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LERAY-VOLEVICH CONDITIONS FOR SYSTEMS OF ABSTRACT EVOLUTION EQUATIONS OF NIRENBERG/NISHIDA TYPE

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Let us consider the following Cauchy problem for a linear differential equation of first order with analytic coefficients and data in a neighbourhood of the origin:

$$\partial_t u(t, x)-a_1(t, x, \partial_x)u(t, x)=f(t, x),\ u(0, x)=u_0(x).$$

The Cauchy-Kovalevsky theorem asserts that if the order of a_1 with respect to ∂_x fulfils ord $a_1\leqq 1$, then there exists a unique analytic solution $u=u(t, x)$ in a neighbourhood of the origin. In [5] it is shown that ord $a_1\leqq 1$ is necessary, too. The situation is quite different for linear systems with analytic coefficients and data, for example, for

$$\partial_t u_1(t, x)-a_{1,1}(t, x, \partial_x)u_1(t, x)-a_{1,2}(t, x, \partial_x)u_2(t, x)=f_1(t, x),$$

$$\partial_t u_2(t, x)-a_{2,1}(t, x, \partial_x)u_1(t, x)-a_{2,2}(t, x, \partial_x)u_2(t, x)=f_2(t, x),$$

$$u_1(0, x)=u_1(x),\ u_2(0, x)=u_2(x).$$

Local well-posedness is valid under more general conditions, the so-called Leray-Volevich conditions [4, 13]

$$\text{ord}\, a_{i,j}\leqq q_i-q_j+1, \qquad i, j\in\{1, 2\}, \tag{1}$$

where ord $a_{i,j}$ denotes the order of $a_{i,j}(t, x, \partial_x)$ with respect to ∂_x, q_j are arbitrary natural numbers ($a_{i,j}(t, x, \partial_x)\equiv 0$ if $q_i-q_j+1<0$). Setting $q_1=q_2$ then we obtain the Cauchy-Kovalevsky conditions. The conditions (1) are in general not necessary for local well-posedness [1, 4]. But under the condition (1) the system can always be reduced to a first order system [7]. In the case dim $x=1$ a necessary and sufficient condition for the local (uniformly at every point) well-posedness of the Cauchy problem is that the system can be reduced to a one satisfying the Leray-Volevich conditions (1) within meromorphic functions [6].

Besides the nonlinear classical Cauchy-Kovalevsky theorem abstract nonlinear

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versions were given in [8, 9, 10]. These abstract versions imply an existence- and uniqueness result for $d_t u = F(u, t)$, $u(0)=0$, where the nonlinear operator F satisfying suitable assumptions in a scale of Banach spaces is a singular operator of first order (see [11]). These assumptions are of Cauchy-Kovalevsky type.

The goal of the present paper is to show Leray-Volevich conditions which guarantee conical evolution of the solutions (see [1, 12, 14]) can be formulated for the following Cauchy problem for a system of nonlinear abstract evolution equations of Nirenberg/Nishida type:

$$d_t u_i = F_i(u_1, \cdots, u_n, t),\ u_i(0)=0, \tag{2}$$

$i=1, \cdots, n$. Let us assume

$$\{B_s, |\vec{u}|_s\}_{0<s<s_0} = \left\{(\times H_s)^n,\ |\vec{u}|_s^2 = \sum_{i=1}^{n} \|u_i\|_s^2\right\}_{0<s<s_0},\ \{H_s, \|\cdot\|_s\}_{0<s<s_0}$$

is a scale of Banach spaces, this means, $H_s \subset H_{s'}$, $\|\cdot\|_{s'} \leqq \|\cdot\|_s$ for $0<s' \leqq s<s_0$, $s_0 \leqq 1$; η, q_i, $C_{i,j}$, R and K are positive constants, $p_{i,j}$ are nonnegative constants for i, $j=1, \cdots, n$):

For any $0<s'<s<s_0<\eta$ the mappings

$F_i:(u_1, \cdots, u_n, t) \longrightarrow F_i(u_1, \cdots, u_n, t)$ are continuous of

$$\{\vec{u} \in B_s : \|u_i\|_s < R\} \times [0, \eta) \text{ into } B_{s'}. \tag{3}$$

For any $0<s'<s<s_0$, all $\vec{u}=(u_1, \cdots, u_n)$, $\vec{v}=(v_1, \cdots, v_n) \in B_s$ with $\|u_i\|_s<R$, $\|v_i\|_s<R$ and for any $t \in [0, \eta)$ the mappings F_i are nonlinear singular operators of order $p_{i,j}$ with respect to u_j in $\{\vec{u} \in B_s : \|u_i\|_s < R\}$, that is,

$$\|F_i(u_1, \cdots, u_n, t) - F_i(v_1, \cdots, v_n, t)\|_{s'} \leqq \sum_{j=1}^{n} C_{i,j} \frac{\|u_j - v_j\|_s}{(s-s')^{p_{i,j}}}. \tag{4}$$

Choosing $u_j=0$ the mappings $F_i(0, \cdots, 0, t)$ satisfy for all $0<s<s_0$

$$\|F_i(0, \cdots, 0, t)\|_s \leqq K/(s_0-s)^{q_i}. \tag{5}$$

The numbers $p_{i,j}$ and q_i fulfil the Leray-Volevich conditions

$$p_{i,j} \leqq q_i - q_j + 1. \tag{6}$$

THEOREM 1. *Under these assumptions there exists a positive constant b and there exists in a subscale $\{B_s, |\cdot|_s\}_{0<s<s_\infty}$ a uniquely determined solution $\vec{u} \in C^1([0, b(s_\infty - s)), B_s)_{0<s<s_\infty}$ of* (2) *possessing conical evolution, this means,*

$$\sup_{0<s<s_\infty,\ t\in[0, b(s_\infty - s))} \|u_i(t)\|_s (b(s_\infty - s) - t)^{q_i} < \infty$$

for all $i=1, \cdots, n$. Moreover, $\|u_i(t)\|_s < R$ for all admissible t.

REMARK 1. In the special case $q_i=p_{i,j}=1$ which is studied in [8, 9, 10] the Leray-Volevich conditions are obviously satisfied.

For the proof of Theorem 1 we need the next lemma.

LEMMA. *For a fixed positive constant $h\leqq 1$ we have for all $t\in[0, h)$*

$$\int_0^t (h-\tau)^{-(q_j+p_{i,j})}d\tau \leqq D(h-t)^{-q_i}, \tag{7}$$

where D depends only on q_j and $p_{i,j}$.

PROOF. i) $0<q_j+p_{i,j}<1$,

$$\int_0^t (h-\tau)^{-(q_j+p_{i,j})}d\tau \leqq (1-(q_j+p_{i,j}))^{-1} \leqq (1-(q_j+p_{i,j}))^{-1}(h-t)^{-q_i},$$

ii) $q_j+p_{i,j}=1$, using $q_i>0$ then

$$\int_0^t (h-\tau)^{-1}d\tau \leqq \ln(h-t)^{-1} \leqq C(q_i)(h-t)^{q_i},$$

iii) $q_j+p_{i,j}>1$, using $q_j+p_{i,j}-1\leqq q_i$, $h\leqq 1$ then

$$\int_0^t (h-\tau)^{-(q_j+p_{i,j})}d\tau \leqq (q_j+p_{i,j}-1)^{-1}(h-t)^{-(q_j+p_{i,j}-1)}$$
$$\leqq (q_j+p_{i,j}-1)^{-1}(h-t)^{-q_i}.$$

PROOF OF THEOREM 1. Let us consider the problem (2). The change of variables $t=t'b$, $0<b\leqq 1$ is determined later, transforms (2) in

$$d_{t'}u_i=bF_i(u_1, \cdots, u_n, t'b)=G_i(u_1, \cdots, u_n, t'),\ u_i(0)=0, \tag{8}$$

$\eta'=\eta/b$, the relation (4) in

$$\|G_i(u_1, \cdots, u_n, t')-G_i(v_1, \cdots, v_n, t')\|_{s'} \leqq \sum_{j=1}^n C_{i,j}b\frac{\|u_j-v_j\|_s}{(s-s')^{p_{i,j}}}, \tag{9}$$

and, finally, the relation (5) in

$$\|G_i(0, \cdots, 0, t')\|_s \leqq bK/(s_0-s)^{q_i}. \tag{10}$$

As in [8, 9] the solution $\vec{u}$ of (8) will be obtained as the limit of a sequence $\vec{u}_k=(u_{1,k}, \cdots, u_{n,k})$ defined recursively by $u_{i,0}=0$, $u_{i,k+1}=u_{i,k}+v_{i,k}$, where $\|u_{i,k}(t')\|_s\leqq R/2$ for $t'\in[0, s_k-s)$, $s_k<\eta$, $s_{k+1}=s_k(1-(k+2)^{-2})$, and $u_{i,k}$ is defined by

$$u_{i,k}(t')=\int_0^{t'} G_i(u_{1,k-1}(\tau), \cdots, u_{n,k-1}(\tau), \tau)d\tau,\ k=0, 1, 2, \cdots.$$

Now let us introduce the functionals

$$M_k[v_i]=\sup_{0<s<s_k,\, t'\in[0,s_k-s)}\|v_{i,k}(t')\|_s(s_k-s-t')^{q_i}.$$

Using (10) we get for $M_0[v_i]$

$$M_0[v_i]=\sup_{0<s<s_0,\, t'\in[0,s_0-s)}\|v_{i,0}\|_s(s_0-s-t')^{q_i}$$

$$=\sup_{0<s<s_0,\, t'\in[0,s_0-s)}\left\|\int_0^{t'}G_i(0,\cdots 0,\tau)d\tau\right\|_s(s_0-s-t')^{q_i}$$

$$\leqq\sup_{0<s<s_0,\, t'\in[0,s_0-s)}\frac{bKs_0}{(s_0-s)^{q_i}}(s_0-s-t')^{q_i}\leqq bKs_0.$$

Now we suppose that $\|u_{i,k}(t')\|_s<R$ for $t'\in[0,s_k-s)$.

Using (3) from the iteration process $u_{i,k+1}(t')$, respectively, $v_{i,k}(t')$ are well-defined for $t'\in[0,s_k-s)$. Consequently, we can estimate by (9)

$$\|v_{i,k}(t')\|_s=\left\|\int_0^{t'}(G_i(u_{1,k},\cdots,u_{n,k},\tau)-G_i(u_{1,k-1},\cdots,u_{n,k-1},\tau))d\tau\right\|_s$$

$$\leqq\int_0^{t'}\sum_{j=1}^n C_{i,j}b\frac{\|v_{j,k-1}(\tau)\|_{s_k(\tau)}}{(s_k(\tau)-s)^{p_{i,j}}}d\tau,$$

where $s_k(\tau)=(s_k+s-\tau)/2$ for $\tau\in[0,s_k-s)$. Obviously, $s<s_k(\tau)<s_k$ for all τ and $0<s<s_k$. From the definition of $M_{k-1}[v_j]$ we obtain ($s_k<s_{k-1}$, $s_k(\tau)+\tau<s_k$ for all $\tau\in[0,s_k-s)$)

$$\|v_{j,k-1}(\tau)\|_{s_k(\tau)}\leqq\frac{M_{k-1}[v_j]}{(s_{k-1}-s_k(\tau)-\tau)^{q_j}}\leqq\frac{M_{k-1}[v_j]}{(s_k-s_k(\tau)-\tau)^{q_j}}.$$

These estimates lead to

$$\|v_{i,k}(t')\|_s\leqq\sum_{j=1}^n M_{k-1}[v_j]C_{i,j}b2^{q_j+p_{i,j}}\int_0^{t'}(s_k-s-\tau)^{-(q_j+p_{i,j})}d\tau.$$

Taking into consideration the statement of the lemma gives

$$\|v_{i,k}(t')\|_s\leqq D(s_k-s-t')^{-q_i}\sum_{j=1}^n C_{i,j}b2^{q_j+p_{i,j}}M_{k-1}[v_j],$$

equivalently, $M_k[v_i]\leqq\sum_{j=1}^n C_{i,j}Db2^{q_j+p_{i,j}}M_{k-1}[v_j]$. Choosing now the positive constant $b\leqq 1$ such that $\sum_{j=1}^n C_{i,j}Db2^{q_j+p_{i,j}}=\lambda<1$ we arrive step by step at $M_1[v_i]\leqq\lambda bKs_0$, $M_2[v_i]\leqq\lambda^2 bKs_0$, $\cdots$, $M_k[v_i]\leqq\lambda^k bKs_0$.

From the definition of the functionals $M_k[v_i]$ we obtain

$$\|v_{i,k}(t')\|_s\leqq\frac{M_k[v_i]}{(s_k-s_{k+1})^{q_i}}=\frac{M_k[v_i](k+2)^{2q_i}}{s_k^{q_i}}\leqq\frac{\lambda^k bKs_0(k+2)^{2q_i}}{s_\infty^{q_i}},$$

respectively,

$$\|u_{i,k+1}(t')\|_s\leqq\sum_{l=0}^k\|v_{i,l}(t')\|_s\leqq\frac{bKs_0}{s_\infty^{q_i}}\sum_{l=0}^k(l+2)^{2q_i}\lambda^l$$

for all k and all $t'\in[0,s_\infty-s)$, where s_∞ is the limit of the sequence $\{s_k\}$. Now a suitable choice of b yields $s_\infty^{-q_i}bKs_0\sum_{l=0}^{\infty}(l+2)^{2q_i}\lambda^l \leqq R/2$ for all $i=1, \cdots, n$. From this follows $\|u_{i,k+1}(t')\|_s \leqq R/2$ for all $k \in N_0$, $t' \in [0, s_\infty - s)$ and $0<s<s_\infty$.

The sequence $\{u_{i,k}(t')\}$ converges to a function $u_i(t')$ in H_s for all $t' \in [0, s_\infty - s)$. The vector $\vec{u}(t')=(u_1(t'), \cdots, u_n(t')) \in B_s$ belonging to $C^1([0, s_\infty - s), B_s)_{0<s<s_\infty}$, $\|u_i(t')\|_s < R$, represents a solution of (8). Hence, $\vec{u}(t)$ belongs to $C^1([0, b(s_\infty - s)), B_s)_{0<s<s_\infty}$ and is a solution of (2). Moreover we obtain for all $t' \in [0, s_\infty - s)$ from

$$\|v_{i,k}(t')\|_s (s_\infty - s - t')^{q_i} \leqq \|v_{i,k}(t')\|_s (s_k - s - t')^{q_i} \leqq \lambda^k bKs_0,$$

immediately,

$$\|u_i(t')\|_s (s_\infty - s - t')^{q_i} \leqq bKs_0 \sum_{l=0}^{\infty} \lambda^l < \infty .$$

But this implies

$$\sup_{0<s<s_\infty,\, t\in[0, b(s_\infty - s))} \|u_i(t)\|_s (b(s_\infty - s) - t)^{q_i} < \infty ,$$

consequently, the solution possesses conical evolution.

Let us now suppose the existence of two different solutions

$$\vec{u},\ \vec{v} \in C^1([0, b(s_\infty - s)), B_s)_{0<s<s_\infty},\ \|u_i(t)\|_s,\ \|v_i(t)\|_s < R ,$$

possessing conical evolution, that is,

$$\sup_{0<s<s_\infty,\, t\in[0, b(s_\infty - s))} \|u_i(t)\|_s (b(s_\infty - s) - t)^{q_i} < \infty ,$$

$$\sup_{0<s<s_\infty,\, t\in[0, b(s_\infty - s))} \|v_i(t)\|_s (b(s_\infty - s) - t)^{q_i} < \infty .$$

After the transformation $t=t'b$ we conclude

$$M = \sup_{0<s<s_\infty,\, t'\in[0, s_\infty - s),\, i=1,\cdots,n} \|u_i(t') - v_i(t')\|_s (s_\infty - s - t')^{q_i} < \infty .$$

Applying this relation to

$$u_i(t') - v_i(t') = \int_0^{t'} (G_i(u_1, \cdots, u_n, \tau) - G_i(v_1, \cdots, v_n, \tau)) d\tau$$

implies

$$\|(u_i - v_i)(t')\|_s \leqq \int_0^{t'} \sum_{j=1}^{n} C_{i,j} b \frac{M}{(s_\infty - s(\tau) - \tau)^{q_j} (s(\tau) - s)^{p_{i,j}}} d\tau .$$

Choosing $s(\tau) = (s_\infty + s - \tau)/2$ gives

$$\|(u_i - v_i)(t')\|_s \leqq \int_0^{t'} \sum_{j=1}^{n} C_{i,j} b \frac{M 2^{q_j + p_{i,j}}}{(s_\infty - s - \tau)^{q_j + p_{i,j}}} d\tau ,$$

and, finally, $M \leqq \lambda M$. Thus, $M=0$, equivalently, $u_i(t') = v_i(t')$ in H_s for $t' \in [0, s_\infty - s)$. From this follows $\vec{u} = \vec{v}$ in contradiction to the assumption.

Besides the conical evolution of solutions in [1] the notion cylindrical evolution of solutions of special systems of partial differential equations is introduced. The next theorem expresses the possibility to transfer this type of evolution to solutions of systems of Nirenberg/Nishida type (2).

THEOREM 2. *Instead of* (6) *let us suppose the condition*

$$p_{i,j}\leqq q_i-q_j. \tag{11}$$

Under this assumption and the assumptions of Theorem 1 *there exists in a subscale* $\{B_s, |\cdot|_s\}_{0<s<s_\infty}$ *a uniquely determined solution* $\vec{u}\in C_1([0, T], B_s)_{0<s<s_\infty}$ *of* (2) *possessing cylindrical evolution, this means,*

$$\sup_{0<s<s_\infty,\, t\in[0,T]}\|u_i(t)\|_s(s_\infty-s)^{q_i}<\infty \qquad \textit{for all} \quad i=1, \cdots, n,$$

where $T<\eta$ *is a certain positive constant independent on* $0<s<s_\infty$. *Moreover, it holds* $\|u_i(t)\|_s<R$ *for all* $t\in[0, T]$.

SKETCH OF THE PROOF. Let us define the functionals

$$M_k[v_i]=\sup_{0<s<s_k,\, t\in[0,T]}\|v_{i,k}(t)\|_s(s_k-s)^{q_i}, \qquad \text{where } T<\eta$$

is determined later. Using

$$M_0[v_i]=\sup_{0<s<s_0,\, t\in[0,T]}\|v_{i,0}(t)\|_s(s_0-s)^{q_i}$$
$$\leqq \sup_{0<s<s_0,\, t\in[0,T]}\frac{KT}{(s_0-s)^{q_i}}(s_0-s)^{q_i}=KT$$

and applying (4) to the formulas

$$v_{i,k}(t)=\int_0^t(F_i(u_{1,k}, \cdots, u_{n,k}, \tau)-F_i(u_{1,k-1}, \cdots, u_{n,k-1}, \tau))d\tau$$

one obtains

$$\|v_{i,k}(t)\|_s\leqq\int_0^t\sum_{j=1}^n C_{i,j}\frac{M_{k-1}[v_j]}{(s_k-\tilde{s})^{q_j}(\tilde{s}-s)^{p_{i,j}}}d\tau.$$

Setting $\tilde{s}=s+(s_k-s)/2$ yields with (11)

$$\|v_{i,k}(t)\|_s(s_k-s)^{q_i}\leqq\sum_{j=1}^n C_{i,j}2^{q_j+p_{i,j}}TM_{k-1}[v_j]$$

and, finally, $M_k[v_i]\leqq\sum_{j=1}^n C_{i,j}2^{q_j+p_{i,j}}T\ M_{k-1}[v_j]$. A suitable choice of T implies $\sum_{j=1}^n C_{i,j}2^{q_j+p_{i,j}}T=\lambda<1$. The same reasoning as in the proof of Theorem 1 gives $M_k[v_i]\leqq\lambda^k KT$,

$$\|u_{i,k+1}(t)\|_s\leqq\sum_{l=0}^k\|v_{i,l}(t)\|_s\leqq\frac{KT}{s_\infty^{q_i}}\sum_{l=0}^\infty(l+2)^{2q_i}\lambda^l\leqq R/2$$

for all $i=1, \cdots, n$ and

$$\sup_{0<s<s_\infty, t\in[0,T]} \|u_i(t)\|_s(s_\infty-s)^{q_i} \leqq \sum_{l=0}^{\infty} \sup_{0<s<s_l, t\in[0,T]} \|v_{i,l}(t)\|_s(s_l-s)^{q_i}$$

$$\leqq \sum_{l=0}^{\infty} M_l[v_i] \leqq KT \sum_{l=0}^{\infty} \lambda^l .$$

Thus, all statements concerning the existence of the solution are proved. The uniqueness follows from Theorem 1.

REMARK 2. One should refer to the correspondence of the type of evolution with the weights are used in the definition of the functionals. In the case of conical evolution the interval of existence with respect to t depends on the parameter s, $0<s<s_\infty$. The set $\{(s, t)\in R^2: (s, t)\in((0, s_\infty)\times(-(s_\infty-s), s_\infty-s))\}$ forms a conical set in R^2. Contrary to this case in the case of cylindrical evolution the interval of existence with respect to t does not depend on the parameters s. The set $\{(s, t)\in R^2: (s, t)\in((0, s_\infty)\times[0, T])\}$ forms a cylindrical set in R^2.

At the end of this paper we deal with some examples for the case of conical evolution. The statements can be easy transfered to the case of cylindrical evolution.

EXAMPLE 1. Let G be a bounded domain in R^n. Then we define as in [2]

$$\{H_s, \|u\|_s\}_{s>0} = \left\{u\in C^\infty(G): \sup_{G, |\alpha|\in N_0} |\partial_x^\alpha u| \prod_{j=1}^{n} \frac{(sr_j)^{\alpha_j r_j}}{\Gamma_{r_j}(\alpha_j)} = \|u\|_s\right\}_{s>0},$$

where $\gamma_i \geqq 1$ and $\Gamma_{\gamma_j}(\alpha_j)=\lambda_0 \alpha_j!^{\gamma_j}/\alpha_j^{\gamma_j+2}$ if $\alpha_j>0$, $\Gamma_{\gamma_j}(0)=\lambda_0$. With a suitable choice of λ_0 the spaces H_s become Banach algebras. One can show, that the differential operators ∂_x^β are singular operators of order $\langle\beta, \gamma\rangle=\sum_{j=1}^n \beta_j\gamma_j$ in the scale $\{H_s, \|u\|_s\}_{0<s<s_0}$. Thus, is the case of quasilinear systems of the form

$$\partial_t u_i = \sum_{k=1}^{n} F_{i,k}(t, x, u_1, \cdots, u_n)\partial_x^{\beta_{i,k}} u_k$$

we obtain the Leray-Volevich conditions $\langle\beta_{i,k}, \gamma\rangle \leqq q_i - q_k + 1$ were derived for the nonlinear case in [2].

EXAMPLE 2. Let $\{G_s\}_{0<s<s_0}=\{z: |z|<s\}_{0<s<s_0}$ be a family of domains generating the scale of holomorphic functions

$$\{H_s, \|u\|_s\}_{0<s<s_0} = \{H(G_s)\cap C(\bar{G}_s), \sup|u| = \|u\|_s\}_{0<s<s_0},$$

where $H(G_s)$ denotes the space of holomorphic functions in G_s. The differential operators ∂_z^j are singular operators of order j in the scale $\{H_s, \|u\|_s\}_{0<s<s_0}$. Thus, for quasilinear systems of the form

$$\partial_t u_i = \sum_{k=1}^{n} F_{i,k}(t, z, u_1, \cdots, u_n)\partial_z^{j_{i,k}} u_k$$

we obtain the Leray-Volevich conditions $j_{i,k} \leqq q_i - q_k + 1$ were derived for the linear case of higher order in [1].

The solution $\vec{u} = (u_1, \cdots, u_n)$ satisfies the conditions

$$\sup_{G_s,\, 0<s<s_\infty,\, t\in[0, b(s_\infty - s))} |u_j(t, z)|(b(s_\infty - s) - t)^{q_i}$$
$$= \sup_{G_{s_\infty},\, t\in[0, b(s_\infty - |z|))} |u_j(t, z)|(b(s_\infty - |z|) - t)^{q_i}.$$

The set $\{(t, z): z \in G_{s_\infty}$ and $t \in [0, b(s_\infty - |z|))\}$ forms a conical set with the base G_{s_∞}(see [12, 14]). This motivates the notion conical evolution. As in [12] one can also define scales of generalized analytic functions in the sense of I. N. Vekua and study linear systems of Leray-Volevich type in such scales.

EXAMPLE 3. As in [1] we consider scales of Banach spaces of entire functions of exponential type

$$\{H_s, \|u\|_s\}_{0<s<s_0\leq 1}$$
$$= \left\{u \in H(C^n): \sup_{C^n} |u(z)| \exp\left(-\left(\sum_{j=1}^{n} (2-s) r_j (1+|z_j|^{q_j})\right)\right) = \|u\|_s\right\}_{,0<s<s_0\leq 1},$$

where $q_j \geqq 1$ and r_j are positive constants. The multiplication operator $z^\alpha \cdot$, respectively, the differential operator ∂_z^β are singular operators of order $\sum_{i=1}^{n} \alpha_i / q_i$, respectively, of order $\sum_{i=1}^{n} \beta_i (q_i - 1)/q_i$ in the scale $\{H_s, \|u\|_s\}_{0<s<s_0}$. After introducing the functionals

$$M_k[v_i] = \sup_{0<s<s_k,\, t'\in[0, s_k - s)} \|v_{i,k}(t', z)\|_s (s_k - s - t')^{\sum_{k=1}^{n} m_{j,k}/q_k}$$

the solution $\vec{u} = (u_1, \cdots, u_n)$ of the linear system

$$\partial_t u_i = \sum_{j=1}^{n} b_{i,j}(t) \left(\prod_{k=1}^{n} z_k^{a_{i,j,k}} \partial_z^{b_{i,j,k}}\right) u_j$$

satisfies

$$\sup_{0<s<s_\infty,\, t\in[0, b(s_\infty - s))} \|u_j(t, z)\|_s (b(s_\infty - s) - t)^{\sum_{k=1}^{n} m_{j,k}/q_k} < \infty.$$

The Leary-Volevich conditions are representable in the form

$$\sum_{k=1}^{n} (a_{i,j,k} + b_{i,j,k}(q_k - 1))/q_k \leqq 1 + \sum_{k=1}^{n} (m_{i,k} - m_{j,k})/q_k.$$

One can prove that all entire functions $u_j(z)$ satisfying

$$|u_j(z)| \leqq M_j \prod_{k=1}^{n} (1+|z_k|)^{m_{j,k}} \exp((2-s) r_k (1+|z_k|)^{q_k}$$

for all $0<s<1$ fulfil

$$\|u_j(z)\|_{s'} \leqq M_j \prod_{k=1}^{n} (1+|z_k|)^{m_{j,k}} \exp(-(s-s')r_k(1+|z_k|)^{q_k})$$

$$\leqq M_j C/(s-s')^{\sum_{k=1}^{n} m_{j,k}/q_k} \qquad \text{for all} \quad 0<s'<s<1.$$

Hence we obtain the same growth conditions as in [1]. The Leray-Volevich conditions correspond to the conditions from [1].

It seems to be interesting that we can conclude for all examples the special Leray-Volevich conditions from the same abstract result (Theorem 1). Hence, the Theorem 1 in the case of conical evolution and Theorem 2 in the case of cylindrical evolution lead to new qualitative results for systems of abstract evolution equations.

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