

# A modified inspector leadership game with belief-dependent payoffs \*

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## Abstract

We formulate a monitoring model of a situation where a principal (an inspector) verifies that an agent (an inspectee) adheres to a level of effort. We incorporate belief-dependent parsimonious payoffs, guilt feelings and reciprocity, into the payoff of the agent. We examine the impact of the incorporation of each belief-dependent payoff on an error probability that the principal conducts a *costly* investigation into the level of effort chosen by the agent *although* the agent chooses a desirable level of effort for the principal and on the behavior of the agent. It is known that theories endowed with these belief-dependent payoffs well explain stylized facts of a wide range of experimental games and anomalous phenomena of economics. We find that in our monitoring model, however, the incorporations of these belief-dependent payoffs have quite different impacts on both the error probability and the behavior of the agent from each other.

**Keywords:** behavioral economics, psychological game theory, inspection game, guilt, reciprocity

**JEL classification numbers:** C19, C72, J33.

## 1 Introduction

We formulate a simple monitoring model which is a modified inspector leadership game.<sup>1</sup> The game consists of four stages. At the first stage of the game,

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<sup>1</sup>Inspector leadership game is classified as an *inspection game*. The inspection game is a mathematical model of a situation where a player verifies that the opponent player adheres to a legal rule. The inspection game is applied to analyses of a safeguard system against nuclear weapons, of material accountancy systems and of auditing systems. In this paper, we modify the inspection game to deal with workers' moral hazard problems. For details of inspection games, see Avenhaus, Olsada and Zamir (1991), Avenhaus and Olsada (1992) and Avenhaus, von Stengel and Zamir (2003).

a principal (an inspector) decides *critical output*  $\bar{y}$ . At the second stage of the game, given the critical output, an agent (an inspectee) chooses either a high level of effort  $e_H$  or a low level of effort  $e_L$ . Each level of effort  $e_l$  ( $l = H, L$ ) is a nonnegative real number. At the third stage of the game, output  $y$  is realized according to a probability distribution  $F_l(y)$  which is conditioned by the level of effort  $e_l$  which was chosen by the agent at the previous stage. We assume that both players know each conditional probability distribution  $F_l(y)$  of the random variable  $y$ . Finally, at the fourth stage of the game, the principal observes the realized output but does not observe the level of effort which was chosen by the agent at the second stage of the game. If the realized output is below or at the critical output  $\bar{y}$  which was decided by the principal at the first stage of the game, then the principal conducts a *costly* investigation that provides the principal with *correct* information on the level of effort which was chosen by the agent at the second stage. Otherwise, that is, if the realized output is above the critical output which was decided at the first stage of this game, then the principal does not conduct the investigation into the level of effort which was chosen by the agent and gives a fixed wage  $w_H$  to the agent. If the investigation reveals that the agent chose the low level of effort  $e_L$  at the second stage, the principal gives the agent a fixed lower wage  $w_L < w_H$ ; namely the principal imposes penalty on the agent. Since the principal knows the conditional distribution  $F_H(y)$  of output, the principal's decision on the critical output  $\bar{y}$  at the first stage means that he decides an error probability  $F_H(\bar{y})$  that he conducts the costly investigation into the level of effort chosen by the agent although the agent chooses the high level of effort  $e_H$ . We denote by  $\alpha$  this error probability in the following sections.

The principal's payoff is defined as follows: If the principal conducts the investigation into the level of effort which was chosen by the agent at the second stage, the principal's payoff is output realized according to the probability distribution conditional on the level of effort chosen by the agent minus the sum of the wage paid for the agent and the cost for conducting the investigation. If the principal does not conduct the investigation into the level of effort which was chosen by the agent, the principal's payoff is the output realized according to the level of effort chosen by the agent minus the wage paid for the agent. The payoff of the agent is the wage given by the principal minus the level of effort chosen by the agent. We call this monitoring model formulated above a benchmark model.

In *behavioral game theory*, the theories of parsimonious payoff functions are classified into two groups. One is a class of theories where the domain of the payoff function of each player consists only of outcomes of the game. Fehr and Schmidt (1999) and Bolton and Ockenfels (2000) proposed typical theories which belong to the group; they proposed models of inequality aversion in which players care about their own payoff and their relative payoff with the cosequentialistic way. The other is a class of theories where the domain of the payoff function of each player consists not only of outcomes of the game but of the belief of the players, i.e. players have the *belief-dependent payoff* functions.

The game theory endowed with the belief-dependent payoff functions is called the *psychological game theory* which was proposed by Geanakoplos, Pearce and Stacchetti (1989) and Rabin (1993).<sup>2</sup>

After finding out the subgame perfect equilibrium points of the benchmark model, we incorporate the belief-dependent parsimonious payoffs – a guilt feeling, an impulse for deceiving, and reciprocity – into the agent's payoff of the benchmark model. To analyse our model with the belief-dependent parsimonious payoff, we employ an equilibrium concept, called the *psychological equilibrium point*, which is the profile of behavior strategies which constitutes the subgame perfect equilibrium point of the model with belief-dependent payoffs.

Let  $q$  be the agent's behavior strategy which is the probability that the agent chooses the *low* level of effort  $e_L$  at the second stage of our model and  $q''$  be a priori belief that the agent holds about the belief that the principal holds about the behavior strategy  $q$  chosen by the agent. We call the latter probability  $q''$  the second order belief of the agent. We consider a scenario where the principal does not conduct the investigation into the level of effort although the agent has chosen the *low* level of effort  $e_L$  in our model. In this scenario, we assume that the smaller the second order belief  $q''$  becomes, the smaller payoff the agent with the *guilt feeling* would get; the larger payoff the agent with the *impulse for deceiving* would get. In Section 3 and in Section 4, we formulate this guilt feeling and this impulse for deceiving, and then investigate the impacts of the incorporation of these belief-dependent payoffs into the agent's payoff on the equilibrium point of our model. While the best response correspondence of the agent with each belief-dependent payoff has complicated features, we find the *unique* psychological equilibrium point for each game with each belief-dependent payoff. We show that in both cases, the guilt feeling and the impulse for deceiving, the agent chooses the high level of effort  $e_H$  in the psychological equilibrium points; but if the belief-dependent guilt feeling is incorporated into the payoff of the agent, then the critical output chosen by the principal becomes smaller than that in the subgame perfect equilibrium point of our benchmark model, and if the belief-dependent impulse for deceiving is incorporated into the payoff of the agent, then the critical output chosen by the principal becomes larger than that in the subgame perfect equilibrium point of the benchmark model.

After investigating the properties of the psychological equilibrium points of our model with the guilt feeling and the impulse for deceiving, we examine the impact of the incorporation of the belief-dependent *reciprocity* into the agent's payoff on the equilibrium point of our model. Given the critical output which has been chosen by the principal at the first stage, the agent cares about the difference between his own expected payoff and the principal's expected payoff, in which the agent calculates these expected payoffs based on the *second order*

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<sup>2</sup>For the recent progress of the research on the behavioral game theory and on experimental games, see Camerer (2003). Our monitoring model belongs to a class of models where the moral hazard problem is dealt with. Engelmaier (2005) is a survey of behavioral game theoretic models of the workers' moral hazard problems.

belief  $q''$  defined above and on the critical output given by the principal. We call these expected payoffs which the agent calculates based on the second order belief  $q''$  the *fictitious* expected payoffs. Then, the reciprocal behavior of the agent is assumed as follows: If the fictitious expected payoff of the agent is larger than that of the principal, then the agent given the critical output chooses an *actual* behavior strategy  $q$  to raise the principal's *actual* expected payoff up. If the fictitious expected payoff of the agent is smaller than that of the principal, then the agent given the critical output chooses an *actual* behavior strategy  $q$  to lower the principal's *actual* expected payoff down. In Section 5, we formulate this reciprocal behavior of the agent and investigate the property of the psychological equilibrium point of the model. We find that whatever the principal has decided at the first stage, the principal can not prevent the agent with the belief-dependent reciprocity from choosing the low level of effort  $e_L$  in the psychological equilibrium point.

It is known that theories endowed with these belief-dependent parsimonious payoffs, guilt feelings and reciprocity, well explain stylized facts of a wide range of experimental games and anomalous phenomena of economics (see Dufwenberg (2002) and Falk and Fischbacher (2006)). In our model, however, the incorporations of these belief-dependent payoffs have very different impacts on the equilibrium behaviors from each other.

## 2 The model

We consider a dynamic game with two players,  $P$  (principal) and  $A$  (agent). The game consists of four stages as following.

1. At the first stage of the game, player  $P$  chooses a probability  $\alpha \in [0, 1]$  that player  $P$  monitors player  $A$  *ex post* by conducting the investigation that provides player  $P$  with information on the level of effort chosen by player  $A$ .<sup>3</sup>
2. At the second stage of the game, player  $A$  given the probability  $\alpha$  chooses his level of effort  $e_l$  from a set  $\{e_H, e_L\}$ . Each level of effort  $e_l$  is a nonnegative real number and  $e_H > e_L$ . We consider the behavior strategy given by the probability  $q \in [0, 1]$  for choosing the low level of effort  $e_L$ .
3. At the third stage of the game, Nature picks up output  $y \in \mathbb{R}_+$ .<sup>4</sup> If player  $A$  has chosen  $e_l$  ( $l=L, H$ ) at the previous stage, the corresponding output  $y$  is realized according to a cumulative distribution function  $F_l(y)$  which has the mean  $\mu_l \in [0, \infty)$  where  $\mu_H > \mu_L$ . Each distribution function  $F_l$  ( $l = L, H$ ) is absolutely continuous and has each inverse function  $F_l^{-1}$  and an identical variance  $\sigma^2$ .

<sup>3</sup> $[0, 1]$  denotes an closed interval with end points 0 and 1. In the literature, we use similar notations. For example,  $(a, b)$  denotes an open interval and  $[a, b]$  a semi closed interval with end points  $a$  and  $b$ .

<sup>4</sup> $\mathbb{R}_+$  denotes the set of non-negative real numbers.

4. At the fourth stage, if output  $y$  realized at the previous stage belongs to a set  $Z_\alpha \equiv \{y \in \mathbb{R}_+ \mid F_H(y) \leq \alpha\}$ , player  $P$  conducts the investigation for the level of effort chosen by player  $A$ . It costs a fixed amount of  $c > 0$  unit of output for player  $P$  to conduct the investigation. After the investigation;

- if player  $A$  has chosen  $e_H$  at the second stage of the game, then player  $P$  gives fixed wage  $w_H \in \mathbb{R}_+$  to player  $A$ ,
- if player  $A$  has chosen  $e_L$  at the second stage of the game, then player  $P$  gives fixed wage  $w_L \in \mathbb{R}_+$  to player  $A$  where  $w_L < w_H$ .

If output  $y$  realized at the previous stage does not belong to the set  $Z_\alpha \equiv \{y \in \mathbb{R}_+ \mid F_H(y) \leq \alpha\}$ , then player  $P$  does not conduct the investigation and gives the fixed wage  $w_H$  to player  $A$ .

### Payoff of each player

Let  $e_l$  be the level of effort chosen by the agent at the second stage of the game. If the output  $y$  realized at the third stage of the game belongs to the set  $Z_\alpha$ , the payoff of player  $P$  and of player  $A$  is given by  $y - w_l - c$  and  $w_l - e_l$ , respectively. If the output  $y$  realized at third stage of the game does *not* belong to the set  $Z_\alpha$ , the payoff of player  $P$  and of player  $A$  is given by  $y - w_H$  and  $w_H - e_l$ , respectively. Let  $z_\alpha \equiv F_H^{-1}(\alpha)$ . Then the expected payoff of each player in our model is given by

$$\begin{aligned} Eu_A(\alpha, q) &= q \left[ \int_{z_\alpha}^{+\infty} (w_H - e_L) dF_L(y) + \int_0^{z_\alpha} (w_L - e_L) dF_L(y) \right] \\ &+ (1 - q) \left[ \int_{z_\alpha}^{+\infty} (w_H - e_H) dF_H(y) + \int_0^{z_\alpha} (w_H - e_H) dF_H(y) \right], \quad (2.1) \end{aligned}$$

$$\begin{aligned} Eu_P(\alpha, q) &= q \left[ \int_{z_\alpha}^{+\infty} (y - w_H) dF_L(y) + \int_0^{z_\alpha} (y - w_L - c) dF_L(y) \right] \\ &+ (1 - q) \left[ \int_{z_\alpha}^{+\infty} (y - w_H) dF_H(y) + \int_0^{z_\alpha} (y - w_L - c) dF_H(y) \right]. \quad (2.2) \end{aligned}$$

We assume that the market-imposed minimal expected payoff for player  $A$  is 0 and  $w_L - e_L = 0$ .

### Relationship between hypothesis testing in statistics and our model

The null hypothesis  $H_0$  in our model is that player  $A$  chooses the high level of effort  $e_H$ , and the alternative hypothesis  $H_1$  in our model is that player  $A$  chooses the low level of effort  $e_L$ . The probability  $\alpha$  chosen by player  $P$  is that

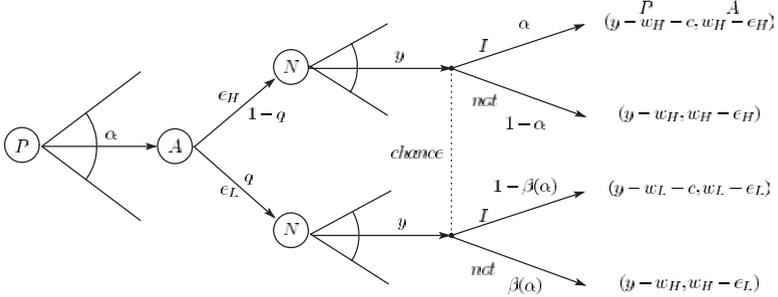


Figure 1: The model.

of the error of the first kind in hypothesis testing. Namely, the value of  $\alpha$  is the probability that the principal conducts the *costly* investigation *althouth* the agent chooses the high level of effort  $\epsilon_H$ .

We denote by  $\beta \in [0, 1]$  the probability of *the error of the second kind* in hypothesis testing. Namely, the value of  $\beta$  is the probability that the player  $P$  does *not* conduct the investigation into the level of effort chosen by player  $A$  *although* player  $A$  chooses the low level of effort  $\epsilon_L$ . Moreover, we obtain the function  $\beta(\alpha) = 1 - F_L(z_\alpha)$  where  $z_\alpha \equiv F_H^{-1}(\alpha)$ .

**Assumption 1.** The function  $\beta(\alpha) \in [0, 1]^{[0,1]}$  is convex and continuous, and fulfills  $\beta(0) = 1$  and  $\beta(1) = 0$ .<sup>5</sup>

This assumption implies that the function  $\beta(\alpha)$  is monotonically decreasing. The game tree of our model is described in Figure 1.

## 2.1 Analysis

### The best response correspondence of player A

In order to investigate subgame perfect equilibrium points of our model, we consider the best response correspondence  $q^*(\alpha)$  of player  $A$  to each  $\alpha \in [0, 1]$  chosen by player  $P$ . From (2.1) and the definition of  $\beta(\alpha)$ , the expected payoff of player  $A$  is given by

$$Eu_A(\alpha, q) = q\{(1 - \beta(\alpha))(w_H - w_L) + (\epsilon_H - \epsilon_L)\} + w_H - \epsilon_H. \quad (2.1.a)$$

<sup>5</sup> $\beta(\alpha) \in [0, 1]^{[0,1]}$  denotes a function  $\beta(\alpha)$  on  $[0, 1]$  into  $[0, 1]$ . In the following, we use similar notations. For example,  $G(\alpha) \in \mathbb{R}^{[0,1]}$  denotes a function  $G(\alpha)$  on  $[0, 1]$  into  $\mathbb{R}$ .

Since we are interested in the case that for some  $\alpha \in (0, 1)$  player  $A$  has an incentive to choose the high effort  $e_H$ , we assume that  $w_H - e_H > w_L - e_L$ . In the following, we assume this inequality without further remark. Proofs of lemmas in this section are relegated to the Appendix A.

**Lemma 2.1.** Let  $\alpha^* \equiv \beta^{-1}(1 - \frac{e_H - e_L}{w_H - w_L})$  where  $\beta^{-1}$  is the inverse function of  $\beta(\alpha)$ . The best response correspondence  $q^*(\alpha)$  of player  $A$  is given by

$$q^*(\alpha) = \begin{cases} 1 & \text{if } \alpha < \alpha^*, \\ [0, 1] & \text{if } \alpha = \alpha^*, \\ 0 & \text{if } \alpha > \alpha^*. \end{cases}$$

### The expected payoff of player $P$ and the subgame perfect equilibrium point

Given the best response correspondence  $q^*(\alpha)$  of player  $A$ , from (2.2) and the definition of  $\beta(\alpha)$ , the expected payoff of player  $P$  is given by

$$\begin{aligned} Eu_P(\alpha, q^*(\alpha)) &= (1 - q^*(\alpha))\{\mu_H - c\alpha - w_H\} \\ &+ q^*(\alpha)\{\beta(\alpha)(w_L + c - w_H) + \mu_L - w_L\} \quad (2.2.a) \end{aligned}$$

The expected payoff  $Eu_P(\alpha, q^*(\alpha))$  of player  $P$  has following properties.

#### Lemma 2.2.

- (1)  $Eu_P(\alpha, q^*(\alpha))$  is an increasing function of  $\alpha$  on  $[0, \alpha^*)$  if and only if  $w_H - w_L > c$ .
- (2)  $Eu_P(\alpha, q^*(\alpha))$  is a decreasing function of  $\alpha$  on  $(\alpha^*, 1]$ .

Let  $Eu_P(\alpha, 0) \equiv \mu_H - c\alpha - w_H$  and  $Eu_P(\alpha, 1) \equiv \beta(\alpha)(w_L + c - w_H) + \mu_L - w_L$ .

#### Lemma 2.3.

- If  $(\mu_H - w_H) - (\mu_L - w_L) > c$ , then
- (1)  $Eu_P(\alpha, 0) > Eu_P(\alpha, 1)$  for each  $\alpha \in [0, 1]$  and
  - (2)  $Eu_P(0, 1) < Eu_P(\alpha^*, 0)$ .

According to Lemma 2.2 and Lemma 2.3, the graphs of the expected payoff  $Eu_P(\alpha, q^*(\alpha))$  of player  $P$  are drawn at Figure 2. This Figure 2 says that player  $A$  must choose the high effort in the subgame perfect equilibrium point due to the inequality  $(\mu_H - w_H) - (\mu_L - w_L) > c$ . In the following, we assume the inequality  $(\mu_H - w_H) - (\mu_L - w_L) > c$  without further remark. We obtain the following benchmark result.

**Theorem 2.1.** The subgame perfect equilibrium point  $(\alpha^*, q^*)$  of our model is given by the pair of  $\alpha^* = \beta^{-1}(1 - \frac{e_H - e_L}{w_H - w_L})$  and  $q^* = 0$ .

Since we have assumed that  $w_H - e_H > w_L - e_L$ , it turns out that  $\alpha^* > 0$ . To prevent player  $A$  from choosing the low effort  $e_L$ , player  $P$  allows the error probability  $\alpha$  to be strictly positive.

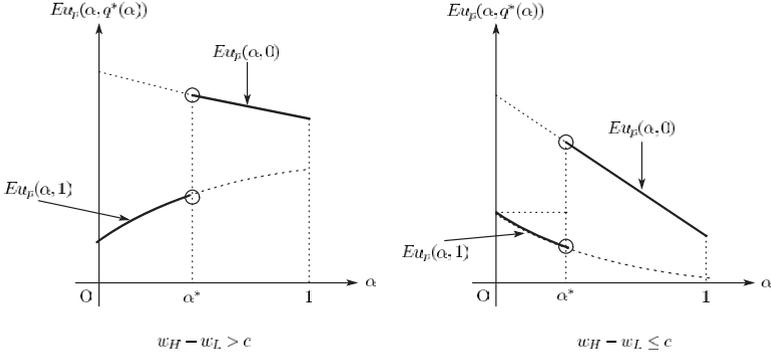


Figure 2: The expected payoff  $Eu_P(\alpha, q^*(\alpha))$  of player  $P$ .

### 3 Belief-dependent guilt feelings

We incorporate belief-dependent guilt feelings into the payoff of player  $A$  à la Dufwenberg (2002).<sup>6</sup> Let  $q'' \in [0, 1]$  be player  $A$ 's belief about player  $P$ 's belief about the behavior strategy  $q \in [0, 1]$  which is the probability that player  $A$  chooses the low level of effort  $e_L$ . We call the belief  $q'' \in [0, 1]$  the *second order belief* of player  $A$ .

Consider a scenario where player  $A$  has chosen the low level of effort  $e_L$  and player  $P$  does *not* conduct the investigation into the level of effort chosen by player  $A$ . In this scenario, the smaller the second order belief  $q''$  of player  $A$  is, the more player  $A$  would feel guilty about his choosing the low level of effort. We add a negative valued and increasing function  $g(q'')$  of the second order belief  $q''$  to player  $A$ 's payoff of the outcome corresponding to this scenario. This change of player  $A$ 's payoff is described at Figure 3.

**Assumption 2.**  $g \in \mathbb{R}^{\mathbb{R}}$  is a continuous and monotonically increasing function and fulfills that  $g(0) = -k$  and  $g(1) = 0$  where  $k > 0$ .<sup>7</sup>

Namely, the value of  $g(q'')$  captures the strength of the guilt feeling of player  $A$  with  $q''$ . We call  $g(q'')$  *belief-dependent payoff* of the guilt feeling.

<sup>6</sup>Dufwenberg (2002) proposed a game of trust with belief-dependent guilt feelings.

<sup>7</sup>Note that the function  $g$  is not defined on  $[0, 1]$  but defined on the set of *all* real numbers  $\mathbb{R}$ , so that we can define an inverse function  $g^{-1}$  of  $g$  where the range of  $g^{-1}$  is the set of all real numbers  $\mathbb{R}$ .

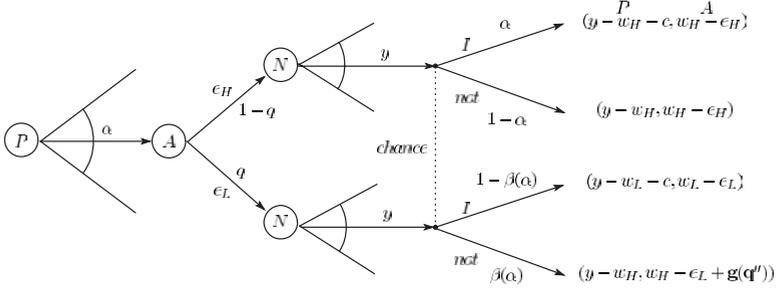


Figure 3: The model with belief-dependent guilt feelings of player A.

In the following, the equilibrium concept for our model is the psychological equilibrium point which was proposed by Geanakoplos, Pearce and Stacchetti (1989) and Rabin (1993).

**Definition 3.1.** The *psychological equilibrium point* of our model is the triplet  $(\alpha^{**}, q^{**}, q'')$  such that

- (1) the pair of  $(\alpha^{**}, q^{**})$  is the subgame perfect equilibrium point of our model with the belief-dependent payoff and
- (2)  $q^{**} = q''$ . (*consistency*)

### 3.1 Analysis

#### The best response correspondence of player A

In order to investigate the psychological equilibrium point of our model with player A's belief-dependent guilt feelings, we consider the best response correspondence  $q^*(\alpha, q'')$  of player A to each pair  $(\alpha, q'') \in [0, 1]^2$ . From (2.1) and Figure 3, the expected payoff  $Eu_A(\alpha, q, q'')$  of player A with  $q''$  is given by

$$Eu_A(\alpha, q, q'') = q\{\beta(\alpha)(w_H - w_L + g(q'')) + (w_L - w_H) + (\epsilon_H - \epsilon_L)\} + (w_H - \epsilon_H). \quad (2.1.b)$$

Let  $G(\alpha) \equiv g^{-1}\left(\frac{(w_H - w_L) + (\epsilon_L - \epsilon_H)}{\beta(\alpha)} - (w_H - w_L)\right)$  where  $g^{-1}$  is the inverse function of  $g$  and  $\alpha \neq 1$ .

**Lemma 3.1.** The best response correspondence  $q^*(\alpha, q'')$  of player A with  $q''$

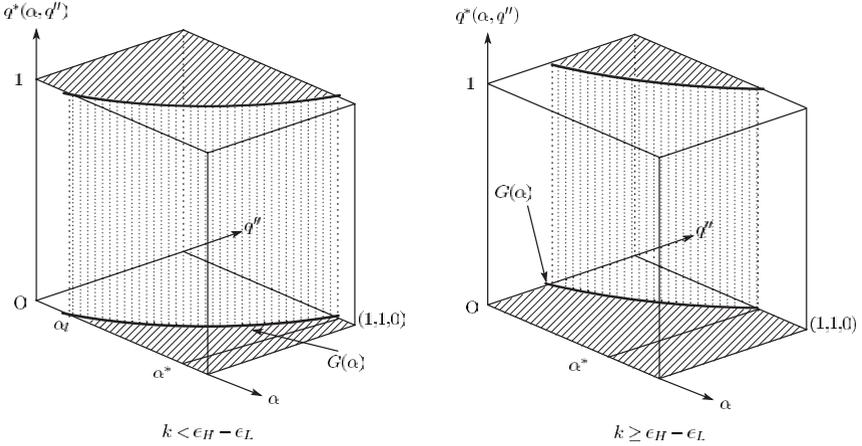


Figure 4: The best response correspondences  $q^*(\alpha, q'')$  of player  $A$ .

is given by

$$q^*(\alpha, q'') = \begin{cases} 1 & \text{if } q'' > G(\alpha), \\ [0, 1] & \text{if } q'' = G(\alpha), \\ 0 & \text{if } q'' < G(\alpha) \text{ or } \alpha = 1. \end{cases}$$

Proofs of lemmas in this section are relegated to the Appendix B.

According to Lemma 3.1 and following Lemma 3.2, the best response correspondence  $q^*(\alpha, q'')$  of player  $A$  with  $q''$  is drawn at Figure 4.

**Lemma 3.2.** There exists the number  $\alpha_l$  such that  $G(\alpha_l) = 0$  and  $0 < \alpha_l < \alpha^*$  if and only if  $k < \epsilon_H - \epsilon_L$ .

Since  $G(\alpha)$  is a monotonically increasing function of  $\alpha$ , for each  $\alpha < \alpha_l$  the optimal strategy of player  $A$  with each  $q'' \in [0, 1]$  is to choose the low effort  $\epsilon_L$ .

### The expected payoff of player $P$ and the psychological equilibrium points

**Lemma 3.3.** Given the best response correspondence  $q^*(\alpha, q'')$  of player  $A$  with a second order belief  $q''$ , the expected payoff  $Eu_P(\alpha, q^*(\alpha, q''))$  of player  $P$  is given by

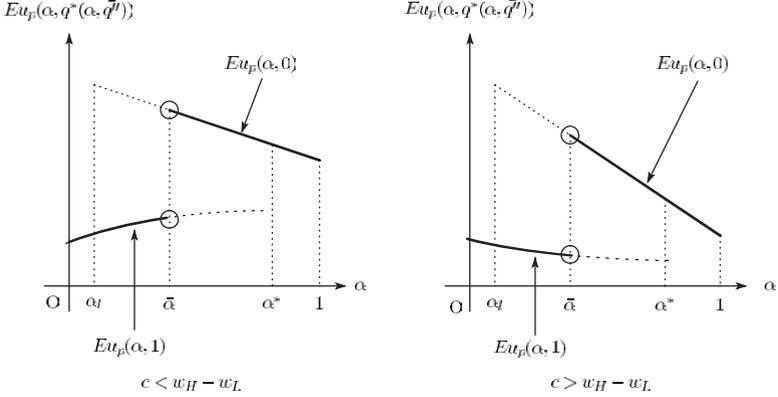


Figure 5: The expected payoff of player  $P$  for a fixed second order belief  $\bar{q}'' = G(\bar{\alpha})$ .

- (1)  $\beta(\alpha)(w_L + c - w_H) + \mu_L - w_L$   
 on a set  $S_1 = \{(\alpha, q'') \in [0, 1]^2 \mid 0 \leq \alpha < \alpha_l, 0 \leq q'' \leq 1\}$  and  
 on a set  $S_2 = \{(\alpha, q'') \in [0, 1]^2 \mid \alpha_l \leq \alpha < \alpha^*, q'' > G(\alpha)\}$ ,
- (2)  $\mu_H - c\alpha - w_H$   
 on a set  $S_3 = \{(\alpha, q'') \in [0, 1]^2 \mid \alpha_l < \alpha < \alpha^*, q'' < G(\alpha)\}$  and  
 on a set  $S_4 = \{(\alpha, q'') \in [0, 1]^2 \mid \alpha^* < \alpha < 1, 0 \leq q'' \leq 1\}$ .

Let  $k \leq \epsilon_H - \epsilon_L$ . Then, for each  $\bar{\alpha} \in [\alpha_l, \alpha^*]$ , there is the second order belief  $\bar{q}'' \in [0, 1]$  such that  $\bar{q}'' = G(\bar{\alpha})$  by Lemma 3.2 and the monotonicity of  $G(\alpha)$ . Note that the expected payoff  $Eu_P(\alpha, q^*(\alpha, \bar{q}''))$  for the fixed  $\bar{q}''$  is drawn at Figure 5. The shape of the graph of  $Eu_P(\alpha, q^*(\alpha, \bar{q}''))$  is the same as that of  $Eu_P(\alpha, q^*(\alpha))$  in Section 2 except each point where each graph jumps.

Observing Figure 5, we provide an explanation of why the following result, Theorem 3.1, holds. Let  $c < w_H - w_L$ .<sup>8</sup> Then,  $Eu_P(\alpha, 1)$  is a monotonically increasing function, and  $Eu_P(\alpha, 0)$  is a monotonically decreasing function by Lemma 2.2. Therefore, if player  $A$  chooses  $\bar{q}''$ , there is no incentive for player  $P$  to change  $\bar{\alpha}$  to any other strategies. Since  $q^*(\bar{\alpha}, \bar{q}'') = [0, 1]$  by Lemma 3.1, and  $Eu_P(\alpha, 0) > Eu_P(\alpha, 1)$  for each  $\alpha \in [0, 1]$  by Lemma 2.3, for  $\bar{\alpha}$  to constitute player  $P$ 's strategy of a subgame perfect equilibrium point, the equality  $q^*(\bar{\alpha}, \bar{q}'') = 0$  must hold. By the consistency condition of Definition 3.1, both

<sup>8</sup>To explain intuitively why Theorem 3.1 holds, we assume this inequality  $c < w_H - w_L$ . The assumption  $c < w_H - w_L$  is irrelevant to whether Theorem 3.1 holds or not.

equalities  $q^{**} = \bar{q}'' = 0$  and  $G(\bar{\alpha}) = 0$  hold in the psychological equilibrium point of our model.

**Theorem 3.1.** *If  $k < \epsilon_H - \epsilon_L$ , then the psychological equilibrium point  $(\alpha^{**}, q^{**}, q'')$  of our model with player A's sense of guilty is given by  $(\alpha_l, 0, 0)$  where  $\alpha_l = \beta^{-1} \left( \frac{(u_H - u_L) - (\epsilon_H - \epsilon_L)}{(u_H - u_L) - k} \right)$ .*

(Proof:) Let  $\bar{\alpha} = G^{-1}(\bar{q}'')$  for each  $\bar{q}'' \in [0, 1]$ . By Lemma 3.1, player A's best response correspondence  $q^*(\bar{\alpha}, \bar{q}'')$  is given by  $q^*(\bar{\alpha}, \bar{q}'') = [0, 1]$ . By Lemma 2.3, in a subgame perfect equilibrium point  $q^*(\bar{\alpha}, \bar{q}'')$  must be zero, that is, for each  $\bar{q}'' \in [0, 1]$  the subgame perfect equilibrium point  $(\alpha, q)$  is given by  $(\bar{\alpha}, 0)$ .

By the consistency condition of Definition 3.1, in the psychological equilibrium point  $q^{**} = q'' = 0$  holds. Then  $\alpha^{**} = G^{-1}(0) = \alpha_l$ . Since  $G(\alpha) \equiv g^{-1} \left( \frac{(u_H - u_L) + (\epsilon_L - \epsilon_H)}{\beta(\alpha)} - (w_H - w_L) \right)$  and  $g(0) = -k$ , we obtain that  $\alpha_l = \beta^{-1} \left( \frac{(u_H - u_L) - (\epsilon_H - \epsilon_L)}{(u_H - u_L) - k} \right)$ . ■

Since  $\alpha^* = \beta^{-1} \left( 1 - \frac{\epsilon_H - \epsilon_L}{u_H - u_L} \right)$  and  $k > 0$ , it turns out that  $0 < \alpha_l < \alpha^*$ .

**Corollary 3.1.** *If  $k \geq \epsilon_H - \epsilon_L$ , then the psychological equilibrium point  $(\alpha^{**}, q^{**}, q'')$  of our model with player A's belief-dependent guilt feelings is given by  $(0, 0, 0)$ .*

(Proof:) Let  $\bar{\alpha} = G^{-1}(\bar{q}'')$  for each  $\bar{q}'' \in [0, 1]$ . By Lemma 3.1, player A's best response correspondence  $q^*(\bar{\alpha}, \bar{q}'')$  is given by  $q^*(\bar{\alpha}, \bar{q}'') = [0, 1]$ . By Lemma 2.3, in a subgame perfect equilibrium point  $q^*(\bar{\alpha}, \bar{q}'')$  must be zero, that is, for each  $\bar{q}'' \in [0, 1]$  the subgame perfect equilibrium point  $(\alpha, q)$  is given by  $(\bar{\alpha}, 0)$ .

By the consistency condition of Definition 3.1, in the psychological equilibrium point  $q^{**} = q'' = 0$  holds. Since  $k \geq \epsilon_H - \epsilon_L$ ,  $\alpha^{**} = 0$  by Lemma 3.2. We obtain the result. ■

## 4 Belief-dependent impulses for deceiving

We incorporate player A's belief-dependent impulses for deceiving player P, into our benchmark model. We follow the previous section for the formation of the belief-dependent payoff. Consider the scenario where player A has chosen the level of effort  $\epsilon_L$  and player P does *not* conduct the investigation for the level of effort chosen by player A. In this scenario, the smaller the second order belief  $q''$  of player A for choosing the low effort  $\epsilon_L$  is, the more player A with his belief-dependent impulse for deceiving player P would feel satisfaction with this situation. We add a decreasing function  $g_d(q'')$  to player A's payoff of the outcome corresponding to this scenario.

**Assumption 3.**  $g_d \in \mathbb{R}^{\mathbb{R}}$  is a continuous and monotonically decreasing function and fulfills that  $g_d(0) = k_d$  and  $g_d(1) = 0$  where  $k_d > 0$ .

We investigate the psychological equilibrium point in our model with player  $A$ 's impulse for deceiving.

### The best response correspondence of player $A$

We replace the function  $g$  in the previous Section 3 with the function  $g_d$ , so that obtain following Lemma 4.1. Let  $G_d(\alpha) \equiv g_d^{-1}\left(\frac{(u_H - u_L) + (\epsilon_H - \epsilon_L)}{\beta(\alpha)} - (w_H - w_L)\right)$  where  $g_d^{-1}$  is the inverse function of  $g_d$  where  $\alpha \neq 1$ . Note that this function  $G_d(\alpha)$  is a decreasing one.

**Lemma 4.1** The best response correspondence  $q^*(\alpha, q'')$  of player  $A$  with  $q''$  is given by

$$q^*(\alpha, q'') = \begin{cases} 1 & \text{if } q'' < G_d(\alpha), \\ [0, 1] & \text{if } q'' = G_d(\alpha), \\ 0 & \text{if } q'' > G_d(\alpha) \text{ or } \alpha = 1. \end{cases}$$

According to Lemma 4.1 and following Lemma 4.2, the best response correspondence  $q^*(\alpha, q'')$  of player  $A$  with  $q''$  is drawn at Figure 7.

**Lemma 4.2** There exists a number  $\alpha_m$  such that  $G_d(\alpha_m) = 0$  and  $\alpha^* < \alpha_m < 1$ .

This lemma follows from continuity of  $g_d(\alpha)$ . The proof of this lemma is relegated to the Appendix B.

### The psychological equilibrium point

**Theorem 4.1.** The psychological equilibrium point  $(\alpha^{**}, q^{**}, q'')$  of our model with player  $A$ 's impulse to deceive is given by  $(\alpha_m, 0, 0)$  where

$$\alpha_m = \beta^{-1}\left(\frac{(u_H - u_L) - (\epsilon_H - \epsilon_L)}{(u_H - u_L) + k_d}\right).$$

(Proof:) Let  $\bar{\alpha} = G_d^{-1}(\bar{q}'')$  for each  $\bar{q}'' \in [0, 1]$ . By Lemma 4.1, player  $A$ 's best response correspondence  $q^*(\bar{\alpha}, \bar{q}'')$  is given by  $q^*(\bar{\alpha}, \bar{q}'') = [0, 1]$ . By Lemma 2.3, in a subgame perfect equilibrium point  $q^*(\bar{\alpha}, \bar{q}'')$  must be zero, that is, for each  $\bar{q}'' \in [0, 1]$  the subgame perfect equilibrium point  $(\alpha, q)$  is given by  $(\bar{\alpha}, 0)$ .

By the consistency condition of Definition 3.1, in the psychological equilibrium point  $q^{**} = q'' = 0$  must hold. Then  $\alpha^{**} = G_d^{-1}(0) = \alpha_m$ . ■

Since  $\alpha^* = \beta^{-1}\left(1 - \frac{\epsilon_H - \epsilon_L}{u_H - u_L}\right)$  and  $k_d > 0$ , it turns out that  $\alpha^* < \alpha_m$ .

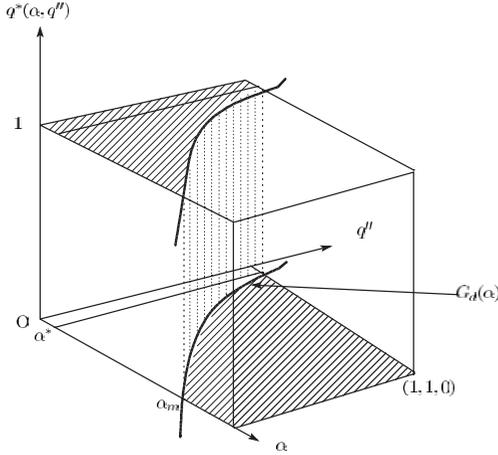


Figure 6: The best response correspondence  $q^*(\alpha, q'')$  of player  $A$  with his belief-dependent impulse for deceiving.

## 5 Belief-dependent reciprocity

We incorporate belief-dependent reciprocity into the payoff of player  $A$  of our model à la Dufwenberg and Kirchsteiger (2004) and Falk and Fischbacher (2006).

<sup>5</sup> Let  $Eu_i(\alpha, q)$ , ( $i = 1, 2$ ), be each player  $i$ 's expected payoff function formulated in Section 2 of this paper (expressions (2.1) and (2.2)) and  $q''$  the second order belief of player  $P$ . A formulation  $\{Eu_A(\alpha, q'') - Eu_P(\alpha, q'')\}$  measures player  $P$ 's kindness as perceived by player  $A$ . A formulation  $\{Eu_P(\alpha, q) - Eu_P(\alpha, q'')\}$  measures how much player  $A$  alters player  $P$ 's payoff with his own behavior strategy  $q$ . Thus, the product of  $\{Eu_A(\alpha, q'') - Eu_P(\alpha, q'')\}$  and  $\{Eu_P(\alpha, q) - Eu_P(\alpha, q'')\}$  measures the reciprocity utility of player  $A$ .

We assume that the expected payoff  $Eu_A^R(\alpha, q, q'')$  of player  $A$  with belief-dependent reciprocity is

$$Eu_A^R(\alpha, q, q'') = Eu_A(\alpha, q) + \rho_A \cdot \{Eu_A(\alpha, q'') - Eu_P(\alpha, q'')\} \cdot \{Eu_P(\alpha, q) - Eu_P(\alpha, q'')\}, \quad (2.1c)$$

where  $\rho_A$  is a positive parameter that represents the impact of reciprocation on player  $A$ 's payoff.

The second term,  $\rho_A \cdot \{Eu_A(\alpha, q'') - Eu_P(\alpha, q'')\} \cdot \{Eu_P(\alpha, q) - Eu_P(\alpha, q'')\}$ , is the payoff caused by player  $A$ 's belief-dependent reciprocity.

<sup>5</sup> Bolton and Gekkenfels (2000) proposes a theory of consequentialistic reciprocity.

### The psychological equilibrium point

Dufwenberg and Kirchsteiger (2004) and Falk and Fischbacher (2006) prove the existence theorems of the psychological equilibrium points of games with belief-dependent reciprocity. Their existence theorems hold in the setting of our model.

Suppose that in the psychological equilibrium point of our model the player  $A$  with belief-dependent reciprocity chooses the high level of effort  $e_H$ . That is, we assume that the equilibrium behavior strategy of player  $A$  is given by  $q = 0$ . By the consistency condition of Definition 3.1, the second order belief  $q''$  must be zero in this equilibrium. We verify whether the behavior strategy,  $q = 0$ , of player  $A$  becomes the best response for some  $\alpha \in [0, 1]$  or not.

Since the expected payoff function  $Eu_A^R(\alpha, q, q'')$  is a linear function of  $q$ , the coefficient of  $q$  in the function  $Eu_A^R(\alpha, q, q'')$  is given by

$$\begin{aligned} \frac{\partial Eu_A^R}{\partial q}(\alpha, q, q'') &= \{(1 - \beta(\alpha))(w_H - w_L) + (\epsilon_H - \epsilon_L)\} + \\ \rho_A[q''\{(1 - \beta(\alpha))(w_H - w_L) + (\epsilon_H - \epsilon_L)\} &+ (w_H - w_L) - \\ (1 - q'')(\mu_H - c\alpha - w_H) - q''\{\beta(\alpha)(w_L + c - w_H) &+ \mu_L - w_L\}] \\ [-\{\mu_H - c\alpha - w_H\} + \{\beta(\alpha)(w_L + c - w_H) &+ \mu_L - w_L\}. \end{aligned}$$

By simple calculations, we find that if  $(\mu_H - w_H) - (\mu_L - w_L) > c$ , then  $\frac{\partial Eu_A^R}{\partial q}(\alpha, 0, 0) > 0$  for each  $\alpha \in [0, 1]$ .<sup>10</sup> That is, for each  $\alpha \in [0, 1]$ , player  $A$  with belief-dependent reciprocity has an incentive for choosing the low level of effort  $e_L$ . This fact contradicts our assumption that the behavior strategy  $q$  of player  $A$  is zero in the psychological equilibrium point. Thus, we obtain the following result.

**Theorem 5.1.** Player  $P$  (an inspector) can not prevent player  $A$  (an inspectee) with belief-dependent reciprocity from choosing the low level of effort  $e_L$  in our monitoring model.

This theorem contrasts sharply with those in the former sections. Moreover, in Theorem 1.1, Theorem 2.1, and Theorem 3.1, player  $P$  (an inspector) can prevent player  $A$  (an inspectee) from choosing the low level of effort  $e_L$  in equilibrium points.

## 6 Concluding remarks

Exploiting the consistency condition required by the definition of psychological equilibrium points, we have found a *unique* equilibrium point of our model in Section 3 and in Section 4. The consistency condition is the key to specifying behaviors of players with belief-dependent payoffs. Inside the sphere of the psychological game theory, we can not explain how the consistent beliefs are

<sup>10</sup>Note that we have assumed the inequality  $(\mu_H - w_H) - (\mu_L - w_L) > c$  in Section 2.

enforced on the players. We need to rely on several ways outside the sphere of the psychological game theory to model the mechanism for enforcing the consistent beliefs on the players. For example, the evolutionary game theory would be the hopeful way.

Our benchmark model is formulated by means of inspection games. It is widely recognized that if the inspector (principal) takes leadership announcing his strategy and making it known to the inspectee (agent), the inspector (principal) can prevent the inspectee (agent) from choosing undesirable action for the inspector (see Avenhaus, Okada and Zamir (1991) and Avenhaus, von Stengel and Zamir (2002)). We have shown, however, that when the agent has belief-dependent reciprocity, the inspector can not prevent the inspectee from choosing undesirable action for the principal. Inspection games deal with a situation which is more or less competitive. In the competitive situation, the agent feels unkindness of the principal. When we apply the theory of inspector leadership game to our real life, we must pay attention to this feeling of unkindness perceived by the inspectee.

We have investigated a simple monitoring system. Only two distinct levels of wage, high or low, is considered. Dye (1986) and Kanodia (1985) constructs monitoring models similar to ours. They assume more flexible wage than that of our models. We need to construct a monitoring model which has more levels of wage than that of our model.

## Appendix A

This appendix A provides proofs of lemmas in Section 2.

**Proof of Lemma 2.1.** From (2.1.a), if  $(1 - \beta(\alpha))(w_H - w_L) + (\epsilon_H - \epsilon_L) > 0$  then  $q^*(\alpha) = 1$ . Since  $w_H - w_L > 0$ , the former inequality becomes  $\beta(\alpha) > 1 - \frac{\epsilon_H - \epsilon_L}{w_H - w_L}$ . Due to  $w_H - \epsilon_H > w_L - \epsilon_L = 0$ , inequalities  $0 < \frac{\epsilon_H - \epsilon_L}{w_H - w_L} < 1$  hold. Hence, by Assumption 1, the function  $\beta(\alpha)$  has the inverse function  $\beta^{-1}(1 - \frac{\epsilon_H - \epsilon_L}{w_H - w_L})$  which is decreasing. It follows that the inequality  $(1 - \beta(\alpha))(w_H - w_L) + (\epsilon_H - \epsilon_L) > 0$  is equivalent to  $\alpha < \beta^{-1}(1 - \frac{\epsilon_H - \epsilon_L}{w_H - w_L})$ .

From (2.1.a), if  $(1 - \beta(\alpha))(w_H - w_L) + (\epsilon_H - \epsilon_L) = 0$ , then  $q^*(\alpha) \in [0, 1]$ . Since the inverse function  $\beta^{-1}$  exists due to Assumption 1, the former equality becomes  $\alpha = \beta^{-1}(1 - \frac{\epsilon_H - \epsilon_L}{w_H - w_L})$ .

If  $(1 - \beta(\alpha))(w_H - w_L) + (\epsilon_H - \epsilon_L) < 0$ , then  $q^*(\alpha) = 0$ . Due to Assumption 1, the function  $\beta(\alpha)$  has the inverse function  $\beta^{-1} \in [0, 1]^{[0, 1]}$  which is decreasing. Hence, the condition  $(1 - \beta(\alpha))(w_H - w_L) + (\epsilon_H - \epsilon_L) < 0$  becomes  $\alpha > \beta^{-1}(1 - \frac{\epsilon_H - \epsilon_L}{w_H - w_L})$ . ■

**Proof of Lemma 2.2.** (1) By Lemma 2.1 and (2.2.a),  $E u_F(\alpha, q^*(\alpha)) = \mu_L + \beta(\alpha)(w_L + c - w_H) - w_L$  for each  $\alpha \in [0, \alpha^*]$ . Since  $\beta(\alpha)$  is a decreasing function of  $\alpha$ ,  $w_H - w_L > c$  if and only if  $E u_F(\alpha, q^*(\alpha))$  is an increasing function on  $[0, \alpha^*]$ .

(2) By Lemma 2.1,  $E_{uF}(\alpha, q^*(\alpha)) = \mu_H - c\alpha - w_H$  for each  $(\alpha^*, 1]$ . Since  $c > 0$ , we obtain the result. ■

**Proof of Lemma 2.3.** (1)  $E_{uF}(\alpha, 0) > E_{uF}(\alpha, 1)$  is  $\mu_H - c\alpha - w_H > \mu_L + \beta(\alpha)(w_L + c - w_H) - w_L$  from (2.2). This inequality becomes  $\mu_H - \mu_L + (\beta(\alpha) - 1)(w_H - w_L) > (\alpha + \beta(\alpha))c$ . Noting a fact that for each  $\alpha \in [0, 1]$   $\alpha + \beta(\alpha) < 1$  which is implied from the definition of  $\beta(\alpha)$ , we obtain that  $\mu_H - \mu_L + (\beta(\alpha) - 1)(w_H - w_L) \geq \mu_H - \mu_L - \alpha(w_H - w_L) \geq \mu_H - \mu_L - w_H + w_L = (\mu_H - w_H) - (\mu_L - w_L) > c > 0$ .

(2) When  $w_H - w_L > c$ , the proof of this part (2) of Lemma 2.3 is trivial. We concentrate ourselves on the case  $w_H - w_L \leq c$ . Suppose, by contradiction,  $E_{uF}(0, 1) \geq E_{uF}(\alpha^*, 0)$ . Then, there is a number  $\alpha' \in [0, \alpha^*]$  such that  $E_{uF}(0, 1) = E_{uF}(\alpha', 0)$  by part (2) of Lemma 2.2. We have  $\alpha' = \frac{(\mu_H - w_H) - (\mu_L - w_L)}{c} + 1$ . Since  $(\mu_H - w_H) - (\mu_L - w_L) > 0$ , we obtain  $\alpha' > 1$ . This is a contradiction. ■

## Appendix B

This appendix B provides proofs of lemmas in Section 3 and in Section 4.

**Proof of Lemma 3.1.** From (2.1.b), if an inequality  $\beta(\alpha)(w_H - w_L + g(q'')) + (w_L - w_H) + (\epsilon_H - \epsilon_L) > 0$  holds, then  $q^*(\alpha, q'') = 1$ . Let  $\beta(\alpha) \neq 0$ . Then, the inequality becomes  $g(q'') > \frac{(w_H - w_L) + (\epsilon_L - \epsilon_H)}{\beta(\alpha)} - (w_H - w_L)$ . Noting that  $g$  has the monotonically increasing inverse function  $g^{-1} \in \mathbb{R}^{\mathbb{R}}$  by Assumption 2, the inequality becomes  $q'' > g^{-1}\left(\frac{(w_H - w_L) + (\epsilon_L - \epsilon_H)}{\beta(\alpha)} - (w_H - w_L)\right)$ .

From (2.1.b), if an inequality  $\beta(\alpha)(w_H - w_L + g(q'')) + (w_L - w_H) + (\epsilon_H - \epsilon_L) < 0$  holds, then  $q^*(\alpha, q'') = 0$ . Let  $\beta(\alpha) \neq 0$ . Then, the inequality becomes  $g(q'') < \frac{(w_H - w_L) + (\epsilon_L - \epsilon_H)}{\beta(\alpha)} - (w_H - w_L)$ . Since  $g$  has the monotonically increasing inverse function  $g^{-1} \in \mathbb{R}^{\mathbb{R}}$ , the inequality becomes  $q'' < g^{-1}\left(\frac{(w_H - w_L) + (\epsilon_L - \epsilon_H)}{\beta(\alpha)} - (w_H - w_L)\right)$ .

From (2.1.b), if an equality  $\beta(\alpha)(w_H - w_L + g(q'')) + (w_L - w_H) + (\epsilon_H - \epsilon_L) = 0$  holds, then player  $A$  randomly chooses the level of effort. Let  $\beta(\alpha) \neq 0$ . Then the equality becomes  $g(q'') = \frac{(w_H - w_L) + (\epsilon_L - \epsilon_H)}{\beta(\alpha)} - (w_H - w_L)$ . Since  $g$  has its inverse function  $g^{-1} \in \mathbb{R}^{\mathbb{R}}$ , we obtain  $q'' = g^{-1}\left(\frac{(w_H - w_L) + (\epsilon_L - \epsilon_H)}{\beta(\alpha)} - (w_H - w_L)\right)$ .

When  $\beta(\alpha) = 0$ , that is,  $\alpha = 1$ ,  $\beta(\alpha)(w_H - w_L + g(q'')) + (w_L - w_H) + (\epsilon_H - \epsilon_L) = (w_L - w_H) + (\epsilon_H - \epsilon_L) < 0$  due to the inequality  $w_H - \epsilon_H > w_L - \epsilon_L$  that we have assumed in Section 2. Hence,  $q^*(1, q'') = 0$  for each  $q'' \in [0, 1]$ . ■

**Proof of Lemma 3.2.** Since  $\beta(0) = 1$ ,  $G(0) = g^{-1}(\epsilon_L - \epsilon_H) \leq 0$ . By Assumption 2,  $\epsilon_L - \epsilon_H < -k$  if and only if  $G(0) < 0$ .  $G(\alpha) \in \mathbb{R}^{[0, 1]}$  is a differentiable and monotonically increasing function due to Assumption 1 and

Assumption 2. Since  $\lim_{\alpha \rightarrow 1^-} G(\alpha) = +\infty$ , there is a number  $\alpha_t \in (0, 1)$  such that  $G(\alpha_t) = 0$  if and only if  $\epsilon_L - \epsilon_H < -k$ .

Since  $G(\alpha^*) = G(\beta^{-1}(1 - \frac{\epsilon_H - \epsilon_L}{w_H - w_L})) = 1$  and  $G(\alpha) \in \mathbb{R}^{(0,1]}$  is monotonically increasing, we obtain  $\alpha_t < \alpha^*$ . ■

**Proof of Lemma 3.3.** (1) Since  $G(\alpha)$  is monotonically increasing,  $q'' \geq 0 \geq G(\alpha)$  for each point  $(\alpha, q'') \in S_1$ . Hence  $q^*(\alpha, q'') = 1$  for each point  $(\alpha, q'') \in S_1$  by Lemma 3.1. From (2.2.a), the expected payoff of player  $P$  is given by  $Eu_P(\alpha, 1) = \beta(\alpha)(w_L + c - w_H) + \mu_L - w_L$  for each point  $(\alpha, q'') \in S_1$ .

For each point  $(\alpha, q'') \in S_2$ ,  $q^*(\alpha, q'') = 1$  by Lemma 3.1. From (2.2.a), the expected payoff of player  $P$  is given by  $Eu_P(\alpha, 1) = \beta(\alpha)(w_L + c - w_H) + \mu_L - w_L$  for each point  $(\alpha, q'') \in S_2$ .

(2) For each point  $(\alpha, q'') \in S_3$ ,  $q^*(\alpha, q'') = 0$  by Lemma 3.1. From (2.2.a), the expected payoff of player  $P$  is given by  $\mu_H - c\alpha - w_H$  for each point  $(\alpha, q'') \in S_3$ .

Since  $G(\alpha)$  is monotonically increasing,  $q'' < 1 < G(\alpha)$  for each point  $(\alpha, q'') \in S_4$ . Hence  $q^*(\alpha, q'') = 0$  for each point  $(\alpha, q'') \in S_4$  by Lemma 3.1. From (2.2.a), the expected payoff of player  $P$  is given by  $\mu_H - c\alpha - w_H$  for each point  $(\alpha, q'') \in S_4$ . ■

**Proof of Lemma 4.2.** Since  $G_d(\alpha^*) = G_d(\beta^{-1}(1 - \frac{\epsilon_H - \epsilon_L}{w_H - w_L})) = g_d^{-1}(0) = 1$ ,  $G_d(\alpha^*) = 1$ . Since  $\lim_{\alpha \rightarrow 1^-} \beta(\alpha) = 0$ ,  $\lim_{\alpha \rightarrow 1^-} G_d(\alpha) = -\infty$ .  $G_d$  is a continuous function on  $[0, 1)$ . Hence, there exists a number  $\alpha_m$  such that  $G_d(\alpha_m) = 0$  and  $\alpha^* < \alpha_m < 1$ . ■

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