TSUKUBA J. MATH.
Vol. 24 No. 1 (2000), 209–219

ON THE MICROLOCAL STRUCTURE OF REGULAR SIMPLE PREHOMOGENEOUS VECTOR SPACE $(\mathrm{GL}(1)^2 \times \mathrm{SL}(7), \Lambda_3 + \Lambda_1^*)$

By

Shin-ichi KASAI

Abstract. The purpose of this paper is to calculate the b-function of the regular simple prehomogeneous vector space $(\mathrm{GL}(1)^2 \times \mathrm{SL}(7), \Lambda_3 + \Lambda_1^*)$ by the aid of microlocal method using the holonomy diagram of relative invariants.

1. Main Results

In the present paper, we consider a special reducible prehomogeneous vector space, namely the regular simple prehomogeneous vector space $(\mathrm{GL}(1)^2 \times \mathrm{SL}(7), \Lambda_3 + \Lambda_1^*)$ (A(14) of §3 in [5]). Following the ideas of [7], we shall determine the holonomy diagram and compute the b-function. From the result of Kimura [5], there are twelve regular simple prehomogeneous vector spaces which have two algebraically independent relative invariants. The b-functions of four of them are reduced to the case of irreducible prehomogeneous vector spaces. In [1, 2], we have studied three of the remaining nontrivial eight cases. We use the same notations in [1, 4, 7].

By the microlocal calculus on analysis of prehomogeneous vector spaces, we obtain the holonomy diagram (Figure 1) and the following theorem.

THEOREM. *The b-function of* $(\mathrm{GL}(1)^2 \times \mathrm{SL}(7), \Lambda_3 + \Lambda_1^*)$ *is given as follows.*

$$b_\chi(s_1, s_2) = [s_1 + 1]_{n_1} [s_1 + 2]_{n_2} \left[s_1 + \frac{5}{2}\right]_{n_1} [s_1 + s_2 + 3]_{n_1+n_2}$$

Received April 28, 1999
Revised November 12, 1999

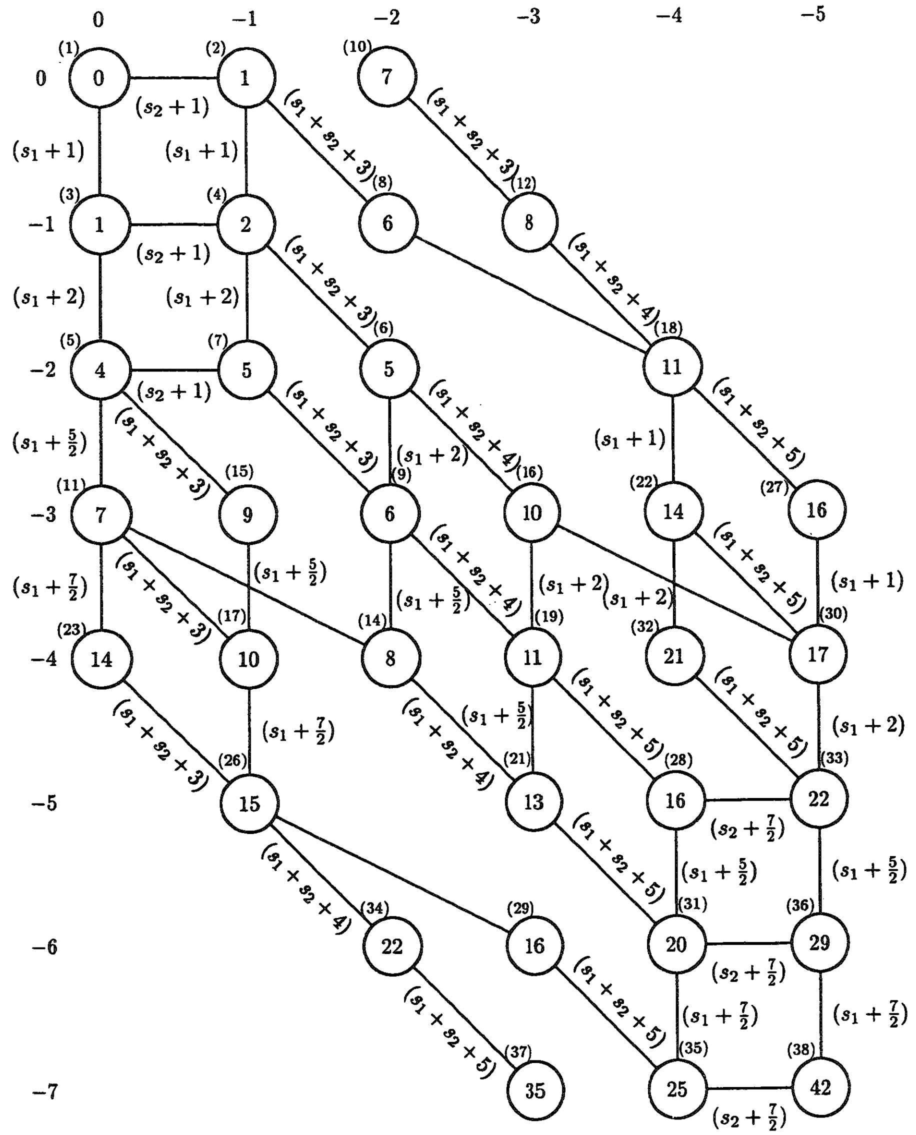

Figure 1. The holonomy diagram of $(GL(1)^2 \times SL(7), \Lambda_3 + \Lambda_1^*)$.

$$\left[s_1 + \frac{7}{2}\right]_{n_1} [s_1 + s_2 + 4]_{n_1+n_2} [s_1 + s_2 + 5]_{n_1+n_2}$$

$$[s_2 + 1]_{n_2} \left[s_2 + \frac{7}{2}\right]_{n_2}$$

for $\chi = \chi_1^{n_1} \chi_2^{n_2}$ $(n_i > 0)$ *where* $[\alpha]_k = \alpha(\alpha+1)\cdots(\alpha+k-1)$.

2. Preliminary results

In the following, we denote by G the group $GL(1)^2 \times SL(7)$ and by ρ the representation $\Lambda_3 + \Lambda_1^*$ of G. We define an element e_i of C^7 by $e_i = {}^t(0, \dots, 0, \overset{i}{\check{1}}, 0, \dots, 0)$ for $0 \leq i \leq 7$. The representation space is identified with $V = \{\tilde{x} = (x, y); x = \sum_{1 \leq i < j < k \leq 7} x_{ijk} e_i \wedge e_j \wedge e_k \in \bigwedge^3 C^7, y = \sum_{l=1}^7 y_l e_l \in C^7\}$. Then the action ρ is given by $\rho(\tilde{g})\tilde{x} = (\alpha\rho_3(g)x, \beta {}^t g^{-1} y)$ for $\tilde{g} = (\alpha, \beta; g) \in G = GL(1)^2 \times SL(7)$ and $\tilde{x} = (x, y) \in V$ where $\rho_3(g)x = \sum x_{ijk}(ge_i) \wedge (ge_j) \wedge (ge_k)$.

We define $\partial/\partial e_l$ by $\partial/\partial e_l(e_i \wedge e_j \wedge e_k) = \delta_{il} e_j \wedge e_k$ for $j, k \neq l$. Let $\varphi(x) = (\varphi_{ij}(x))$ be the 7×7 symmetric matrix obtained by $\varphi_{ij}(x) e_1 \wedge \cdots \wedge e_7 = x \wedge \partial x / \partial e_i \wedge \partial x / \partial e_j (i, j = 1, \dots, 7)$. Then $\varphi_{ij}(x)$ is a homogeneous polynomial of degree 3 and $\varphi(\rho_3(g)x) = g\varphi(x){}^t g$ for $g \in SL(7)$ and $x \in \bigwedge^3 C^7$. We define $f^i_{jk}(x)$ by $f^i_{jk} e_1 \wedge \cdots \wedge e_7 = x \wedge \partial x / \partial e_i \wedge e_j \wedge e_k$ $(i, j, k = 1, \dots, 7)$. Let $\varphi^*(x) = (\varphi^*_{ij}(x))$ be the 7×7 symmetric matrix where $\varphi^*_{ij}(x) = \sum_{s,t=1}^7 f^s_{it}(x) f^t_{sj}(x)$. Then $\varphi^*_{ij}(x)$ is homogeneous polynomial of degree 4 and $\varphi^*(\rho_3(g)x) = {}^t g^{-1} \varphi^*(x) g^{-1}$ for $g \in SL(7)$ and $x \in \bigwedge^3 C^7$ (see [3]).

Proposition 1([5]). (1) *The triplet* (G, ρ, V) *is a regular P.V.*

(2) *The algebraically independent relative invariants are given by* $f_1(\tilde{x}) = tr\varphi(x)\varphi^*(x)$ *and* $f_2(\tilde{x}) = {}^t y \varphi(x) y$ *for* $\tilde{x} = (x, y) \in V$ (See [3]).

(3) *Let* χ_i *be the caharacter of* f_i. *Then their infinitesimal characters are given by* $\delta\chi_1(\tilde{A}) = 7\alpha$ *and* $\delta\chi_2(\tilde{A}) = 3\alpha + 2\beta$ *for* $\tilde{A} = (\alpha, \beta; A) \in \mathfrak{gl}(1)^2 \oplus \mathfrak{sl}(7)$.

Let Λ be the conormal bundle of an orbit S in V and Λ^* that of an orbit S^* in V^*. When $\Lambda = \Lambda^*$, we say that S and S^* are the dual orbits of each other. We identify the dual space V^* with V as usual. Since G is reductive, we have $(G, \rho, V) \cong (G, \rho^*, V^*)$ and hence (G, ρ, V) and (G, ρ^*, V^*) have the same number of G-orbits.

Put $x_0 = 234 + 567 + 1(25 + 36 + 47)$, $x_1 = 235 + 346 + 1(27 - 45)$, $x_4 = 134 + 256 + 127$, $x_7 = 234 + 1(25 + 36 + 47)$, $x_9 = 123 + 456$, $x_{10} = 126 - 135 + 234$, $x_{14} = 1(25 + 36 + 47)$, $x_{15} = 1(24 + 35)$, $x_{22} = 123$ and $x_{35} = 0$ where ijk stands for $e_i \wedge e_j \wedge e_k$.

Proposition 2([6]). *The triplet* (G, ρ, V) *has the following thirty-eight orbits.*

Representative point	Codim	Dual orbit
(1) $\tilde{x}_1 = (x_0, e_1)$	0	$\tilde{x}_{38}$
(2) $\tilde{x}_2 = (x_0, e_2)$	1	$\tilde{x}_{35}$

Representative point	Codim	Dual orbit
(3) $\tilde{x}_3 = (x_1, e_1 + e_4)$	1	$\tilde{x}_{36}$
(4) $\tilde{x}_4 = (x_1, e_1)$	2	$\tilde{x}_{31}$
(5) $\tilde{x}_5 = (x_4, e_1 + e_2)$	4	$\tilde{x}_{33}$
(6) $\tilde{x}_6 = (x_1, e_5)$	5	$\tilde{x}_{21}$
(7) $\tilde{x}_7 = (x_4, e_1)$	5	$\tilde{x}_{28}$
(8) $\tilde{x}_8 = (x_1, e_6)$	6	$\tilde{x}_{29}$
(9) $\tilde{x}_9 = (x_4, e_3 + e_5)$	6	$\tilde{x}_{19}$
(10) $\tilde{x}_{10} = (x_0, 0)$	7	$\tilde{x}_{37}$
(11) $\tilde{x}_{11} = (x_7, e_1)$	7	$\tilde{x}_{30}$
(12) $\tilde{x}_{12} = (x_1, 0)$	8	$\tilde{x}_{34}$
(13) $\tilde{x}_{13} = (x_4, e_3)$	8	$\tilde{x}_{20}$
(14) $\tilde{x}_{14} = (x_7, e_2)$	8	$\tilde{x}_{16}$
(15) $\tilde{x}_{15} = (x_9, e_1 + e_4)$	9	$\tilde{x}_{32}$
(16) $\tilde{x}_{16} = (x_4, e_7)$	10	$\tilde{x}_{14}$
(17) $\tilde{x}_{17} = (x_{10}, e_1)$	10	$\tilde{x}_{22}$
(18) $\tilde{x}_{18} = (x_4, 0)$	11	$\tilde{x}_{26}$
(19) $\tilde{x}_{19} = (x_7, e_5)$	11	$\tilde{x}_9$
(20) $\tilde{x}_{20} = (x_9, e_1)$	12	$\tilde{x}_{13}$
(21) $\tilde{x}_{21} = (x_{10}, e_4)$	13	$\tilde{x}_6$
(22) $\tilde{x}_{22} = (x_7, 0)$	14	$\tilde{x}_{17}$
(23) $\tilde{x}_{23} = (x_{14}, e_1)$	14	$\tilde{x}_{27}$
(24) $\tilde{x}_{24} = (x_9, e_7)$	15	$\tilde{x}_{25}$
(25) $\tilde{x}_{25} = (x_{14}, e_2)$	15	$\tilde{x}_{24}$
(26) $\tilde{x}_{26} = (x_{15}, e_1)$	15	$\tilde{x}_{18}$
(27) $\tilde{x}_{27} = (x_9, 0)$	16	$\tilde{x}_{23}$
(28) $\tilde{x}_{28} = (x_{10}, e_7)$	16	$\tilde{x}_7$
(29) $\tilde{x}_{29} = (x_{15}, e_2)$	16	$\tilde{x}_8$
(30) $\tilde{x}_{30} = (x_{10}, 0)$	17	$\tilde{x}_{11}$
(31) $\tilde{x}_{31} = (x_{15}, e_6)$	20	$\tilde{x}_4$
(32) $\tilde{x}_{32} = (x_{14}, 0)$	21	$\tilde{x}_{15}$
(33) $\tilde{x}_{33} = (x_{15}, 0)$	22	$\tilde{x}_5$
(34) $\tilde{x}_{34} = (x_{22}, e_1)$	22	$\tilde{x}_{12}$
(35) $\tilde{x}_{35} = (x_{22}, e_4)$	25	$\tilde{x}_2$
(36) $\tilde{x}_{36} = (x_{22}, 0)$	29	$\tilde{x}_3$
(37) $\tilde{x}_{37} = (x_{35}, e_1)$	35	$\tilde{x}_{10}$
(38) $\tilde{x}_{38} = (x_{35}, 0)$	42	$\tilde{x}_1$

3. Holonomy diagram

For a point $\tilde{x}$ of V, let $G_{\tilde{x}} = \{g \in G; \rho(g)\tilde{x} = \tilde{x}\}$ be the isotropy subgroup of G at $\tilde{x}$, and let $V_{\tilde{x}}^*$ be the conormal vector space. Then $G_{\tilde{x}}$ acts on $V_{\tilde{x}}^*$ by $\rho_{\tilde{x}} = \rho^*|_{G_{\tilde{x}}}$. If the triplet $(G_{\tilde{x}}, \rho_{\tilde{x}}, V_{\tilde{x}}^*)$ is a P.V., then we denote by y_0 its generic point, and if there is one one-codimensional orbit, then y_1 denotes a point of that orbit. When there exist several one-codimensional orbits, we denote representative points of their orbits by y_1, y_1', etc. Let $\mathfrak{G}_{\tilde{x}}$ (resp. $\mathfrak{G}$) be the Lie algebra of $G_{\tilde{x}}$ (resp. G), and $d\rho_{\tilde{x}}$ the infinitesimal representation of $\rho_{\tilde{x}}$. We denote by A_0 an element of $\mathfrak{G}_{\tilde{x}} = \{A \in \mathfrak{G}; d\rho(A)\tilde{x} = 0\}$ such that $d\rho^*(A_0)y_0 = y_0$. We denote by Λ_i the conormal bundle $T(\rho(G)\tilde{x}_i)^{\perp}$ of a orbit $\rho(G)\tilde{x}_i$.

(1) The case for $\tilde{x}_1$. Since $\tilde{x}_1$ is a generic point of (G, ρ, V), the isotropy subalgebra $\mathfrak{G}_{\tilde{x}_1}$ is isomorphic to $\mathfrak{sl}(3)$ (see $A(14)$ of §3 in [5]). Since $\Lambda_1 = V \times \{0\}$, we have $ord_{\Lambda_1} f^s = 0$.

(2) The case for $\tilde{x}_2$. $V_{\tilde{x}_2}^* = \boldsymbol{C}\langle v_1 \rangle$ where $v_1 = (267, e_5)$. $y_0 = v_1 \in \rho^*(G)\tilde{x}_{35}$, $y_1 = 0 \in \rho^*(G)\tilde{x}_{38}$. $ord_{\Lambda_2} f^s = -s_2 - 1/2$.

(3) The case for $\tilde{x}_3$. $V_{\tilde{x}_3}^* = \boldsymbol{C}\langle v_1 \rangle$ where $v_1 = (567, 0)$. $y_0 = v_1 \in \rho^*(G)\tilde{x}_{36}$, $y_1 = 0 \in \rho^*(G)\tilde{x}_{38}$. $ord_{\Lambda_3} f^s = -s_1 - 1/2$.

(4) The case for $\tilde{x}_4$. $V_{\tilde{x}_4}^* = \boldsymbol{C}\langle v_1, v_2 \rangle$ where $v_1 = (567, 0)$ and $v_2 = (136, -e_4)$. $y_0 = v_1 + v_2 \in \rho^*(G)\tilde{x}_{31}$. $y_1 = v_1 \in \rho^*(G)\tilde{x}_{36}$, $y_1' = v_2 \in \rho^*(G)\tilde{x}_{35}$. $ord_{\Lambda_4} f^s = -s_1 - s_2 - 1$.

(5) The case for $\tilde{x}_5$. $V_{\tilde{x}_5}^* = \boldsymbol{C}\langle v_1, \dots, v_4 \rangle$ where $v_1 = (357, 0)$, $v_2 = (367, 0)$, $v_3 = (457, 0)$ and $v_4 = (467, 0)$. $(G_{\tilde{x}_5}, \rho_{\tilde{x}_5}, V_{\tilde{x}_5}^*) \cong (SL(2) \times GL(2), \Lambda_1 \otimes \Lambda_1, V(2) \otimes V(2))$. $y_0 = v_1 + v_4 \in \rho^*(G)\tilde{x}_{33}$. $y_1 = v_1 \in \rho^*(G)\tilde{x}_{36}$. $ord_{\Lambda_5} f^s = -2s_1 - 2$.

(6) The case for $\tilde{x}_6$. $V_{\tilde{x}_6}^* = \boldsymbol{C}\langle v_1, \dots, v_5 \rangle$ where $v_1 = (567, 0)$, $v_2 = (-257 - 467, e_1)$, $v_3 = (157 + 367, e_2)$, $v_4 = (267 - 456, e_3)$ and $v_5 = (-167 + 356, e_4)$. $y_0 = v_3 + v_4 \in \rho^*(G)\tilde{x}_{21}$, $y_1 = v_3 + v_5 \in \rho^*(G)\tilde{x}_{31}$. $ord_{\Lambda_6} f^s = -2s_1 - 2s_2 - 7/2$.

(7) The case for $\tilde{x}_7$. $V_{\tilde{x}_7}^* = \boldsymbol{C}\langle v_1, \dots, v_5 \rangle$ where $v_1 = (357, 0)$, $v_2 = (367, 0)$, $v_3 = (457, 0)$, $v_4 = (467, 0)$ and $v_5 = (156, e_2)$. $(G_{\tilde{x}_7}, \rho_{\tilde{x}_7}, V_{\tilde{x}_7}^*) \cong (GL(1) \times SO(4) \times GL(1), \Lambda_1 \otimes \Lambda_1 \otimes 1 + 1 \otimes 1 \otimes \Lambda_1, V(4) + V(1))$. $y_0 = v_1 + v_4 + v_5 \in \rho^*(G)\tilde{x}_{28}$. $y_1 = v_1 + v_4 \in \rho^*(G)\tilde{x}_{33}$, $y_1' = v_1 + v_5 \in \rho^*(G)\tilde{x}_{31}$. $ord_{\Lambda_7} f^s = -2s_1 - s_2 - 5/2$.

(8) The case for $\tilde{x}_8$. $V_{\tilde{x}_8}^* = \boldsymbol{C}\langle v_1, \dots, v_6 \rangle$ where $v_1 = (567, 0)$, $v_2 = (-256, e_3)$, $v_3 = (156, e_4)$, $v_4 = (-267, e_1)$, $v_5 = (167, e_2)$ and $v_6 = (126, e_7)$. $y_0 = v_1 + v_6 \in \rho^*(G)\tilde{x}_{29}$. $y_1 = v_6 \in \rho^*(G)\tilde{x}_{35}$, $y_1' = v_3 + v_4 \in \rho^*(G)\tilde{x}_{29}$. $ord_{\Lambda_8} f^s = -s_1 - 2s_2 - 3$.

(9) The case for $\tilde{x}_9$. $V^*_{\tilde{x}_9} = C\langle v_1, \dots, v_6\rangle$ where $v_1 = (357, 0)$, $v_2 = (367, 0)$, $v_3 = (457, 0)$, $v_4 = (467, 0)$, $v_5 = (-237 + 345, e_1)$ and $v_6 = (157 + 356, e_2)$. $y_0 = v_4 + v_5 + v_6 \in \rho^*(G)\tilde{x}_{19}$. $y_1 = v_5 + v_6 \in \rho^*(G)\tilde{x}_{21}$, $y'_1 = v_4 + v_5 \in \rho^*(G)\tilde{x}_{28}$ and $y''_1 = v_4 + v_6 \in \rho^*(G)\tilde{x}_{28}$. $ord_{\Lambda_9} f^s = -3s_1 - 2s_2 - 5$.

(10) The case for $\tilde{x}_{10}$. $V^*_{\tilde{x}_{10}} = C\langle (0, e_i); 1 \le i \le 7\rangle$. $(G_{\tilde{x}_{10}}, \rho_{\tilde{x}_{10}}, V^*_{\tilde{x}_{10}}) \cong (GL(1) \times G_2, \Lambda_1 \otimes \Lambda_2, V(7))$. $y_0 = (0, e_1) \in \rho^*(G)\tilde{x}_{37}$, $y_1 = (0, e_2) \in \rho^*(G)\tilde{x}_{37}$. $ord_{\Lambda_{10}} f^s = -2s_2 - 7/2$.

(11) The case for $\tilde{x}_{11}$. $V^*_{\tilde{x}_{11}} = C\langle v_1, \dots, v_7\rangle$ where $v_1 = (567, 0)$, $v_2 = (267, 0)$, $v_3 = (456, 0)$, $v_4 = (-357, 0)$, $v_5 = (257 - 367, 0)$, $v_6 = (457 - 356, 0)$ and $v_7 = (256 + 467, 0)$. $y_0 = v_2 + v_3 - v_4 \in \rho^*(G)\tilde{x}_{30}$, $y_1 = v_2 + v_3 \in \rho^*(G)\tilde{x}_{33}$. $ord_{\Lambda_{11}} f^s = -3s_1 - 4$.

(12) The case for $\tilde{x}_{12}$. $V^*_{\tilde{x}_{12}} = C\langle v_1, \dots, v_8\rangle$ where $v_1 = (567, 0)$ and $v_{1+i} = (0, e_i)$, $1 \le i \le 7$. $y_0 = v_1 + v_6 \in \rho^*(G)\tilde{x}_{34}$. $y_1 = v_6 \in \rho^*(G)\tilde{x}_{31}$, $y'_1 = v_1 + v_7 \in \rho^*(G)\tilde{x}_{37}$. $ord_{\Lambda_{12}} f^s = -s_1 - 3s_2 - 6$.

(13) The case for $\tilde{x}_{13}$. $V^*_{\tilde{x}_{13}} = C\langle v_1, \dots, v_8\rangle$ where $v_1 = (357, 0)$, $v_2 = (367, 0)$, $v_3 = (457, 0)$, $v_4 = (467, 0)$, $v_5 = (-237, e_1)$, $v_6 = (356, e_2)$, $v_7 = (236, e_5)$ and $v_8 = (-235, e_6)$. $y_0 = v_3 + v_7 \in \rho^*(G)\tilde{x}_{20}$. $y_1 = v_1 + v_4 + v_7 \in \rho^*(G)\tilde{x}_{21}$. $A_0 = (\alpha, \alpha; diag(-3\alpha - 2, 2, \alpha, \alpha + 2, -\alpha/2 - 1, -\alpha/2 - 1, 2\alpha)) \in \mathfrak{G}_{\tilde{x}_{13}}$. Then $\delta\chi_1(A_0) = 7\alpha$ and $\delta\chi_2(A_0) = 5\alpha$.

(14) The case for $\tilde{x}_{14}$. $V^*_{\tilde{x}_{14}} = C\langle v_1, \dots, v_8\rangle$ where $v_1 = (567, 0)$, $v_2 = (267, 0)$, $v_3 = (256 + 467, 0)$, $v_4 = (257 - 367, 0)$, $v_5 = (456, 0)$, $v_6 = (457 - 356, 0)$, $v_7 = (357, 0)$ and $v_8 = (-167 + 236 + 247, 2e_1)$. $y_0 = v_6 + v_8 \in \rho^*(G)\tilde{x}_{16}$. $y_1 = v_7 + v_8 \in \rho^*(G)\tilde{x}_{19}$, $y'_1 = v_2 + v_6 \in \rho^*(G)\tilde{x}_{30}$. $ord_{\Lambda_{14}} f^s = -4s_1 - 2s_2 - 7$.

(15) The case for $\tilde{x}_{15}$. $V^*_{\tilde{x}_{15}} = C\langle v_1, \dots, v_9\rangle$ where $v_1 = (147, 0)$, $v_2 = (-347, 0)$, $v_3 = (247, 0)$, $v_4 = (-167, 0)$, $v_5 = (157, 0)$, $v_6 = (257, 0)$, $v_7 = (267, 0)$, $v_8 = (357, 0)$ and $v_9 = (367, 0)$. $y_0 = v_1 + v_6 + v_9 \in \rho^*(G)\tilde{x}_{32}$. $y_1 = v_6 + v_9 \in \rho^*(G)\tilde{x}_{33}$. $ord_{\Lambda_{15}} f^s = -3s_1 - s_2 - 9/2$.

(16) The case for $\tilde{x}_{16}$. $V^*_{\tilde{x}_{16}} = C\langle v_1, \dots, v_{10}\rangle$ where $v_1 = (357, 0)$, $v_2 = (367, 0)$, $v_3 = (457, 0)$, $v_4 = (467, 0)$, $v_5 = (347, e_1)$, $v_6 = (567, e_2)$, $v_7 = (-147 + 456, e_3)$, $v_8 = (137 - 356, e_4)$, $v_9 = (-267 - 346, e_5)$ and $v_{10} = (257 + 345, e_6)$. $y_0 = v_4 + v_8 + v_{10} \in \rho^*(G)\tilde{x}_{14}$. $y_1 = v_8 + v_{10} \in \rho^*(G)\tilde{x}_{21}$. $ord_{\Lambda_{16}} f^s = -3s_1 - 3s_2 - 7$.

(17) The case for $\tilde{x}_{17}$. $V^*_{\tilde{x}_{17}} = C\langle v_1, \dots, v_{10}\rangle$ where $v_1 = (-467, 0)$, $v_2 = (457, 0)$, $v_3 = (567, 0)$, $v_4 = (147, 0)$, $v_5 = (-167 - 347, 0)$, $v_6 = (157 + 247, 0)$, $v_7 = (-267 - 357, 0)$, $v_8 = (367, 0)$, $v_9 = (257, 0)$ and $v_{10} = (456, 0)$. $y_0 = v_4 - v_7 +$

$v_{10} \in \rho^*(G)\tilde{x}_{22}$. $y_1 = -v_7 + v_{10} \in \rho^*(G)\tilde{x}_{30}$, $y_1' = v_4 + v_8 + v_9 \in \rho^*(G)\tilde{x}_{32}$. $ord_{\Lambda_{17}} f^s = -4s_1 - s_2 - 13/2$.

(18) The case for $\tilde{x}_{18}$. $V^*_{\tilde{x}_{18}} = C\langle v_1, \dots, v_{11}\rangle$ where $v_1 = (357, 0)$, $v_2 = (367, 0)$, $v_3 = (457, 0)$, $v_4 = (467, 0)$ and $v_{4+i} = (0, e_i)$, $1 \le i \le 7$. $y_0 = v_1 + v_4 + v_{11} \in \rho^*(G)\tilde{x}_{26}$. $y_1 = v_1 + v_4 + v_7 + v_9 \in \rho^*(G)\tilde{x}_{29}$, $y_1' = v_1 + v_{11} \in \rho^*(G)\tilde{x}_{34}$. $ord_{\Lambda_{18}} f^s = -2s_1 - 4s_2 - 19/2$.

(19) The case for $\tilde{x}_{19}$. $V^*_{\tilde{x}_{19}} = C\langle v_1, \dots, v_{11}\rangle$ where $v_1 = (567, 0)$, $v_2 = (456, 0)$, $v_3 = (356 - 457, 0)$, $v_4 = (357, 0)$, $v_5 = (256 + 467, 0)$, $v_6 = (257 - 367, 0)$, $v_7 = (267, 0)$, $v_8 = (-457, e_1)$, $v_9 = (345, e_2)$, $v_{10} = 1/2(156 - 245 + 346, 2e_3)$ and $v_{11} = 1/2(157 + 235 + 347, 2e_4)$. $y_0 = v_4 + v_7 + v_{10} \in \rho^*(G)\tilde{x}_9$. $y_1 = v_4 + v_{10} \in \rho^*(G)\tilde{x}_{14}$, $y_1' = v_7 + v_{10} \in \rho^*(G)\tilde{x}_{19}$. $ord_{\Lambda_{19}} f^s = -4s_1 - 3s_2 - 17/2$.

(20) The case for $\tilde{x}_{20}$. $V^*_{\tilde{x}_{20}} = C\langle v_1, \dots, v_{12}\rangle$ where $v_1 = (147, 0)$, $v_2 = (157, 0)$, $v_3 = (167, 0)$, $v_4 = (247, 0)$, $v_5 = (257, 0)$, $v_6 = (267, 0)$, $v_7 = (347, 0)$, $v_8 = (357, 0)$, $v_9 = (367, 0)$, $v_{10} = (156, e_4)$, $v_{11} = (-146, e_5)$ and $v_{12} = (145, e_6)$. $y_0 = v_4 + v_8 + v_{10} \in \rho^*(G)\tilde{x}_{13}$. There is no one-codimensional orbit. $A_0 \in \mathfrak{G}_{\tilde{x}_{20}}$ with $\beta = -5\alpha - 4$, $A = diag(-5\alpha - 4, 2(\alpha + 1), 2(\alpha + 1), -5\alpha - 3, -5\alpha - 3, 9\alpha + 6, 2\alpha)$. $\delta\chi_1(A_0) = 7\alpha$, $\delta\chi_2(A_0) = -7\alpha - 8$.

(21) The case for $\tilde{x}_{21}$. $V^*_{\tilde{x}_{21}} = C\langle v_1, \dots, v_{13}\rangle$ where $v_1 = (467, 0)$, $v_2 = (-457, 0)$, $v_3 = (567, 0)$, $v_4 = (456, 0)$, $v_5 = (147, 0)$, $v_6 = (-167 - 347, 0)$, $v_7 = (157 + 247, 0)$, $v_8 = (367, 0)$, $v_9 = (-267 - 357, 0)$, $v_{10} = (257, 0)$, $v_{11} = (146, e_2)$, $v_{12} = (-145, e_3)$ and $v_{13} = 1/2(-156 - 246 + 345, 2e_1)$. $y_0 = v_5 + v_9 + v_{13} \in \rho^*(G)\tilde{x}_6$. $y_1 = v_5 + v_{10} + v_{13} \in \rho^*(G)\tilde{x}_9$, $y_1' = v_9 + v_{13} \in \rho^*(G)\tilde{x}_{16}$, and $y_1'' = v_9 + v_{10} + v_{11} \in \rho^*(G)\tilde{x}_{13}$. $ord_{\Lambda_{21}} f^s = -5s_1 - 3s_2 - 21/2$.

(22) The case for $\tilde{x}_{22}$. $V^*_{\tilde{x}_{22}} = C\langle v_1, \dots, v_{14}\rangle$ where $v_1 = (567, 0)$, $v_2 = (267, 0)$, $v_3 = (456, 0)$, $v_4 = (-357, 0)$, $v_5 = (257 - 367, 0)$, $v_6 = (457 - 356, 0)$, $v_7 = (256 + 467, 0)$ and $v_{7+i} = (0, e_i)$, $1 \le i \le 7$. $y_0 = v_2 + v_3 - v_4 + v_{12} \in \rho^*(G)\tilde{x}_{17}$. $y_1 = v_2 + v_3 + v_{13} \in \rho^*(G)\tilde{x}_{26}$, $y_1' = v_2 + v_3 - v_4 + v_{12} + \sqrt{-1}v_{13} \in \rho^*(G)\tilde{x}_{17}$. $ord_{\Lambda_{22}} f^s = -3s_1 - 4s_2 - 10$.

(23) The case for $\tilde{x}_{23}$. $V^*_{\tilde{x}_{23}} = C\langle v_1, \dots, v_{14}\rangle$ where $v_1 = (234, 0)$, $v_2 = (567, 0)$, $v_3 = (345, 0)$, $v_4 = (267, 0)$, $v_5 = (246, 0)$, $v_6 = (357, 0)$, $v_7 = (237, 0)$, $v_8 = (456, 0)$, $v_9 = (236 - 247, 0)$, $v_{10} = (356 - 457, 0)$, $v_{11} = (235 + 347, 0)$, $v_{12} = (256 + 467, 0)$, $v_{13} = (245 - 346, 0)$ and $v_{14} = (257 - 367, 0)$. $(G_{\tilde{x}_{23}}, \rho_{\tilde{x}_{23}}, V^*_{\tilde{x}_{23}}) \cong (GL(1) \times Sp(3), \Lambda_1 \otimes \Lambda_3, V(14))$. $y_0 = v_1 + v_2 \in \rho^*(G)\tilde{x}_{27}$, $y_1 = v_7 + v_{13} \in \rho^*(G)\tilde{x}_{30}$. $ord_{\Lambda_{23}} f^s = -4s_1 - 7$.

(24) The case for $\tilde{x}_{24}$. $V^*_{\tilde{x}_{24}} = \boldsymbol{C}\langle v_1, \dots, v_{15}\rangle$ where $v_1 = (147, 0)$, $v_2 = (157, 0)$, $v_3 = (167, 0)$, $v_4 = (247, 0)$, $v_5 = (257, 0)$, $v_6 = (267, 0)$, $v_7(347, 0)$, $v_8 = (357, 0)$, $v_9 = (367, 0)$, $v_{10} = (237, e_1)$, $v_{11} = (-137, e_2)$, $v_{12} = (127, e_3)$, $v_{13} = (567, e_4)$, $v_{14} = (-467, e_5)$ and $v_{15} = (457, e_6)$. $(G_{\tilde{x}_{24}}, \rho_{\tilde{x}_{24}}, V^*_{\tilde{x}_{24}} \cong (GL(1) \times SL(3) \times SL(3), \Lambda_1 \otimes \Lambda_1^* \otimes \Lambda_1^* + \Lambda_1 \otimes \Lambda_1 \otimes 1 + \Lambda_1 \otimes 1 \otimes \Lambda_1, V(9) + V(3) + V(3))$. Thus the triplet $(G_{\tilde{x}_{24}}, \rho_{\tilde{x}_{24}}, V^*_{\tilde{x}_{24}})$ is a non P.V.

(25) The case for $\tilde{x}_{25}$. $V^*_{\tilde{x}_{25}} = \boldsymbol{C}\langle v_1, \dots, v_{15}\rangle$ where $v_1 = (236 - 247, 0)$, $v_2 = (234, 0)$, $v_3 = (237, 0)$, $v_4 = (246, 0)$, $v_5 = (267, 0)$, $v_6 = (235 + 347, 0)$, $v_7 = (245 - 346, 0)$, $v_8 = (256 + 467, 0)$, $v_9 = (257 - 367, 0)$, $v_{10} = (356 - 457, 0)$, $v_{11} = (-345, 0)$, $v_{12} = (357, 0)$, $v_{13} = (456, 0)$, $v_{14} = (-567, 0)$ and $v_{15} = (247, e_1)$. The triplet $(G_{\tilde{x}_{25}}, \rho_{\tilde{x}_{25}}, V^*_{\tilde{x}_{25}})$ is a non P.V.

(26) The case for $\tilde{x}_{26}$. $V^*_{\tilde{x}_{26}} = \boldsymbol{C}\langle v_1, \dots, v_{15}\rangle$ where $v_1 = (267, 0)$, $v_2 = (367, 0)$, $v_3 = (467, 0)$, $v_4 = (567, 0)$, $v_5 = (167, 0)$, $v_6 = (246 - 356, 0)$, $v_7 = (236, 0)$, $v_8 = (256, 0)$, $v_9 = (346, 0)$, $v_{10} = (456, 0)$, $v_{11} = (247 - 357, 0)$, $v_{12} = (237, 0)$, $v_{13} = (257, 0)$, $v_{14} = (347, 0)$ and $v_{15} = (457, 0)$. $y_0 = v_5 + v_9 + v_{13} \in \rho^*(G)\tilde{x}_{18}$, $y_1 = v_5 + v_6 + v_{12} \in \rho^*(G)\tilde{x}_{22}$, $y_1' = v_9 + v_{13} \in \rho^*(G)\tilde{x}_{27}$. $ord_{\Lambda_{26}} f^s = -5s_1 - s_2 - 19/2$.

(27) The case for $\tilde{x}_{27}$. $V^*_{\tilde{x}_{27}} = \boldsymbol{C}\langle v_1, \dots, v_{16}\rangle$ where $\{v_1, \dots, v_9\} = \{(ij7, 0);\ 1 \leq i \leq 3, 4 \leq j \leq 6\}$ and $v_{9+i} = (0, e_i)$, $1 \leq i \leq 7$. $y_0 = (147 + 257 + 367, e_7) \in \rho^*(G)\tilde{x}_{23}$. $y_1 = (147 + 257, e_7) \in \rho^*(G)\tilde{x}_{26}$, $y_1' = (147 + 257 + 367, e_1 + e_4) \in \rho^*(G)\tilde{x}_{25}$. $ord_{\Lambda_{27}} f^s = -3s_1 - 5s_2 - 14$.

(28) The case for $\tilde{x}_{28}$. $V^*_{\tilde{x}_{28}} = \boldsymbol{C}\langle v_1, \dots, v_{16}\rangle$ where $v_1 = (567, 0)$, $v_2 = (-467, 0)$, $v_3 = (457, 0)$, $v_4 = (456, 0)$, $v_5 = (147, 0)$, $v_6 = (257, 0)$, $v_7 = (367, 0)$, $v_8 = (267 + 357, 0)$, $v_9 = (167 + 347, 0)$, $v_{10} = (157 + 247, 0)$, $v_{11} = \frac{1}{2}(267 - 357, 2e_1)$, $v_{12} = 1/2(-167 + 247, 2e_2)$, $v_{13} = \frac{1}{2}(157 - 247, 2e_3)$, $v_{14} = (237, e_4)$, $v_{15} = (-137, e_5)$ and $v_{16} = (127, e_6)$. $y_0 = v_4 + v_5 + v_{14} \in \rho^*(G)\tilde{x}_7$, $y_1 = v_4 + v_+ v_{14} \in \rho^*(G)\tilde{x}_9$, $y_1' = v_1 + v_5 + v_{14} \in \rho^*(G)\tilde{x}_{25}$. $ord_{\Lambda_{28}} f^s = -5s_1 - 4s_2 - 13$.

(29) The case for $\tilde{x}_{29}$. $V^*_{\tilde{x}_{29}} = \boldsymbol{C}\langle v_1, \dots, v_{16}\rangle$ where $v_1 = (267, 0)$, $v_2 = (236, 0)$, $v_3 = (237, 0)$, $v_4 = (256, 0)$, $v_5 = (257, 0)$, $v_6 = (567, 0)$, $v_7 = (-367, 0)$, $v_8 = (467, 0)$, $v_9 = (167, 0)$, $v_{10} = (-247 + 357, 0)$, $v_{11} = (246 - 356, 0)$, $v_{12} = (456, 0)$, $v_{13} = (457, 0)$, $v_{14} = (346, 0)$, $v_{15} = (347, 0)$ and $v_{16} = (235, e_1)$. $y_0 = v_9 + v_{12} + v_{15} + v_{16} \in \rho^*(G)\tilde{x}_8$, $y_1 = v_3 + v_9 + v_{12} + v_{15} \in \rho^*(G)\tilde{x}_{18}$, $y_1' = v_9 + v_{10} + v_{14} + v_{16} \in \rho^*(G)\tilde{x}_{18}$. $ord_{\Lambda_{29}} f^s = -6s_1 - 3s_2 - 27/2$.

(30) The case for $\tilde{x}_{30}$. $V^*_{\tilde{x}_{30}} = \boldsymbol{C}\langle v_1, \dots, v_{17}\rangle$ where $v_1 = (567, 0)$, $v_2 = (-467, 0)$, $v_3 = (457, 0)$, $v_4 = (147, 0)$, $v_5 = (257, 0)$, $v_6 = (367, 0)$, $v_7 = (267 + 357, 0)$, $v_8 = (167 + 347, 0)$, $v_9 = (157 + 247, 0)$, $v_{10} = (456, 0)$ and $v_{10+i} =$

$(0, e_i)$, $1 \le i \le 7$. $y_0 = v_4 + v_8 + v_{10} + v_{17} \in \rho^*(G)\tilde{x}_{11}$, $y_1 = v_4 + v_8 + v_{10} + v_{14} \in \rho^*(G)\tilde{x}_{14}$, $y_1' = v_8 + v_{10} + v_{17} \in \rho^*(G)\tilde{x}_{17}$, and $y_1'' = v_4 + v_8 + v_{17} \in \rho^*(G)\tilde{x}_{23}$. $ord_{\Lambda_{30}} f^s = -4s_1 - 5s_2 - 29/2$.

(31) The case for $\tilde{x}_{31}$. $V_{\tilde{x}_{31}}^* = C\langle v_1, \ldots, v_{20}\rangle$ where $v_1 = (-467, 0)$, $v_2 = (-567, 0)$, $v_3 = (267, 0)$, $v_4 = (367, 0)$, $v_5 = (167, 0)$, $v_6 = (246 - 356, 0)$, $v_7 = (236, 0)$, $v_8 = (256, 0)$, $v_9 = (346, 0)$, $v_{10} = (456, 0)$, $v_{11} = (247 - 357, 0)$, $v_{12} = (237, 0)$, $v_{13} = (257, 0)$, $v_{14} = (347, 0)$, $v_{15} = (457, 0)$, $v_{16} = (356, e_1)$, $v_{17} = (-146, e_2)$, $v_{18} = (-156, e_3)$, $v_{19} = (126, e_4)$ and $v_{20} = (136, e_5)$. $y_0 = v_8 + v_9 + v_{12} + v_{15} + v_{17} \in \rho^*(G)\tilde{x}_4$, $y_1 = v_8 + v_9 + v_{11} + v_{19} + v_{20} \in \rho^*(G)\tilde{x}_{14}$, $y_1' = v_7 + v_8 + v_{14} + v_{15} + v_{20} \in \rho^*(G)\tilde{x}_7$, and $y_1'' = v_8 + v_{12} + v_{15} + v_{17} \in \rho^*(G)\tilde{x}_4$. $ord_{\Lambda_{31}} f^s = -6s_1 - 4s_2 - 15$.

(32) The case for $\tilde{x}_{32}$. $V_{\tilde{x}_{32}}^* = C\langle v_1, \ldots, v_{21}\rangle$ where $v_1 = (234, 0)$, $v_2 = (567, 0)$, $v_3 = (345, 0)$, $v_4 = (267, 0)$, $v_5 = (246, 0)$, $v_6 = (357, 0)$, $v_7 = (237, 0)$, $v_8 = (456, 0)$, $v_9 = (236 - 247, 0)$, $v_{10} = (356 - 457, 0)$, $v_{11} = (235 + 347, 0)$, $v_{12} = (256 + 467, 0)$, $v_{13} = (245 - 346, 0)$, $v_{14} = (257 - 367, 0)$ and $v_{14+i} = (0, e_i)$, $1 \le i \le 7$. $y_0 = v_1 + v_2 + v_{16} + v_{19} \in \rho^*(G)\tilde{x}_{15}$, $y_1 = v_7 + v_{13} + v_{18} \in \rho^*(G)\tilde{x}_{17}$, $y_1' = v_1 + v_2 + v_{16} + v_{20} \in \rho^*(G)\tilde{x}_{15}$. $ord_{\Lambda_{32}} f^s = -4s_1 - 4s_2 - 23/2$.

(33) The case for $\tilde{x}_{33}$. $V_{\tilde{x}_{33}}^* = C\langle v_1, \ldots, v_{22}\rangle$ where $v_1 = (267, 0)$, $v_2 = (367, 0)$, $v_3 = (467, 0)$, $v_4 = (567, 0)$, $v_5 = (167, 0)$, $v_6 = (246 - 356, 0)$, $v_7 = (236, 0)$, $v_8 = (256, 0)$, $v_9 = (346, 0)$, $v_{10} = (456, 0)$, $v_{11} = (247 - 357, 0)$, $v_{12} = (237, 0)$, $v_{13} = (257, 0)$, $v_{14} = (347, 0)$, $v_{15} = (457, 0)$ and $v_{15+i} = (0, e_i)$, $1 \le i \le 7$. $y_0 = v_5 + v_9 + v_{13} + v_{21} + v_{22} \in \rho^*(G)\tilde{x}_5$. $y_1 = v_5 + v_9 + v_{13} + v_{21} \in \rho^*(G)\tilde{x}_7$, $y_1' = v_5 + v_6 + v_{12} + v_{21} \in \rho^*(G)\tilde{x}_{11}$ and $y_1'' = v_9 + v_{13} + v_{21} + v_{22} \in \rho^*(G)\tilde{x}_{15}$. $ord_{\Lambda_{33}} f^s = -5s_1 - 5s_2 - 16$.

(34) The case for $\tilde{x}_{34}$. $V_{\tilde{x}_{34}}^* = C\langle v_1, \ldots, v_{22}\rangle$ where $v_1 = (567, 0)$, $v_2 = (-467, 0)$, $v_3 = (457, 0)$, $v_4 = (-456, 0)$, $v_5 = (145, 0)$, $v_6 = (146, 0)$, $v_7 = (147, 0)$, $v_8 = (167, 0)$, $v_9 = (-157, 0)$, $v_{10} = (156, 0)$, $v_{11} = (245, 0)$, $v_{12} = (246, 0)$, $v_{13} = (247, 0)$, $v_{14} = (267, 0)$, $v_{15} = (-257, 0)$, $v_{16} = (256, 0)$, $v_{17} = (345, 0)$, $v_{18} = (346, 0)$, $v_{19} = (347, 0)$, $v_{20} = (367, 0)$, $v_{21} = (-357, 0)$ and $v_{22} = (356, 0)$. $y_0 = v_5 + v_8 + v_{12} - v_{21} \in \rho^*(G)\tilde{x}_{12}$. $y_1 = v_8 + v_{11} + v_{12} - v_{21} \in \rho^*(G)\tilde{x}_{18}$. $ord_{\Lambda_{34}} f^s = -6s_1 - 2s_2 - 13$.

(35) The case for $\tilde{x}_{35}$. $V_{\tilde{x}_{35}}^* = C\langle v_1, \ldots, v_{25}\rangle$ where $v_1 = (467, 0)$, $v_2 = (-457, 0)$, $v_3 = (456, 0)$, $v_4 = (567, 0)$, $v_5 = (145, 0)$, $v_6 = (146, 0)$, $v_7 = (147, 0)$, $v_8 = (245, 0)$, $v_9 = (246, 0)$, $v_{10} = (247, 0)$, $v_{11} = (345, 0)$, $v_{12} = (346, 0)$, $v_{13} = (347, 0)$, $v_{14} = (167, 0)$, $v_{15} = (-157, 0)$, $v_{16} = (156, 0)$, $v_{17} = (267, 0)$, $v_{18} =$

$(-257,0)$, $v_{19}=(256,0)$, $v_{20}=(367,0)$, $v_{21}=(-357,0)$, $v_{22}=(356,0)$, $v_{23}=(234,e_1)$, $v_{24}=(-134,e_2)$ and $v_{25}=(124,e_3)$. $y_0=v_5+v_{14}+v_{18}+v_{22}+v_{23}\in\rho^*(G)\tilde{x}_2$. $y_1=v_5+v_{14}+v_{18}+v_{23}\in\rho^*(G)\tilde{x}_4$, $y_1'=v_{14}+v_{18}+v_{22}+v_{23}\in\rho^*(G)\tilde{x}_8$. $ord_{\Lambda_{35}}f^s=-7s_1-4s_2-18$.

(36) The case for $\tilde{x}_{36}$. $V^*_{\tilde{x}_{36}}=C\langle v_1,\dots,v_{29}\rangle$ where $v_1=(567,0)$, $v_2=(-467,0)$, $v_3=(457,0)$, $v_4=(-456,0)$, $v_5=(145,0)$, $v_6=(146,0)$, $v_7=(147,0)$, $v_8=(167,0)$, $v_9=(-157,0)$, $v_{10}=(156,0)$, $v_{11}=(245,0)$, $v_{12}=(246,0)$, $v_{13}=(247,0)$, $v_{14}=(267,0)$, $v_{15}=(-257,0)$, $v_{16}=(256,0)$, $v_{17}=(345,0)$, $v_{18}=(346,0)$, $v_{19}=(347,0)$, $v_{20}=(367,0)$, $v_{21}=(-357,0)$, $v_{22}=(356,0)$ and $v_{22+i}=(0,e_i)$, $1\leq i\leq 7$. $y_0=v_5+v_8+v_{12}-v_{21}+v_{27}+v_{28}\in\rho^*(G)\tilde{x}_3$. $y_1=v_5+v_8+v_{12}-v_{21}+v_{27}\in\rho^*(G)\tilde{x}_4$, $y_1'=v_8+v_{11}+v_{12}-v_{21}+v_{28}+v_{29}\in\rho^*(G)\tilde{x}_5$. $ord_{\Lambda_{36}}f^s=-6s_1-5s_2-18$.

(37) The case for $\tilde{x}_{37}$. $V^*_{\tilde{x}_{37}}=C\langle(ijk,0);\ 1\leq i<j<k\leq 7\rangle$. $y_0=(125+136+147+234+567,0)\in\rho^*(G)\tilde{x}_{10}$. $y_1=(125+136+234+567,0)\in\rho^*(G)\tilde{x}_{12}$, $y_1'=(145+167+234+256+357,0)\in\rho^*(G)\tilde{x}_{12}$. $ord_{\Lambda_{37}}f^s=-7s_1-3s_2-35/2$.

(38) The case for $\tilde{x}_{38}$. $(G_{\tilde{x}_{38}},\rho_{\tilde{x}_{38}},V^*_{\tilde{x}_{38}})\cong(G,\rho,V)$. $y_0=\tilde{x}_1$, $y_1=\tilde{x}_2$, $y_1'=\tilde{x}_3$. $ord_{\Lambda_{38}}f^s=-7s_1-5s_2-21$.

Hence, we obtain the holonomy diagram (Figure 1) where we denote by

the conormal bundle of the orbit $\rho(G)\tilde{x}_i$ which is j-codimensional.

Acknowledgement

The author would like to express his hearty thanks to Professor Tatsuo Kimura for his invaluable advice and encouragement.

References

[1] Kasai, S., The b-function and the holonomy diagram of a regular simple prehomogeneous vector space $(GL(1)^2\times Spin(10)$, *half-spin rep.* + *vector rep.*), to appear in J. Algebra.

[2] Kasai, S., The b-functions of regular simple prehomogeneous vector spaces $(GL(1)^3\times SL(2m), \Lambda_2+(\Lambda_1+\Lambda_1)^{(*)})$

[3] Kimura, T., Remark on some combinatorial construction of relative invariants, Tsukuba J. Math. **5** (1981), 101–115.

[4] Kimura, T., The b-functions and holonomy diagrams of irreducible regular prehomogeneous vector spaces, Nagoya Math. J. **85** (1982), 1–80.

[5] Kimura, T., A classification of prehomogeneous vector spaces of simple algebraic groups with scalar multiplications, J. Algebra **83** (1983), 72–100.
[6] Kimura, T. and Kasai, S., The orbital decomposition of some prehomogeneous vector spaces, Adv. Stud. Pure Math. **6** (1985), 437–480.
[7] Sato, M., Kashiwara, M., Kimura, T. and Oshima, T., Micro-local analysis of prehomogeneous vector spaces, Inv. Math. **62** (1980), 117–179.
[8] Sato, M. and Kimura, T., A classification of irreducible prehomogeneous vector spaces and their relative invariants, Nagoya Math. J. **65** (1977), 1–155.

Department of Mathemetics,
Faculty of Education,
Yamaguchi University,
Yamaguchi, 753-8513, Japan
e-mail address: kasai@edu.yamaguchi-u.ac.jp