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# NOTES ON $P_\kappa\lambda$ AND $[\lambda]^\kappa$

By

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This paper consists of notes on some combinatorial properties. § 1 deals with $\lambda$-ineffability and the partition property of $P_\kappa\lambda$ with $\lambda$ ineffable. In § 2 we combine the flipping property and a filter investigated by Di Prisco and Marek to characterize huge cardinals.

We work in ZFC and the notations are standard. $P_\kappa\lambda=\{x\subset\lambda: |x|<\kappa\}$ $[\lambda]^\kappa=\{x\subset\lambda: |x|=\kappa\}$, $D_\kappa\lambda=\{\{x, y\}: x, y\in P_\kappa\lambda$ and $x\subsetneq y\}$.

## § 1 $P_\kappa\lambda$ when $\lambda$ is ineffable.

$\kappa$ is called $\lambda$-ineffable if for any function $f: P_\kappa\lambda\longrightarrow P_\kappa\lambda$ such that $f(x)\subset x$ for all $x\in P_\kappa\lambda$, there is a subset $A$ of $\lambda$ such that the set $\{x\in P_\kappa\lambda: A\cap x=f(x)\}$ is stationary. We abbreviate the following statement to Part*$(\kappa, \lambda)$;

"For any function $F: D_\kappa\lambda\longrightarrow 2$, there is a stationary homogeneous set $H$ i.e. $|F''([H]^2\cap D_\kappa\lambda)|=1.$"

If Part*$(\kappa, \lambda)$, then $\kappa$ is $\lambda$-ineffable. We shall show the converse is true when $\lambda$ is ineffable.

LEMMA 1. $X\subset P_\kappa\lambda$ is closed unbounded iff $\{\alpha<\lambda: X\cap P_\kappa\alpha$ is closed unbounded in $P_\kappa\alpha\}$ contains a closed unbounded subset of $\lambda$. Hence $S$ is stationary in $P_\kappa\lambda$ if $\{\alpha<\lambda: S\cap P_\kappa\alpha$ is stationary in $P_\kappa\alpha\}$ is a stationary subset of $\lambda$.

THEOREM 2. Suppose that $\lambda$ is ineffable. If Part*$(\kappa, \alpha)$ for all $\alpha<\lambda$, then Part*$(\kappa, \lambda)$.

PROOF. Let $F: D_\kappa\lambda\longrightarrow 2$ and $F_\alpha=F\upharpoonright D_\kappa\alpha$ for every $\alpha<\lambda$. By our assumptions, there is a stationary subset $A_\alpha$ of $P_\kappa\alpha$ such that

$$F''([A_\alpha]^2\cap D_\kappa\alpha)=\{k_\alpha\},\ k_\alpha\in\{0, 1\}.$$

Since $\lambda$ is ineffable, we can find an $A\subset P_\kappa\lambda$ so that

$$S=\{\alpha<\lambda: A_\kappa=A\cap P_\kappa\alpha\} \text{ is stationary in } \lambda.$$

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A is stationary by Lemma 1.

Let $t$, $u\in[A]^2\cap D_\kappa\lambda$. Since $S$ is unbounded in $\lambda$, there is a member of $S$, $\alpha$ such that both $t$ and $u$ are in $[A_\alpha]^2\cap D_\kappa\alpha$. Hence $F(t)=F(u)=k_\alpha$. So, A is a stationary homogeneous set for $F$.

DEFINITION. $\kappa$ is $\lambda$-almost ineffable if for any function $f:P_\kappa\lambda\longrightarrow P_\kappa\lambda$ such that $f(x)\subset x$ for all $x\in P_\kappa\lambda$, there is a subset $A$ of $\lambda$ such that the set $\{x\in P_\kappa\lambda: A\cap x=f(x)\}$ is unbounded.

THEOREM 3. Suppose that $\lambda$ is almost ineffable. Then $\kappa$ is $\lambda$-almost ineffable iff $\kappa$ is $\alpha$-almost ineffable for all $\alpha<\lambda$.

PROOF. $\longrightarrow$ is proved by the same argument as the lemma in Magidor [9] p.p. 281.
($\longleftarrow$) Let $f:P_\kappa\lambda\longrightarrow P_\kappa\lambda$ and $f(x)\subset x$ for all $x\in P_\kappa\lambda$. Considering a function $f\restriction P_\kappa\alpha$ and using $\alpha$-ineffability, we get an $A_\alpha\subset\alpha$ for every $\alpha<\lambda$ such that

$$X_\alpha=\{x\in P_\kappa\alpha: f(x)=x\cap A_\alpha\} \text{ is unbounded in } P_\kappa\alpha.$$

Using now the almost ineffability of $\lambda$, there is an $A\subset\lambda$ so that

$$S=\{\alpha<\lambda: A_\alpha=A\cap\alpha\} \text{ is unbounded in } \lambda.$$

Let $X=\{x\in P_\kappa\lambda: f(x)=x\cap A\}$. If $\alpha\in S$ and $x\in P_\kappa\alpha$, then $x\cap A_\alpha=x\cap A\cap\alpha=x\cap A$. Hence $X_\alpha\subset X\cap P_\kappa\alpha$ for every $\alpha\in S$. This gives

$$\{\alpha<\lambda: X\cap P_\kappa\alpha \text{ is unbounded in } P_\kappa\alpha\} \text{ is unbounded in } \lambda.$$

Thus $X$ is unbounded in $P_\kappa\lambda$.

COROLLARY 4. The following are equivalent for $\kappa<\lambda$ with $\lambda$ ineffable.

(a) Part*$(\kappa, \alpha)$ for all $\alpha<\lambda$.
(b) Part*$(\kappa, \lambda)$.
(c) $\kappa$ is $\lambda$-ineffable.
(d) $\kappa$ is $\alpha$-ineffable for all $\alpha<\lambda$.
(e) $\kappa$ is $\alpha$-almost ineffable for all $\alpha<\lambda$.
(f) $\kappa$ is $\lambda$-almost ineffable.
(g) $\kappa$ is $\alpha$-supercompact for all $\alpha<\lambda$.

PROOF. (a)$\longrightarrow$(b) is Theorem 1. (b)$\longrightarrow$(c) is Theorem 2 in Magidor [9]. (c)$\longrightarrow$(d) is the lemma also in [9]. (d)$\longrightarrow$(e) is trivial. (e)$\longleftrightarrow$(f) is Theorem 3. (e)$\longrightarrow$(g) is by Carr's result: If $\kappa$ is $2^{\alpha^{<\kappa}}$-shelah, then $\kappa$ is $\alpha$-supercompact. ($\kappa$ is $\alpha$-shelah if $\kappa$ is $\alpha$-almost ineffable.) See [3].

On the coding of $P_\kappa\lambda$, there are works of Zwicker [14] and Shelah [12]. The author can not answer this question.

QUESTION 5. Is there a function $t:\lambda\longrightarrow P_\kappa\lambda$ such that for any stationary subset $A$ of $\lambda$, $t''A$ is stationary in $P_\kappa\lambda$.

It is, of course, true if $\kappa=\lambda$. In fact let $t=id.|\kappa$. The question is interesting when $\lambda$ is ineffable.

PROPOSITION 6. If $\lambda$ is ineffable and there is a $t:\lambda\longrightarrow P_\kappa\lambda$ such that $t''A$ is stationary for any stationary subset $A$ of $\lambda$, then $\kappa$ is $\lambda$-ineffable.

PROOF. Suppose that $f:P_\kappa\lambda\longrightarrow P_\kappa\lambda$ and $f(x)\subset x$ for all $x\in P_\kappa\lambda$. Let $A_\alpha=\{\beta<\alpha:\beta\in f(t(\alpha))\}$. Since $\lambda$ is ineffable, there is a stationary subset $S$ of $\lambda$ and $A\subset\lambda$ so that $A_\alpha=A\cap\alpha$ for all $\alpha\in S$.

$B=t''S$ is stationary and for any $x\in B$ there is an $\alpha_x\in S$ such that $x=t(\alpha_x)$. Hence $f(x)\cap\alpha_x=A\cap\alpha_x$.

Let $B'=\{x\in B:f(x)\neq A\cap x\}$ and $\delta_x$=the least ordinal in $f(x)\Delta(A\cap x)$. $\delta_x\in x$ for all $x\in B'$.

Soppose that $B'$ is stationary. There is an ordinal $\delta<\lambda$ such that $C=\{x\in B':\delta_x=\delta\}$ is stationary.

$$\forall x\in C(f(x)\cap(\delta+1)\neq A\cap(\delta+1)).$$

So,

$$\forall x\in C(\alpha_x<\delta).$$

$$|\{\alpha_x:\ x\in C\}|\geq|C|=\lambda^{<\kappa}\geqslant\lambda.$$

Thus there is an $x\in C$ such that $\delta<\alpha_x$.
Hence $\{x\in B:f(x)=A\cap x\}$ is stationary.

REMARK. $t''A$ is a stationary subset which splits into $\lambda$ disjoint stationary subsets. Gitik constructed a model of $ZFC$ in which there is a stationary set that can not be splitted into $\lambda$ disjoint stationary subsets in [6].

## §2 $[\lambda]^\kappa$ when $\kappa$ is huge.

Let $j:V\longrightarrow M$ be a huge embedding with critical point $\kappa$ and $j(\kappa)=\lambda$ in this section.

At first we recall a filter on $[\lambda]^\kappa$ investigated by Di Prisco and Marek in [5]. It is analogous to the closed unbounded filter on $P_\kappa\lambda$.

DEFINITION. For $X\subset P_\kappa\lambda$, define $A_X$, the basic set generated by $X$, as follows: $A_X=\{x\in[\lambda]^\kappa : x$ is the union of an increasing $\kappa$-chain of elements of $X\}$. Define $F_{\kappa,\lambda}$ by

$A\in F_{\kappa,\lambda}$ iff there is a closed unbounded $X\subset P_\kappa\lambda$ such that $A_X\subset A$.

THEOREM (Di Prisco, Marek, Baumgartner)
$F_{\kappa,\lambda}$ is the least $\kappa$-complete, normal, fine filter on $[\lambda]^\kappa$. If $U$ is the normal ultrafilter on $[\lambda]^\kappa$ induced by $j$, then every set in $F_{\kappa,\lambda}$ is in $U$. In this case $F_{\kappa,\lambda}$ is not $\kappa^+$-complete.

$X\subset[\lambda]^\kappa$ is unbounded if $\forall x\in[\lambda]^\kappa\exists y\in X(x\subset y)$. $X$ is $F_{\kappa,\lambda}$ stationary if $X\cap Y\neq 0$ for all $Y\in F_{\kappa,\lambda}$.

PROPOSITION 1. Any $X\in F_{\kappa,\lambda}$ is unbounded.

PROOF. There is a $C\subset P_\kappa\lambda$ that is closed unbounded and $C_X\subset X$. Let $a\in[\lambda]^\kappa$ and $f:\kappa\longrightarrow$ a be a bijection, $x_\alpha=f''\alpha$ for all $\alpha<\kappa$. We can find, using induction, $y_\alpha\in C$ such that $y_\alpha\supsetneqq x_\alpha\cup\{y_\beta:\beta<\alpha\}$ for every $\alpha<\kappa$.

$$\{y_\alpha|\alpha<\kappa\}\subset C \text{ is a } \kappa\text{-chain and } x=\bigcup\{x_\alpha:\alpha<\kappa\}\subset\bigcup\{y_\alpha:\alpha<\kappa\}=y\in C_X\subset X.$$

Next proposition shows the situation is different from $P_\kappa\lambda$.

PROPOSITION 2. If $\kappa$ is huge, there is a $F_{\kappa,\lambda}$-stationary set that is not unbounded.

PROOF. $(\lambda)^\kappa=\{x\in[\lambda]^\kappa$: the order type of $x$ is $\kappa\}$ is in $U$. Clearly $(\lambda)^\kappa$ is not unbounded.

Moreover, we shall show that there is a $F_{\kappa,\lambda}$-stationary set $S$ such that for any $x$, $y$ in $S$, $x\not\subset y$. Thus, partition property may not be directry extended to $[\lambda]^\kappa$ as for $P_\kappa\lambda$.

DEFINITION. $f$ is a $\omega$-Jonsson function over a set $x$ iff $f:{}^\omega x\longrightarrow x$ and whenever $y\subset x$ and $|y|=|x|$, $f''{}^\omega y=x$.

LEMMA 3. Let $U$ be the normal ultrafilter on $[\lambda]^\kappa$ induced by $j$ and $f$ is a $\omega$-Jonsson function over $\lambda$. Then $\{x\in[\lambda]^\kappa : f\restriction{}^\omega x$ is $\omega$-Jonsson over $x\}\in U$.

PROOF. The same argument as a normal ultrafilter on $P_\kappa\lambda$ can be carried out. Let $e: V\longrightarrow N\simeq V^{[\lambda]^\kappa}/U$ and $X\subset e''\lambda$ with $|X|=|e''\lambda|=\lambda$. Since $Y=e^{-1}(X)\subset\lambda$ and $|Y|=\lambda$, $f''{}^\omega Y=\lambda$. So,

$$\forall\alpha<\lambda\exists s\in{}^\omega Y(\alpha=f(s)).$$

This implies

$$\forall\alpha\in e''\lambda\exists s\in{}^{\omega}Y(\alpha=e(f)(e(s))).$$

Since $e(s)=e''s\in{}^{\omega}X$,

$$e(f)''{}^{\omega}X=e''\lambda.$$

Hence $e(f)\restriction{}^{\omega}e''\lambda$ is $\omega$-Jonsson over $e''\lambda$.
Thus $\{x:f\restriction{}^{\omega}x$ is $\omega$-Jonsson over $x\}\in U$.

THEOREM 4. There is an $A\in U$ such that for every pair $x$, $y$ in $A$, $x\not\subset y$.

PROOF. Let $f$ be a $\omega$-Jonsson function over $\lambda$ and $A=\{x\in[\lambda]^\kappa:f\restriction{}^{\omega}x$ is $\omega$-Jonsson over $x\}\in U$.

Suppose $y\subsetneq x\in A$. Since $|x|=|y|$, $f''{}^{\omega}y=x$. But $f''{}^{\omega}y\subset y$.

## §3 Flipping properties and huge cardinals, partition properties of $P_\kappa\lambda$.

Flipping properties wrer first studied by Abramson, Harrington, Kleinberg and Zwicker in [1] and turned out to be another form of large cardinal property. Di Prisco and Zwicker [4] extended this line to supercompactness. More precisely, they gave a new type of flipping properties equivalent to $\lambda$-ineffability and $\lambda$-mildly ineffability. We shall introduce an analogous type properties and discuss the relationship with huge cardinals.

DEFINITION. If $t:\lambda\longrightarrow P([\lambda]^\kappa)$, we call $t'$ a flip of $t(t'\sim t)$ if $t':\lambda\longrightarrow P([\lambda]^\kappa)$ and for all $\alpha<\lambda$, $t'(\alpha)=t(\alpha)$ or $t'(\alpha)=[\lambda]^\kappa-t(\alpha)$. $\mathrm{Flip}(\kappa,\lambda)\equiv\forall t:\lambda\longrightarrow P([\lambda]^\kappa)\exists t'\sim t$ such that $\underset{\alpha<\lambda}{\Delta}t'(\alpha)$ is $F_{\kappa,\lambda}$-stationary. $\mathrm{Inef}(\kappa,\lambda)\equiv$for any function $f:[\lambda]^\kappa\longrightarrow[\lambda]^\kappa$ such that $f(x)\subset x$ for all $x\in[\lambda]^\kappa$, there is a subset $A$ of $\lambda$ such that the set

$$\{x\in[\lambda]^\kappa:A\cap x=f(x)\}\text{ is }F_{\kappa,\lambda}\text{-stationary.}$$

THEOREM 1. (i) $\mathrm{Flip}(\kappa,\lambda)$ iff $\mathrm{Inef}(\kappa,\lambda)$.
(ii) If $\mathrm{Flip}(\kappa,2^{\lambda^\kappa})$, then there is a huge embedding $j$ such that $\kappa$ is the critical point and $j(\kappa)=\lambda$.
(iii) If $j:V\longrightarrow M$ is a huge embedding with the critical point $\kappa$ such that $j(\kappa)=\lambda$, then $\mathrm{Flip}(\kappa,\lambda)$.

PROOF (i) Assume that $\mathrm{Flip}(\kappa,\lambda)$ and $f:[\lambda]^\kappa\longrightarrow[\lambda]^\kappa$ such that $f(x)\subset x$ for all $x\in[\lambda]^\kappa$. Define $t:\lambda\longrightarrow P([\lambda]^\kappa)$ by

$$t(\alpha)=\{x\in[\lambda]^\kappa:\alpha\in f(x)\}.$$

Let $t'\sim t$ be such that $\underset{\alpha<\lambda}{\Delta} t'(\alpha)$ is $F_{\kappa,\lambda}$-stationary.

Put $A=\bigcup\{f(x): x\in \underset{\alpha<\lambda}{\Delta} t'(\alpha)\}$. We shall show that if $x\in\underset{\alpha<\lambda}{\Delta} t'(\alpha)$ then $x\cap A=f(x)$. Obviously $f(x)\subset x\cap A$. If $\alpha\in x\cap A$, then there is a $y\in\underset{\beta<\lambda}{\Delta} t'(\beta)$ so that $\alpha\in f(y)$. Since $\alpha\in f(y)$, $y\in t(\alpha)$ and $\alpha\in y$. Hence $t'(\alpha)=t(\alpha)$. Now $\alpha\in x\in\underset{\beta<\lambda}{\Delta} t'(\beta)$ and $t'(\alpha)=t(\alpha)$. This gives $x\in t(\alpha)$. Hence $\alpha\in f(x)$.

Conversely, let $t:\lambda\longrightarrow P([\lambda]^\kappa)$. Define $f:[\lambda]^\kappa\longrightarrow[\lambda]^\kappa$ by

$$f(x)=\{\alpha\in x: x\in t(\alpha)\}.$$

There is a subset $A$ of $\lambda$ such that $B=\{x\in[\lambda]^\kappa: x\cap A=f(x)\}$ is $F_{\kappa,\lambda}$-stationary. Define $t':\lambda\longrightarrow P([\lambda]^\kappa)$ by $t'(\alpha)=t(\alpha)$ if $\alpha\in A$ and $t'(\alpha)=[\lambda]^\kappa-t(\alpha)$ if $\alpha\notin A$.

Suppose $x\in S$ and $\alpha\in x$. If $\alpha\in A$, then $\alpha\in f(x)$ hence $x\in t(\alpha)=t'(\alpha)$. If $\alpha\notin A$, then $\alpha\notin f(x)$ hence $x\notin t(\alpha)$. So $x\in t'(\alpha)$. Now we have shown $S\subset\underset{\alpha<\lambda}{\Delta} t'(\alpha)$, which must be $F_{\kappa,\lambda}$-stationary.

(ii) Let $\gamma=2^{\lambda^\kappa}$ and $\{A_\alpha:\alpha<\gamma\}$ be an enumeration of $P([\lambda]^\kappa)$. Define $t:\gamma\longrightarrow P([\gamma]^\kappa)$ by $t(\alpha)=\{x\in[\gamma]^\kappa: x\cap\lambda\in A_\alpha\}$. Let $t'\sim t$ be such that $\underset{\alpha<\gamma}{\Delta} t'(\alpha)$ is $F_{\kappa,\gamma}$-stationary.

A filter $U$ on $[\lambda]^\kappa$ is defined by $A_\alpha\in U$ iff $t'(\alpha)=t(\alpha)$. We shall show in fact $U$ is a normal ultrafilter. The fact that for any $a\in P_\kappa\gamma$ the set $\{x\in[\gamma]^\kappa: a\subset x\}$ is a member of $F_{\kappa,\gamma}$ is often used.

(1) $A_\alpha\in U\wedge A_\alpha\subset A_\beta\longrightarrow A_\beta\in U$.

There is a $x\in\underset{\xi<\lambda}{\Delta} t'(\xi)$ such that $\{\alpha,\beta\}\subset x$. Since $x\in t'(\alpha)=t(\alpha)$, $x\cap\lambda\in A_\alpha\subset A_\beta$. Thus $x\in t(\beta)$. Hence $t'(\beta)=t(\beta)$.

(2) $U$ is $\kappa$-complete.

Suppose $\{B_\alpha:\alpha<\delta\}\subset U$ $(\delta<\kappa)$ and $f:\delta\longrightarrow\gamma$ such that $B_\alpha=A_{f(\alpha)}$ for all $\alpha<\delta$. Let $A_\eta=\bigcap_{\alpha<\delta} B_\alpha$.

There is a $x\in\Delta t'(\xi)$ such that $\{\eta\}\cup f''\delta\subset x$. For all $\alpha<\delta$, $x\in t'(f(\alpha))=t(f(\alpha))$, so $x\in\bigcap_{\alpha<\delta} A_{f(\alpha)}$. Hence $x\cap\lambda\in A_\eta$. This shows $x\in t(\eta)$ and $t'(\eta)=t(\eta)$.

(3) For any $\alpha<\lambda$, $\{x\in[\lambda]^\kappa:\alpha\in x\}\in U$.

Let $A_\beta=\{x\in[\lambda]^\kappa:\alpha\in x\}$. $t(\beta)=\{x\in[\gamma]^\kappa:\alpha\in x\cap\lambda\}=\{x\in[\gamma]^\kappa:\alpha\in x\}\in F_{\kappa,\gamma}$. There is a $x\in\underset{\xi<\gamma}{\Delta} t'(\xi)$ such that $x\in t(\beta)$ and $\beta\in x$. Hence $x\in t'(\beta)$ and $t'(\beta)=t(\beta)$.

(4) $U$ is an ultrafilter.

Obviously $\phi\notin U$. So we have to show only that if $A\notin U$, then $[\lambda]^\kappa$-$A\in U$. Suppose that $A_\alpha\notin U$. $t'(\alpha)=[\gamma]^\kappa-t(\alpha)$. Let $[\lambda]^\kappa-A_\alpha=A_\beta$. There is a $x\in\underset{\xi<\gamma}{\Delta} t'(\xi)$ such that $\{\alpha,\beta\}\subset x$. Since $x\in t'(\alpha)=[\gamma]^\kappa-t(\alpha)$, $x\cap\lambda\notin A_\alpha$. Hence $x\cap\lambda\in A_\beta$ and $x\in t(\beta)$. Thus $t'(\beta)=t(\beta)$.

(5) $U$ is normal.

Suppose that $\{B_\alpha:\alpha<\lambda\}\subset U$. Let $f:\lambda\longrightarrow\gamma$ be such that $B_\alpha=A_{f(\alpha)}$ for all $\alpha<\lambda$, and $\underset{\alpha<\lambda}{\Delta} B_\alpha = A_\beta$.

Note that $X=\{x\in P_\kappa\gamma : \forall\alpha\in x\cap\lambda(f(\alpha)\in x)\}$ is a closed unbounded subset of $P_\kappa\gamma$. Let $C=A_X=\{y\in[\gamma]^\kappa : \exists D\subset X(D$ is a $\kappa$-chain, $y=\bigcup D)\}$. Then $C\in F_{\kappa,\gamma}$.

If $y\in C$ and $\alpha\in y\cap\lambda$, there is an $x\in D$ such that $\alpha\in x\cap\lambda$ and $x\subset y$. Hence $f(\alpha)\in x\subset y$. Now we have got that for any $y\in C$, if $\alpha\in y\cap\lambda$ then $f(\alpha)\in y$.

There is a $y\in\underset{\xi<\gamma}{\Delta} t'(\xi)$ such that $y\in C$ and $\beta\in y$. For all $\alpha\in y\cap\lambda$, $f(\alpha)\in y$ and $y\in t'(f(\alpha))=t(f(\alpha))$, hence $y\cap\lambda\in A_{f(\alpha)}$.

If $t'(\beta)=[\gamma]^\kappa-t(\beta)$, $y\cap\lambda\notin A_\beta=\underset{\alpha<\lambda}{\Delta} B_\alpha$. So, there is an $\alpha\in y\cap\lambda$ such that $y\cap\lambda\notin B_\alpha=A_{f(\alpha)}$. Contradiction. Hence $t'(\beta)=t(\beta)$.

(iii) Let $U$ be the normal ultrafilter on $[\lambda]^\kappa$ induced by $j$, $t:\lambda\longrightarrow P([\lambda]^\kappa)$. Define $t':\lambda\longrightarrow P([\lambda]^\kappa)$ as follows. $t'(\alpha)=t(\alpha)$ if $t(\alpha)\in U$, and $t'(\alpha)=[\lambda]^\kappa-t(\alpha)$ if $t(\alpha)\notin U$. Then $t'\sim t$ and for all $\alpha<\lambda$, $t'(\alpha)\in U$. Hence $\underset{\alpha<\lambda}{\Delta} t'(\alpha)\in U$. Every member of $U$ is $F_{\kappa,\lambda}$-stationary.

Next the author tried to express the partition property of $P_\kappa\lambda$ in the form of a flipping propertie. (Though it does not seem successful.)

PROPPOSITION 2. The followings are equivalent.

(a) Part*$(\kappa, \lambda)$.

(b) For any $t:P_\kappa\lambda\longrightarrow P(P_\kappa\lambda)$, there are $t'\sim t$ and a stationary set $X$ such that if $\{x, y\}\in D_\kappa\lambda\cap[X]^2$ then $y\in t'(x)$.

PROOF. $(a)\longrightarrow(b)$. Define $F:D_\kappa\lambda\longrightarrow 2$ by $F(x, y)=0$ if $y\in t(x)$ and $F(x, y)=1$ otherwise. Let $X$ be a stationary homogeneous set for $F$. When $F''([X]^2\cap D_\kappa\lambda)=\{0\}$, $t'=t$. If $F''([X]^2\cap D_\kappa\lambda)=\{1\}$, let $t'(x)=P_\kappa\lambda\text{-}t(x)$ for all $x\in X$.

(b)$\longrightarrow$(a). Put $t(x)=\{y : F(x, y)=0\}$. There are $t'\sim t$ and a stationary set $X$ such that if $x\subsetneqq y\in X$ then $x\in t'(y)$.

Let $X_1=\{x\in X : t'(x)=t(x)\}$ and $X_2=\{x\in X : t'(x)=P_\kappa\lambda-t(x)\}$. Either $X_1$ or $X_2$ is stationary and both of them are homogeneous set for $F$.

We add easy obervations at the end of this paper.

DEFINITION. A stationary coding set for $P_\kappa\lambda$ (an "*SC*") consists of a stationary set $A\subset P_\kappa\lambda$ together with a $1:1$ function $c:A\longrightarrow\lambda$ (called the coding function) satisfying that for each $x$, $y\in A$

$$x\subsetneqq y\longleftrightarrow c(x)\in y.$$

PROPOSITION 3. If Part*$(\kappa, \lambda)$, then an *SC* exists. (This is also seen in Zwicker [14]. The author considered this property without a word an "*SC*".)

PROOF. Let $F(x, y)=0$ if $c(x)\in y$ and $F(x, y)=1$ otherwise, for any $1:1$ function $c: P_\kappa\lambda \longrightarrow \lambda$.

DEFINITION. $X \subset P_\kappa\lambda$ is prestationary iff for any choice function on $X$ is constant on some unbounded set $S \subset X$.

This definition makes sense. In fact,

LEMMA 4. (Menas in [10]) There is a prestationary set that is not stationary.

LEMMA 5. If $X$ is prestationary, then $\{x \in X : a \subset x\}$ is also prestationary for all $a \in P_\kappa\lambda$.

DEFINITION. wPart*($\kappa$, $\lambda$) iff any partition of $P_\kappa\lambda$ has a prestationary homogeous set.

THEOREM 6. If wPart* ($\kappa$, $\lambda$), then $\kappa$ is almost $\lambda$-ineffable.

PROOP. Magidor's proof of Theorem 2 in [9] can be carried out. What we really need is a homogeneous set $H$ such that for any choice function $f$ there is an unbounded subset $T$ of $H$ so that

$$\forall x \in T \exists y \in T (x \not\subseteq y \text{ and } f(x) \geqslant f(y)).$$

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