is

$$\mathrm{Sign}(M) = -\langle 4c_1(\eta), [X]\rangle,$$

where $\mathrm{Sign}(M)$ denotes the signature of M.

Furthermore, assuming $h \geq 3$, any class $c_1(\eta)$ can be realized by a fibration $F \to M \to X$. Here $h \geq 3$ is fixed, but X may vary.

Summarizing we have

PROPOSITION 3.1. *The characteristic classes $c_1(\eta)$ and $e(\xi)$ of the fibration*

$$F \to M \to X$$

are uniquely defined by the signature of M. Moreover, since $\xi \to M$ is a bundle of dimension two, it is also defined by the signature (because it is defined by $e(\xi)$).

Now we can state the following problem: *define characteristic classes of the fibration $F \to M \to X$ using other representation of $\Pi_1(X)$ instead of* (4).

4. The spectral sequence and the intersection form.

The spectral sequence of the fibration $F \xrightarrow{i} M \xrightarrow{\pi} X$ is of the following type:

$$E_2^{pq} = H^p(X; \overline{H^q(F)}) \Rightarrow H^{p+q}(M; \boldsymbol{Z}),$$

where $\overline{H^q(F)} = \tilde{X} \times_{\Pi_1(X)} H^q(F; \boldsymbol{Z})$ is the coefficient system over X induced by the homomorphism $f_* : \Pi_1(X) \to \mathrm{Sp}(2h; \boldsymbol{Z})$.

Since we continue to assume that $F \to M \to X$ is oriented, we have

$$E_2^{0\,2} = H^0(X; \overline{H^2(F)}) \cong H^2(F)^{\Pi_1(X)} \cong H^2(F) \cong \mathbf{Z}$$

and

$$E_2^{p0} = H^p(X; \overline{H^0(F)}) \cong H^p(X; H^0(F)) \cong H^p(X; \mathbf{Z}).$$

The only possible non-trivial differentials are

$$d_2^{0\,2} : E_2^{0\,2} \cong \mathbf{Z} \to E_2^{2\,1} \quad \text{and} \quad d_2^{0\,1} : E_2^{0\,1} \to E_2^{2\,0} \cong H^2(X; \mathbf{Z}).$$

There is the following commutative diagram

$$\begin{array}{ccc} H^2(M;\mathbf{Z}) & \xrightarrow{i^*} & H^2(F;\mathbf{Z}) \cong \mathbf{Z} \\ {\scriptstyle \text{epi}}\downarrow & & \| \\ E_\infty^{0\,2} & \longrightarrow & E_2^{0\,2} \cong \mathbf{Z}. \end{array}$$

Since $i^*(e(\xi)) = e(F_h) \neq 0$, $h \geq 2$, it follows that $E_\infty^{0\,2} \cong \mathbf{Z}$, which implies $d_2^{0\,2} = 0$.
From this we obtain

(5) $$H^2(M;\mathbf{Z}) \cong H^2(F;\mathbf{Z}) \oplus H^1(X; \overline{H^1(F)}) \oplus E_2^{2\,0}/\operatorname{Im} d_2^{0\,1}.$$

Since $E_2^{2\,0} = H^2(X;\mathbf{Z}) \cong \mathbf{Z}$, there are three possibilities:

$$E_2^{2\,0}/\operatorname{Im} d_2^{0\,1} \cong \begin{cases} \mathbf{Z} \\ \mathbf{Z}/k\mathbf{Z} \\ 0. \end{cases}$$

Meyer has proved in [11] that

$$\operatorname{Sign}(M) = \operatorname{Sign}(H^1(X; \overline{H^1(F)}), \text{with respect to the obvious pairing}) = 4m$$

for some integer m. This implies that rank $H^2(M;\mathbf{Z})$ and rank $H^1(X; \overline{H^1(F)})$ are even. Then it follows from (5) that $E_2^{2\,0}/\operatorname{Im} d_2^{0\,1} \cong \mathbf{Z}$, hence $d_2^{0\,1} = 0$.
So we have proved the following result.

THEOREM 4.1. *If $F \to M \to X$ is an oriented fibration (i.e. the vertical subbundle $\xi \subset TM$ is oriented), then the spectral sequence*

$$E_2^{pq} = H^p(X; \overline{H^q(F)}) \Rightarrow H^{p+q}(M; \mathbf{Z})$$

collapses. In particular, we have

$$H^1(M;\mathbf{Z}) \cong H^1(F;\mathbf{Z})^{\Pi_1(X)} \oplus H^1(X;\mathbf{Z})$$

and

$$H^2(M;\mathbf{Z}) \cong H^2(F;\mathbf{Z}) \oplus H^1(X;\overline{H^1(F)}) \oplus H^2(X;\mathbf{Z}).$$

We remark that in [12] Morita proved that the spectral sequence of the rational cohomology of any surface bundle collapses.

As a consequence, the homomorphism $\pi^* : H^2(X;\mathbf{Z}) \to H^2(M;\mathbf{Z})$ is not trivial, and hence the quadratic form on $H^2(M;\mathbf{Z})$ is *indefinite*. Then we must distinguish two cases according to the intersection form is even or odd (in the second case there exists an element $x \in H^2(M;\mathbf{Z})$ such that $x^2 \neq 0 \pmod 2$).

Let us consider now the second Stiefel-Whitney class

$$w_2(M) = w_2(TM) \in H^2(M;\mathbf{Z}_2).$$

Let $\xi \subset TM$ be as above the subbundle of vectors tangent to the fibers of $M \xrightarrow{\pi} X$. Then we have $TM/\xi \cong \pi^*(TX)$, i.e. $w_2(M) = w_2(\xi) + \pi^*(w_2(X)) = w_2(\xi)$. Since $w_2(\xi) \equiv e(\xi) \pmod 2$, we conclude with the following implications:

$$\begin{aligned} w_2(M) = 0 &\Rightarrow e(\xi) = 2x \in H^2(M;\mathbf{Z}) \\ &\Rightarrow e_1(\xi) = \mathcal{G}_*(e(\xi)^2) \equiv 0 \pmod 4 \text{ in } H^2(X;\mathbf{Z}) \cong \mathbf{Z} \\ &\Rightarrow c_1(\eta) \equiv 0 \pmod 4 \\ &\Rightarrow \mathrm{Sign}(M) = -4\langle c_1(\eta), [X]\rangle \equiv 0 \pmod{16}. \end{aligned}$$

In particular, one reobtains Rohlin's theorem for the special class of 4-manifolds fibering over surfaces.

THEOREM 4.2. *The closed TOP or DIFF 4-manifolds M considered above satisfy the Rohlin theorem, i.e. $w_2(M) = 0$ implies that* $\mathrm{Sign}(M) \equiv 0$ (*mod* 16). *In this case the integral intersection form μ_M is always even.*

On the other hand, if $w_2(M) \neq 0$, then $e(\xi)^2 \neq 0 \pmod 2$, hence the intersection form μ_M is odd, i.e. it is of type:

$$\mu_M \cong (1) \oplus \cdots \oplus (1) \oplus (-1) \oplus \cdots \oplus (-1).$$

Finally, let us calculate the Euler characteristics for oriented fibrations

$$F \to M \to X,$$

i.e. ξ is oriented.

We have

$$\chi(M) = \chi(F)\chi(X) = (2-2h)(2-2g)$$

and

$$\chi(M) = 2 - 2(\operatorname{rank} H^1(F)^{\Pi_1(X)} + 2g) + \operatorname{rank} H^1(X; \overline{H^1(F)}) + 2,$$

hence

$$4gh - 4h = \operatorname{rank} H^1(X; \overline{H^1(F)}) - 2\operatorname{rank} H^1(F)^{\Pi_1(X)} \geq \operatorname{rank} H^1(X; \overline{H^1(F)}) - 4h.$$

Thus we have the following

PROPOSITION 4.3. rank $H^1(X; \overline{H^1(F)}) \leq 4gh$ *and equality holds if and only if* $H^1(F)^{\Pi_1(X)} \cong H^1(F)$, *i.e. if and only if the classifying map*

$$\Pi_1(X) \xrightarrow{f_*} \Gamma_h \xrightarrow{\sigma_*} \operatorname{Sp}(2h; \boldsymbol{Z})$$

vanishes.

5. The Pontryagin and Euler classes.

In dimension four the Hirzebruch formula for the signature writes as follows:

$$\operatorname{Sign}(M) = (1/3)\langle p_1(M), [M]\rangle.$$

Now the formula of Meyer [11]

$$\operatorname{Sign}(M) = -4\langle c_1(\eta), [X]\rangle$$

and the above calculated relations

$$\mathscr{G}_*(e(\xi)^2) = e_1(\xi) = -12c_1(\eta)$$

give

$$\operatorname{Sign}(M) = (1/3)\langle e_1(\xi), [X]\rangle.$$

The commutative diagram in Section 3 implies that

$$\pi_*\langle e(\xi)^2, [M]\rangle = \langle e_1(\xi), [X]\rangle,$$

where $\pi_* : H_0(M; \boldsymbol{Z}) \xrightarrow{\cong} H_0(X; \boldsymbol{Z})$.

But $\pi_* =$ identity via the identification $H_0(M; \boldsymbol{Z}) \cong \boldsymbol{Z} \cong H_0(X; \boldsymbol{Z})$, hence

$$(1/3)\langle e(\xi)^2, [M]\rangle = \operatorname{Sign}(M) = (1/3)\langle p_1(M), [M]\rangle,$$

i.e.

$$p_1(M) = e(\xi)^2.$$

This is a well known relation between p_1 and e^2.

For the Euler class we have

$$e(M) = e(\xi)\pi^*(e(X)).$$

Now recall the product BSO(4) = BSO(3) × BSU(2) induced by the fibration (which has a section):

$$\mathrm{SO}(3) \longrightarrow \mathrm{SO}(4) \longrightarrow S^3 = \mathrm{SU}(2).$$

Hence we have $p_1 \in H^4(\mathrm{BSO}(3); \boldsymbol{Z}) \cong \boldsymbol{Z}$ and $H^4(\mathrm{BSO}(4); \boldsymbol{Z}) \cong \boldsymbol{Z}[p_1] \oplus \boldsymbol{Z}[e]$, so $p_1(M)$ and $e(M)$ determine the tangent bundle TM.

Remark 1: If $e(\xi) = 0$, then $p_1(M) = e(M) = 0$. Since $i^*(e(\xi)) = e(\xi|_F) = 0$, it follows that $h = 1$, i.e. $F = S^1 \times S^1$. In this case we have a map

$$\Pi_1(X) \to \Gamma_1 = \mathrm{SL}(2; \boldsymbol{Z}) = \mathrm{Sp}(2; \boldsymbol{Z}).$$

Assume that the composition $\Pi_1(X) \to \Gamma_1 = \mathrm{SL}(2; \boldsymbol{Z}) \to \mathrm{SL}(2; \boldsymbol{R})$ induces the constant map. Does it follow that $\Pi_1(X) \to \mathrm{SL}(2; \boldsymbol{Z})$ is trivial? (in other words: Is then $M = X \times F$?). As remarked by Hillman in [7], M need not be a product. Let N be an orientable S^1-bundle over the torus T with nonzero Euler class. Assume that $\Pi_1(T)$ acts trivially on the fibre. Then $N \times S^1$ is a T-bundle over T of the requested type.

Remark 2: It is well-known that an aspherical 4-manifold M which fibres over a surface admits the geometry $H^2 \times H^2$ if and only if the map $f_* : \Pi_1(M) \to \Pi_1(\mathrm{BG})$ (our notation in Section 2) has finite image (see for example [6]).

Open problem (J. A. Hillman) Find examples of aspherical surface bundles over surfaces which admit one of the geometries H^4 *or* $H^2(C)$ (note that f_* must be injective in these cases).

References

[1] Cavicchioli, A., Hegenbarth, F. and Repovš, D., On the stable classification of certain 4-manifolds, Bull. Austral. Math. Soc. **52** (1995), 385–398.

[2] Cavicchioli, A., Hegenbarth, F. and Repovš, D., Four-manifolds with surface fundamental groups, Trans. Amer. Math. Soc., **349** (10) (1997), 4007–4019.
[3] Earle, C. J. and Eells, J., The diffeomorphism group of a compact Riemann surface, Bull. Amer. Math. Soc. **73** (1967), 557–559.
[4] Earle, C. J. and Eells, J., A fibre bundle description of Teichmüller theory, J. Diff. Geometry **3** (1969), 19–43.
[5] Hillman, J. A., On 4-manifolds homotopy equivalent to surface bundles over surfaces, Topology and its Appl. **40** (1991), 275–286.
[6] Hillman, J. A., The algebraic characterization of geometric 4-manifolds, London Math. Soc. Lect. Note Ser. 198, Cambridge Univ. Press, Cambridge, 1994.
[7] Hillman, J. A., Personal communication (1996).
[8] Husemoller, D., Fibre bundles, Springer Verlag, Berlin-Heidelberg-New York, 1973.
[9] Johnson, F. E. A., A group theoretic analogue of the Parshin-Arakelov rigidity theorem, Arch. Math. **63** (1994), 354–361.
[10] Mac Lane, S., Homology, Springer Verlag, Berlin-Heidelberg-New York, 1963.
[11] Meyer, W., Die signatur von flächenbündeln, Math. Ann. **201** (1973), 239–264.
[12] Morita, S., Characteristic classes of surface bundles, Invent. Math. **90** (1987), 551–577.
[13] Morita, S., Characteristic classes of surface bundles and the Casson invariant, Sugaku Exp. **7** (1994), 59–79.

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