

Topological low-energy modes in $N = 0$ Landau levels of graphene: a possibility of a quantum-liquid ground state

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Abstract

We point out that the zero-energy Landau level of Dirac fermions in graphene can be, in the presence of a repulsive electron-electron interaction, split into two associated with a “bond ordering” formation having a “Kekulé pattern”, which respects the chiral symmetry. Since the Kekulé pattern has a three-fold degeneracy, domain structures are implied, for which we show that in-gap states localized along the domain boundaries exist as topological states. Based on this a possibility of a quantum-liquid ground state of graphene in magnetic fields is discussed.

Key words: Graphene, $N = 0$ Landau level, dimerization
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1. Introduction

Graphene, for which an unconventional quantum Hall effect has recently been discovered,[1] is a peculiar condensed-matter realization of massless Dirac fermions. While the quantum Hall effect is generally characterized by a topological (Chern) number, the peculiarity of Landau levels (LL) in graphene appears as existence of the exactly zero energy Landau level. We have established topological aspects in the graphene QHE[2], among which is a bulk-edge correspondence coming from the topological nature of bulk and edge states. All the topological features are intimately related with the chiral (bipartite) symmetry of the honeycomb lattice, which, in addition to being responsible

for the zero energy Landau level, gives a characteristic coexistence of extended and edge states at $E = 0$ [2].

Now, an interesting question is: can the zero energy Landau level of the Dirac fermion’s be lifted when E_F is situated right in the zero-th Landau level. In the present paper we propose that an intriguing candidate for triggering the splitting should be a “bond ordering”, in which the electronic bonds have an ordering with a “Kekulé pattern”, arising either from a Jahn-Teller type distortion or electronic bond-order ($\langle\langle c_i^\dagger c_j \rangle\rangle$) formation due to electron-electron interactions[3]. Reflecting the hexagonal symmetry there are three equivalent ordering (“Kekulé”) patterns (see Fig.1), and the bond ordering is one way to break the degeneracy. The ordering, when static, produces a mass in the Dirac fermion’s dispersion (in zero magnetic field), despite the fact that the bond ordering does *not* break the chi-

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ral symmetry. In magnetic fields, the bond ordering acts to split the zero-energy Landau level of the Dirac fermions, which may be viewed as a kind of the Peierls instability since the one-particle density of states diverges.

2. Bond ordering and the split zero-energy Landau levels

We now examine the above idea in the simplest manner, i.e., we assume that the ordering is static and adopt a Hamiltonian that has the hopping t_{ij} with a Kekulé-type modulation, which represents, in a mean-field sense, the bond ordering, say, originating from the electron-electron interaction. We first take the case in which the ordering pattern has a translational symmetry as in Fig.1. The massless Dirac fermions then acquire a mass which opens a gap (Fig. 2). As stressed the bond ordering does not destroy the chiral symmetry (although the size of the Brillouin zone is expanded), which implies that the particle-hole symmetry is preserved as well for any (including random) ordering patterns. This contrasts with other types of modulation (such as site dependent potentials) which break the chiral symmetry. Correspondingly, the zero-energy Landau level in uniform magnetic fields, which is a special Landau level sitting right at the Dirac cone vertex, also splits into two [4].

This implies that the bond ordering, which is here assumed, is expected to spontaneously induced in a self-consistent treatment as a kind of Peierls instability in the zero-energy LL[3].

3. Topological states localized along domain boundaries as 2D analogue of solitons

We have seen that the bond ordering, when translationally symmetric, splits the zero-energy Landau level accompanied by an energy gap at $E = 0$. However, there are a vast number of possible realizations of bond-ordering patterns that have no translational symmetry. Specifically, there is an important class of such states that consist of “domains”. Namely, there are three, equivalent ‘Kekulé patterns (differently colored in Fig. 1), and there are a huge number

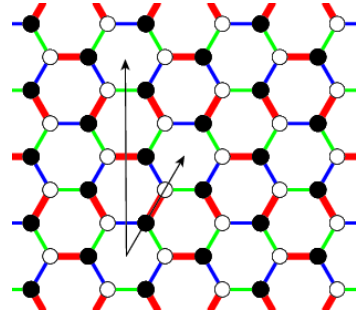


Fig. 1. Kekulé (static bond-ordering) patterns have three, equivalent realizations, where either the red, blue or green bonds are $r(> 1)$ times stronger than the others. The two arrows are unit vectors of the enlarged unit cell. (color online)

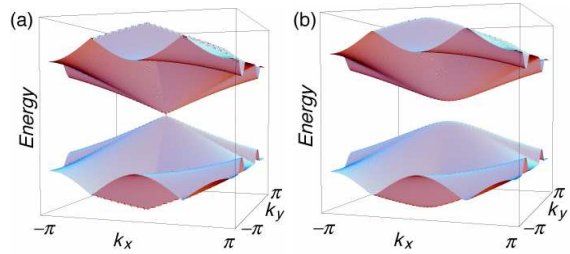


Fig. 2. (a) The band dispersion around $E = 0$ in graphene before (a) and after (b) a (static) bond ordering is introduced, as displayed in the folded Brillouin zone for $r = 1.2$ here.

of configurations where, e.g., a bond-ordered phase with strong red bonds sits next to another phase with strong blue bonds (Fig. 3). We can then expect that “boundary states” that are localized along the *domain boundaries* should appear as generalized boundary states in graphene, which have topological origin and stability.[2] The situation reminds us of the well-known *soliton* modes across the different conjugated patterns in 1D polyacetylene. So the boundary states considered here is a 2D extension of solitons which are topologically protected.

The boundary states, whose energies reside in the gap of the split $N = 0$ Landau levels, still do not violate the chiral symmetry. Physically the situation is similar to the appearance of the zero-mode edge states along the zig-zag edges in graphene in zero magnetic field, which also have a topological origin. These loop-like boundary states (“strings”) may have different realization in 2D as further exemplified in Fig. 4. In each case topologically stable boundary states arise near the

zero energy as displayed in Figs. 3(b),4(b).

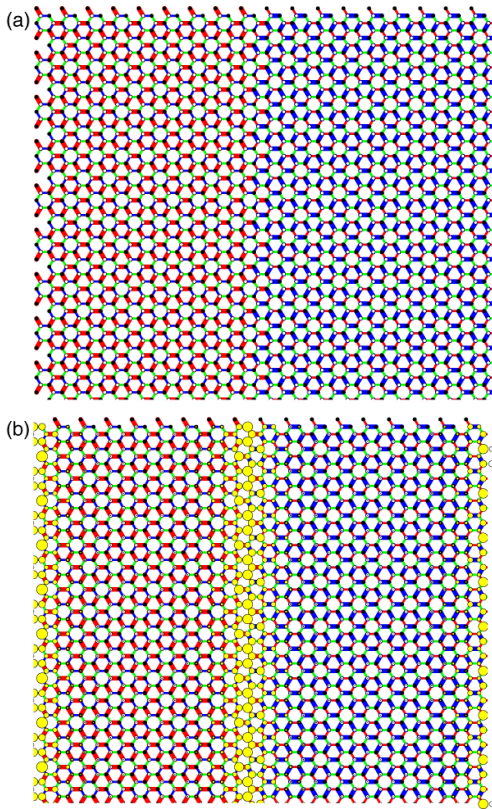


Fig. 3. (a) A bond ordering with straight domain boundaries, where the red (blue) hopping is stronger than the others (by a factor of 1.2) in the left (right) domain. (b) The boundary state whose energy sits between the split $N = 0$ LL is shown with the charge density represented by yellow circles for the magnetic flux of $1/6$ times the flux quantum per hexagon. (color online)

In reality the bond ordering configuration can be dynamical, which may be described in terms of the fluctuating strings[3]. We can then conjecture that the true state may possibly be a quantum liquid realized as a condensate of such “strings”. The physics may have an analogy with the “string net condensation” considered for spin models[5]. A self-consistent treatment of the effect of the electron-electron interaction will be given elsewhere[3].

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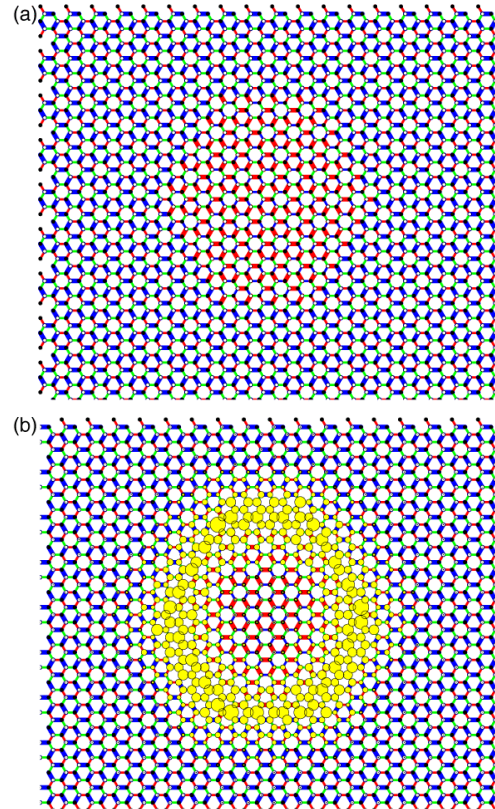


Fig. 4. A bond-ordering pattern with a closed-loop boundary, where the red (blue) hopping is stronger than the others (by a factor of 1.2) in the inner (outer) domain. (b) The boundary state shown as the charge density represented by yellow circles for the same magnetic flux as in Fig.3. (color online)

References

- [1] K. S. Novoselov et al., Nature **438** (2005) 197 ; Y. Zhang et al., *ibid.* **438** (2005) 201; A. K. Geim and K. S. Novoselov, Nature Mater. **6**, 183 (2007).
- [2] Y. Hatsugai, T. Fukui and H. Aoki, Phys. Rev. B **74**, 205414 (2006).
- [3] Y. Hatsugai, T. Fukui and H. Aoki, to be published.

- [4] T. Nakajima and H. Aoki, in these proceedings, propose another mechanism for the splitting in a one-particle picture.
- [5] M. A. Levin and X. G. Wen, Phys. Rev. B **71**, 045110 (2005).