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# Development of curved two-dimensional photonic crystal waveguides

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## Abstract

A two-dimensional photonic crystal waveguide with a novel geometry is introduced. The center line of this waveguide is bent along a free-curve such that the direction of the propagating light can be changed without scattering or reflection losses. The design method is described for a triangular lattice, its optical properties such as transmission spectrum and dispersion relation are calculated, and actual devices are then fabricated and demonstrated that they worked as optical waveguides.

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## 1 Introduction

Photonic crystals (PhCs) are optical structures that have a spatially periodic dielectric constant. Important characteristics of photonic crystals include the existence of photonic bandgaps and unusual dispersion relations for the propagating light, thereby supporting applications such as high  $Q$  cavities and ultrafine waveguides.

A line defect in a two-dimensional (2D) PhC functions as an optical waveguide. The incident light is strongly confined by the photonic bandgap and propagates along the defect. The confinement of light by the bandgap is distinct from total reflection. It enables waveguides having sharp bending points [1].

In this paper, we point out some of the problems of traditional bent waveguides and propose free-curve waveguides as a solution. Their transmission characteristics are calculated numerically, indicating that a free-curve waveguide can confine light just as well as a conventional straight waveguide. We fabricate such free-curve

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4 waveguides and characterize their transmission spectra. We also demonstrate an  
5 air-bridged 2D-PhC fabricated with circular holes arranged in a triangular lattice  
6 on a GaAs slab (having a thickness of 190 nm).  
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## 10 11 12 **2 Issues with bent waveguides** 13 14

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16 One critical problem with standard bent waveguides is that a significant fraction  
17 of the light leaks out of them at the bending points. This radiation can couple to  
18 neighboring waveguides. For example, a directional coupler based on a 2D-PhC [2]  
19 has two pairs of sharply bent points near each other. Consequently its transmission  
20 properties (such as the extinction ratio) are degraded. Another example is a ring  
21 cavity based on a 2D-PhC waveguide [3]. It contains at least six bending points,  
22 resulting in a substantial decrease in the  $Q$  value of the cavity.  
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26 A second problem with bent waveguides is that some of the propagating light is  
27 reflected backward at the bending points. This effect not only causes losses but also  
28 fluctuations in the transmission spectrum due to Fabry-Perot interference.  
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31 A third problem is the geometrical inflexibility of traditional bent PhC waveguides.  
32 For a triangular lattice, one can only change direction by 60 or 120 degrees. Other-  
33 wise one could not maintain the periodicity of the PhC, which is required to confine  
34 and propagate light through the waveguide.  
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37 Many approaches have been proposed to deal with these issues, such as structural  
38 modulation of dielectric materials at bending points [4–6] and topological opti-  
39 mization [7,9,8]. These approaches improve the transmission, but geometrical in-  
40 flexibility and fabrication difficulties (dielectric rod type of PhC or complicated  
41 shapes of air holes) remain.  
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45 Our approach is to change the direction of propagation adiabatically, whereby one  
46 propagation mode is converted to another without coupling to back-propagating or  
47 free-space radiating modes. We realize this effect by fabricating a waveguide whose  
48 center line is bent along a gentle curve.  
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## 55 **3 Design of free-curve waveguides** 56 57 58

59 Define a free curve by position vector  $f(t)$ . As shown in Fig. 1, this free curve is  
60 the center line of the waveguide. Anchor points  $f(t_n)$  are indicated by dots along  
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the curve  $f(t)$  in Fig. 1, where  $t_n$  are given by the recurrence equation,

$$\frac{1}{2}a = \int_{t_{n-1}}^{t_n} \left| \frac{d}{dt} f(t) \right| dt. \quad (1)$$

Air-filled circular holes are arranged with a periodicity of  $\sqrt{3}a$  along lines normal to  $f(t)$  that pass through the points  $f(t_n)$ , as shown in Fig. 1 by dashed lines. The center positions ( $r(m, n)$ ) of the air holes are given by

$$r(m, n) = f(t_n) + m \sqrt{3}a \frac{\frac{d}{dt} f(t_n)}{\left| \frac{d}{dt} f(t_n) \right|} \times \hat{z} \quad (2)$$

if  $n$  is an even integer, and

$$r(m, n) = f(t_n) + \left(m + \frac{1}{2}\right) \sqrt{3}a \frac{\frac{d}{dt} f(t_n)}{\left| \frac{d}{dt} f(t_n) \right|} \times \hat{z} \quad (3)$$

if  $n$  is an odd integer, where  $m$  is all integers from -10 to 10 except there are no air holes at  $r(0, 2n')$  (where  $n'$  is any integer) so as to make a line defect.

Although circular waveguides have been previously proposed [10,11], our design enables waveguides to follow more general curves.

## 4 Optical properties of free-curve waveguides

The transmission spectra and dispersion relation are quantified for the TE mode of a circular bent waveguide as an example.

### 4.1 Transmission properties

We modeled four waveguides with different curvatures: (a) 0 degree (straight waveguide), (b) 0.5 degree/ $a$ , (c) 1.0 degree/ $a$ , and (d) 2.0 degree/ $a$ . The angles between the input and output ports for (b), (c), and (d) are the same, namely 60 degrees, and thus the length of the waveguide is  $120a$ ,  $60a$ , and  $30a$ , respectively. The length of the straight waveguide was  $60a$ . At the input port of each waveguide, oscillating dipoles were placed, and the component of the Poynting vector parallel to the waveguide was observed at the output port. Next, the transmission spectra were normalized to that of a straight waveguide of length  $10a$ . The results are shown in Fig. 2.

These spectra show that incident light can be confined by and propagate along a waveguide bent into a circular arc. If the curvature of the waveguide is less than

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4 0.5 degree/ $a$ , the transmittance is nearly equal to that of a straight waveguide. For  
5 larger curvatures, we see that, although the normalized cut-off frequency remains  
6 0.273, the useful transmission band starts at somewhat higher frequencies. This  
7 effect results from changes in the photonic bandgap as the crystalline structure of  
8 the waveguide is distorted.  
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11 In order to confirm the reduction in the reflectance at the bending points, we calcu-  
12 lated the transmittance of a straight waveguide (of length  $100a$ ) with bent waveg-  
13 uides connected to both of its ends. The reflection can be observed as a fluctuation  
14 in the transmittance due to Fabry-Perot interferences between two bending points.  
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18 For this purpose, we prepared three configurations of bent waveguides as illustrated  
19 in Fig. 3: A - a sharply cornered waveguide, B - a circular arc waveguide, and C -  
20 a sinusoidally curved waveguide. The angle between the input and output ports are  
21 60 degrees in all three cases, and the maximum curvature of waveguide C is equal  
22 to the (constant) curvature of waveguide B, namely 1 degree/ $a$ .  
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25 The graphs in Fig. 3(a), (b), and (c) plot the transmittance of waveguide A, B,  
26 and C, respectively. The sharp periodic peaks in Fig. 3(a) are due to Fabry-Perot  
27 interferences between the two sharp corners.  
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30 On the other hand, in Fig. 3(b), only a small fluctuation remains due to a weak  
31 reflection from the connecting point between the circular and straight waveguides.  
32 Finally, for waveguide C, fluctuations cannot be observed because it was designed  
33 to have matching waveguide curvatures at the connecting points.  
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## 38 4.2 Dispersion relation

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41 The plane-wave expansion method and 2D-FDTD (Finite-Difference Time-Domain)  
42 method with Bloch boundary conditions are standard techniques for calculating dis-  
43 persion relations for a photonic band. The PhC free-curve waveguide, however, has  
44 no translational symmetry, and thus Bloch boundaries cannot be defined. Instead  
45 the following method was adopted, as illustrated in Fig. 4. First, the field distribu-  
46 tion of light propagating with a frequency  $\omega$  is calculated. Second, the magnetic  
47 field perpendicular to the 2D-PhC slab is determined along the center line of the  
48 waveguide. It is Fourier transformed to find the peaks. We obtained four peaks,  
49 derived from two peaks folded into the first Brillouin zone. These two peaks cor-  
50 respond to backward and forward traveling waves having the same propagation  
51 mode. We thereby find a relation between frequency and wavenumber for the con-  
52 fined light, and so obtain the dispersion relation.  
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58 To calculate the field distributions, as shown in Fig. 4(a), we set the length and  
59 angle between the input and output ports to be  $120a$  and 120 degrees, respectively.  
60 The normalized wavenumber resolution is therefore  $1/120$ . The resulting dispersion  
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4 relation is graphed in Fig. 5.  
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7 The dispersion relation is in excellent agreement with that for a straight waveguide  
8 calculated by the 2D-FDTD method. This result shows that the dispersion relation  
9 of a PhC waveguide is conserved even if the guide is bent along a free-curve. By  
10 finding the frequency of the point of intersection of the dispersion curve with the  
11 light line (dashed in Fig. 5), a normalized cut-off wavelength of 0.294 is found, in  
12 agreement with that of a straight waveguide.  
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## 15 16 17 18 19 **5 Fabrication and testing of free-curve waveguides** 20 21 22

23 Waveguides were fabricated on GaAs epitaxial wafers, with a 1- $\mu\text{m}$  AlGaAs sacri-  
24 ficial layer on the substrate and 190 nm of GaAs on top of that. We lithographically  
25 fabricated PhC waveguides using an electron beam (EB) followed by dry etching of  
26 the GaAs top layer using an inductively coupled plasma. Then the sacrificial layer  
27 was removed using hydrofluoric acid. Scanning electron microscope (SEM) images  
28 of the resulting waveguides are shown in Fig. 6. The curvatures of the waveguides  
29 in panels (a) and (b) are 1.0 degree/ $a$  and 2.0 degree/ $a$ , respectively.  
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33 Next we measured the transmission spectra of these fabricated waveguides. The  
34 output waveguide needs to be parallel to the input waveguides due to the alignment  
35 of the measuring system. Therefore, the devices under test had two oppositely bent  
36 waveguides. A straight waveguide with a length of  $100a$  was inserted between these  
37 bent waveguides, as shown in the insets in Fig. 7. Figure 7(a) is a spectrum of a PhC  
38 waveguide with sharp corners, while Figs. 7(b) and (c) are for free-curve wave-  
39 guides with curvatures of 1.0 degree/ $a$  and 2.0 degree/ $a$ , respectively. The results  
40 confirm that light is confined and propagates through the waveguides. However,  
41 we see that the transmittance at long wavelengths decreases as the curvature of a  
42 waveguide increases. The spectrum of the waveguide with sharp bends in Fig. 7(a)  
43 oscillates with a period of about 4 nm at short wavelengths and with a period of  
44 about 2 nm at long wavelengths. These periods agree with the peak interval for  
45 Fabry-Perot resonances across a length of  $100a$ . We conclude that the light reflects  
46 at the corners. On the other hand, the spectra of the free-curve waveguides in Figs.  
47 7(b) and (c) exhibit minimal oscillations.  
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51 Next, we imaged the waveguides from above to look for light leakage at the bending  
52 points, as shown in Fig. 8. The incident light propagates rightward. One sees bright  
53 spots at the sharp corners indicated by the arrows in Fig. 8(a), demonstrating that  
54 light is radiated at those bends. On the other hand, in the case of the free-curve  
55 bends in Figs. 8(b) and (c), no bright spots are observed, indicating a considerable  
56 reduction in lost light from free-curve waveguides.  
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## 6 Conclusion

A conventional PhC waveguide with sharp corners exhibits problems such as radiation and reflection at the bending points. As a solution to those problems, we developed a method to fabricate a PhC waveguide that is bent along a free-curve. Their transmittances and dispersion relation were calculated, showing that a free-curve waveguide can confine and propagate light and that the reflection at the bends is diminished. It was found that a free-curve waveguide has nearly the same dispersion relation as a straight waveguide.

Free-curve waveguides were fabricated and it was experimentally demonstrated that radiation and reflection at the bending points were negligible. Such free-curve waveguides afford flexibility for the design of nanowire circuits and could help with the development of optical integrated PhC circuits.

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4 **Figure captions**  
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8 **Figure 1**

9 The structure of a PhC free-curve waveguide. A curve  $f(t)$  shown as a solid line  
10 is defined and anchor points  $t_n$  are laid out along it. The distance between adjacent  
11 anchor points is equal to half of the lattice constant  $a/2$ . Air-filled circular holes  
12 are then arranged along the dashed normal lines with a spacing of  $\sqrt{3}a$ .  
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17 **Figure 2**

18 Transmission spectra of bent waveguides. The curvature of each waveguide is (a)  
19 0 degree/ $a$ , (b) 0.5 degree/ $a$ , (c) 1.0 degree/ $a$ (60 $a$ ), and (d) 2.0 degree/ $a$ . The trans-  
20 mittances are normalized to that of a straight waveguide of length  $10a$ .  
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24 **Figure 3**

25 Transmission spectra of a waveguide with two bent guides at its ends. (a) Wave-  
26 guide A has 60-degree cornered waveguides. (b) Waveguide B has circular ends with  
27 a curvature of 2.0 degree/ $a$ . (c) Waveguide C is a sinusoidal guide that connects  
28 smoothly to the straight waveguide. The length of the straight waveguide between  
29 the bending points is  $100a$ . The corresponding spectra are shown to the right of  
30 each geometrical sketch.  
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35 **Figure 4**

36 Procedure to obtain the dispersion relation for propagation in a free-curve wave-  
37 guide: (a) The field distribution is calculated by the 2D-FDTD method. (b) Its value  
38 is extracted along the center line of the waveguide. (c) That value is Fourier trans-  
39 formed to obtain the peak location.  
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44 **Figure 5**

45 Dispersion relation for propagation along a straight waveguide (filled circles) and  
46 along a free-curve waveguide (open circles and solid line). The dashed curve is the  
47 light line.  
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51 **Figure 6**

52 SEM images of PhC circular waveguides with curvatures of (a) 1.0 degree/ $a$  and  
53 (b) 2.0 degree/ $a$ .  
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57 **Figure 7**

58 Transmission spectra of a waveguide having (a) cornered waveguides, (b) circular  
59 waveguides with a curvature of 1.0 degree/ $a$ , and (c) circular waveguides with a  
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4 curvature of 2.0 degree/ $a$ .  
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8 **Figure 8**

9 The top view of (a) sharply bent waveguide, (b) circular bent waveguide (curvature  
10 of 1.0 degree/ $a$ ) and (c) circular bent waveguide (curvature of 2.0 degree/ $a$ ). Each  
11 image is the top view taken with an infrared vidicon. In the case of (a), bright spots  
12 (radiation) can be observed (shown by the arrows). On the other hand in the case of  
13 free-curve bent waveguide ((b) and (c)), radiations were not observed.  
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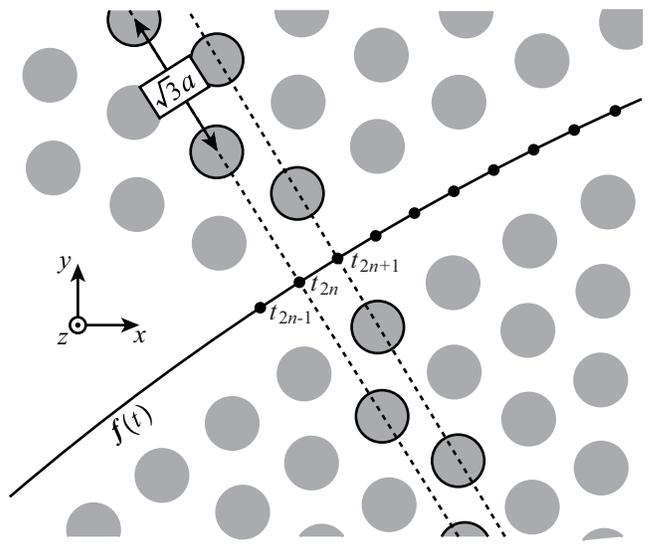


Fig. 1.

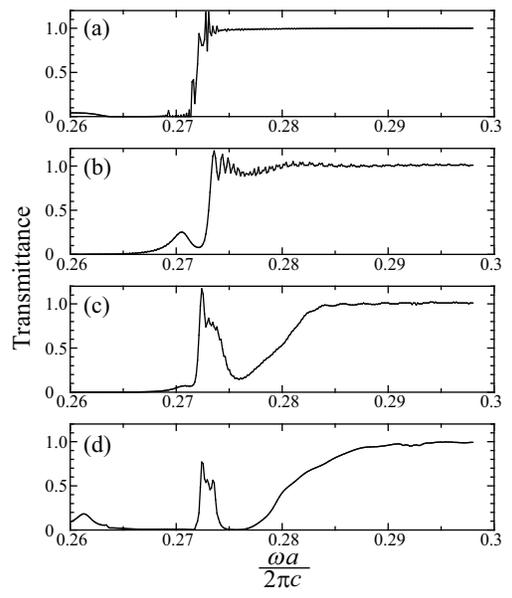


Fig. 2.

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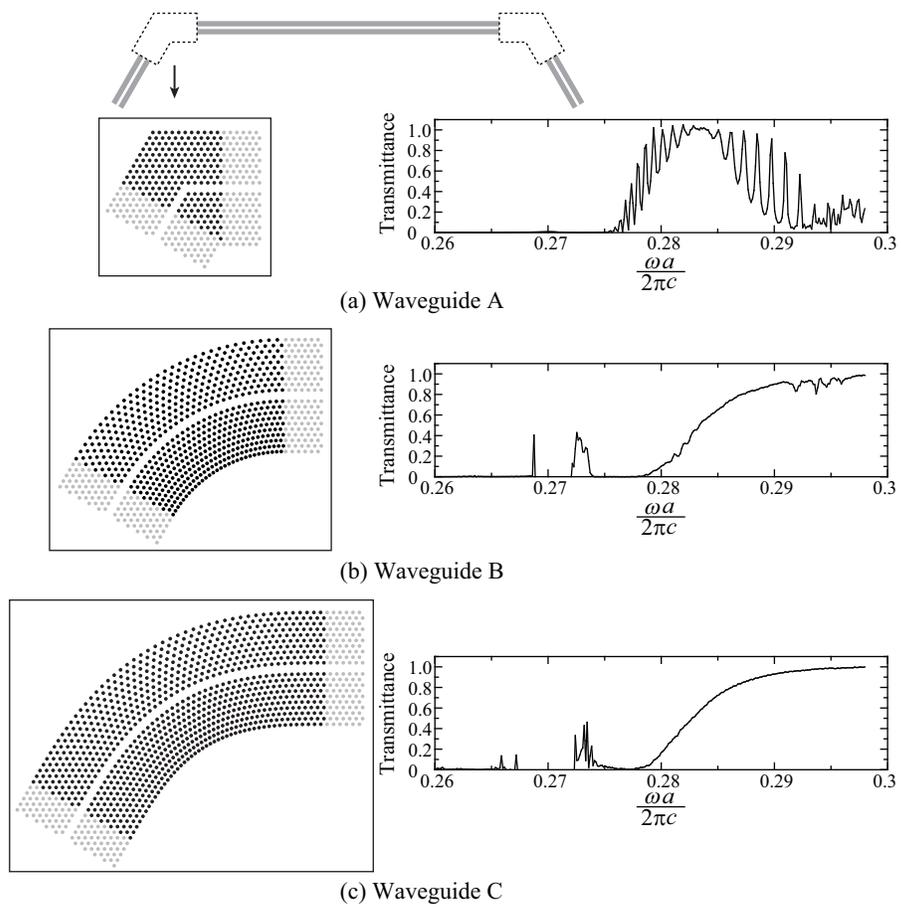


Fig. 3.

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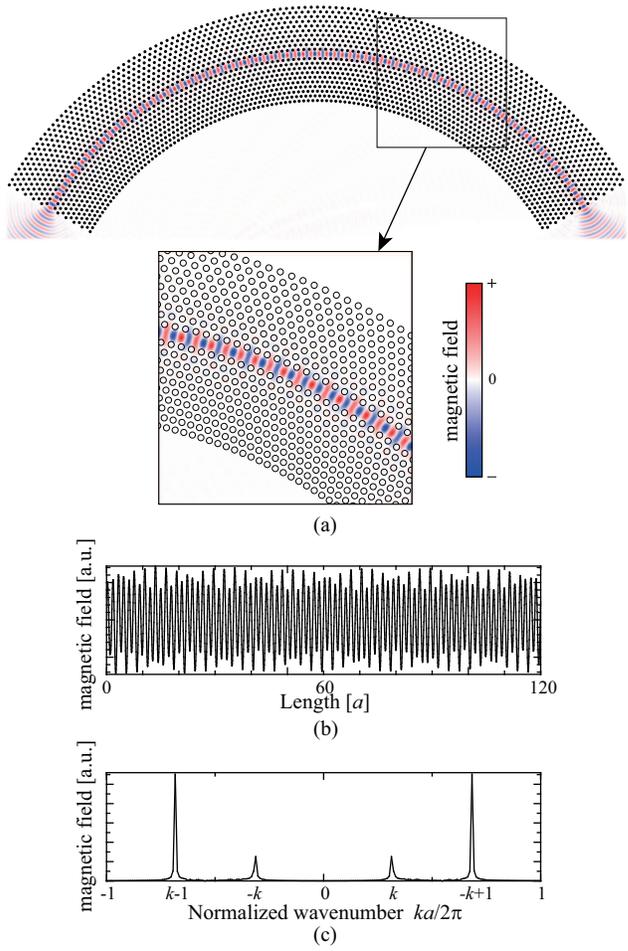


Fig. 4.

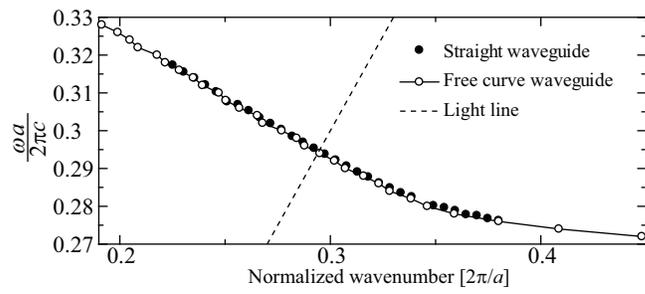


Fig. 5.

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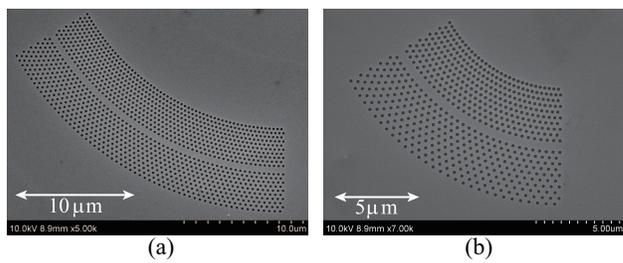


Fig. 6.

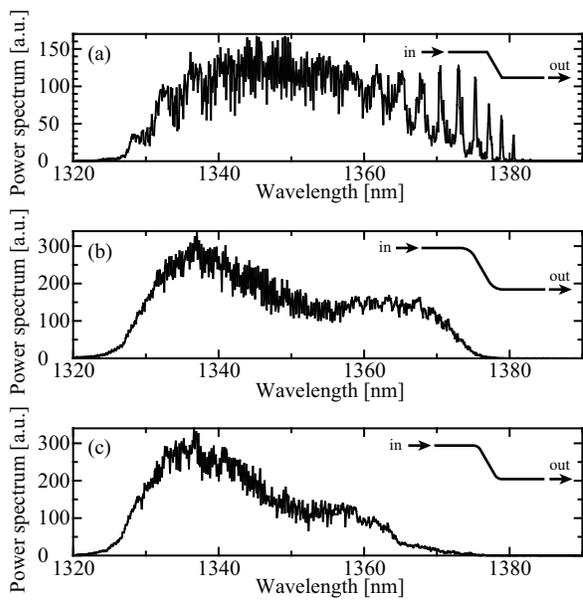


Fig. 7.

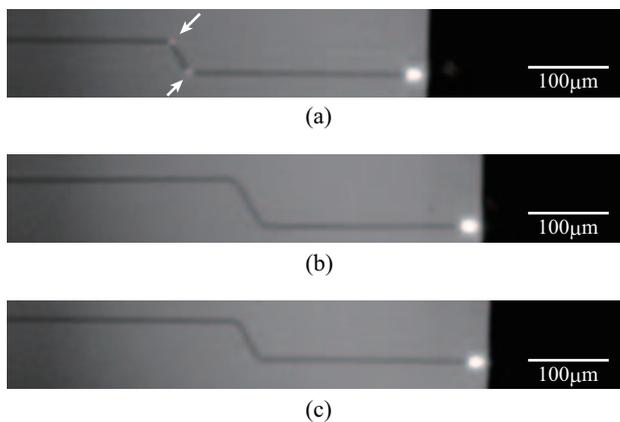


Fig. 8.