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ON REPRESENTATIONS OF THE BIMODULE *DA*

By

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Abstract. Let A be a finite-dimensional algebra over an algebraically closed field k. A representation of the A-A bimodule $DA=\mathrm{Hom}_k(A, k)$ is a module over the matrix algebra:

$$\bar{A}=\begin{bmatrix} A & 0 \\ DA & A \end{bmatrix}$$

We first prove that $\bar{A}$ is representation-finite (and in fact simply connected) whenever A is an iterated tilted algebra of Dynbin type. We then assume that A is a tilted algebra of Dynkin type, and characterise $\bar{A}$ by its Auslander-Reiten quiver.

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Introduction

Let A be a basic, connected, finite-dimensional algebra over an algebraically closed field k, and $T(A)=A\ltimes DA$ be its trivial extension by its minimal injective cogenerator $DA=\mathrm{Hom}_k(A, k)$. It was proved by Hughes and Waschbüsch in [13] (see also [12], [9]) that if A is a tilted algebra of Dynkin type Δ, then $T(A)$ is representation-finite of Cartan class Δ, and conversely, if $T(A)$ is representation-finite of Cartan class Δ, there exists a tilted algebra B of Dynkin type Δ such that $T(B)\simeq T(A)$. It was then shown in [2] that $T(A)$ is representation-finite of Cartan class Δ if and only if A is an iterated tilted algebra of Dynkin type Δ. Moreover, the construction in [13] suggested that the representations of $T(A)$ were related to the representations of the A-A bimodule DA, or, what amounts to the same, the modules over the matrix algebra:

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$$\bar{A}=\begin{bmatrix} A & 0 \\ DA & A \end{bmatrix}.$$

The aim of this paper is to study the representations of the matrix algebra $\bar{A}$ in the case where A is (iterated) tilted of Dynkin type. We shall first prove that $\bar{A}$ is representation-finite (and even simply connected) whenever A is an iterated tilted algebra of Dynkin type. If A is in fact tilted of Dynkin type Δ, we shall describe a functor F: mod $\bar{A} \to$ mod $T(A)$ which is surjective on the indecomposables, and whose restriction on a full subcategory of mod $\bar{A}$ preserves the Auslander-Reiten sequences and the irreducible maps, thus providing us with a simple combinatorial description of the Auslander-Reiten quiver $\Gamma_{\bar{A}}$ of $\bar{A}$: let $\mathcal{S}$ be an (arbitrary) complete slice in the Auslander-Reiten quiver Γ_A of A, then $\mathcal{S}$ generates a configuration $(\boldsymbol{Z}\Delta)_C$ of $\boldsymbol{Z}\Delta$ [9], which is stable under the action of τ^{-m_Δ} (here, τ denotes the translation of $(\boldsymbol{Z}\Delta)_C$, and m_Δ denotes the Coxeter number of Δ minus one, thus $m_{A_n}=n$, $m_{D_n}=2n-3$, $m_{E_6}=11$, $m_{E_7}=17$ and $m_{E_8}=29$) and in which $\mathcal{S}$ embeds fully; let now $[\mathcal{S}, \tau^{-m_\Delta}\mathcal{S}]$ denote the full connected subquiver of $(\boldsymbol{Z}\Delta)_C$ consisting of all the vertices lying between $\mathcal{S}$ and $\tau^{-m_\Delta}\mathcal{S}$, then $\Gamma_{\bar{A}}$ is constructed by glueing the full connected subquiver of Γ_A consisting of the predecessors (respectively, successors) of $\mathcal{S}$ to the left (respectively, to the right of $[\mathcal{S}, \tau^{-m_\Delta}\mathcal{S}]$.

The above description yields a characterisation of $\bar{A}$ in terms of its Auslander-Reiten quiver. Recall first that a slice [11] in a simply connected translation quiver is a full convex subquiver $\mathcal{S}$ such that, if x is a predecessor of $\mathcal{S}$, then $\mathcal{S}$ contains precisely one vertex from the τ-orbit of x. We may now state:

THEOREM *Let B a basic, connected, finite-dimensional k-algebra. There exists a tilted algebra A of Dynkin type Δ such that $B \simeq \bar{A}$ if and only if Γ_B is simply connected and contains a slice $\mathcal{S}$ of underlying graph Δ such that:*

(1) *All projective B-modules which are not injective are predecessors of $\mathcal{S}$.*

(2) *All projective-injective B-modules lie between $\mathcal{S}$ and $\tau^{-m_\Delta}\mathcal{S}$.*

(3) *All injective B-modules which are not projective are successors of $\tau^{-m_\Delta}\mathcal{S}$.*

Throughout this paper, k will denote a fixed algebraically closed field. We shall freely use properties of the Auslander-Reiten sequences and the Auslander-Reiten quiver such as can be found in [4] and [10]. For tilted algebras and their properties, we refer to [7] and [11]. We shall use essentially the results of [13].

1. Definitions and preliminary results:

1.1. Let A be a finite-dimensional k-algebra. Recall that a (finite-dimensional) representation of the A-A-bimodule $DA=\mathrm{Hom}_k(A, k)$ is a triple (U_A, V_A, ϕ), where U_A and V_A are right (finite-dimensional) modules, and ϕ is an A-linear map from $U_A \otimes_A DA_A$ to V_A. A morphism of representations $f: (U, V, \phi) \to (U', V', \phi')$ consists of a pair of A-linear maps $g: U_A \to U'_A$, $h: V_A \to V'_A$ such that $h\phi=\phi'(g\otimes 1)$. It is well-known that the category of (finite-dimensional) representations of the bimodule DA is equivalent to the category of (finite-dimensional) right modules over the matrix algebra:

$$\bar{A}=\begin{bmatrix} A & 0 \\ DA & A \end{bmatrix}=\left\{\begin{bmatrix} a & 0 \\ q & b \end{bmatrix} \mid a, b\in A, q\in DA\right\}$$

endowed with the ordinary matrix addition, and the multiplication induced by the bimodule structure of DA. Indeed, writing 1 for the identity of A, and setting:

$$e:=\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \quad e':=\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix},$$

any right $\bar{A}$-module M can be written in the form (U, V, ϕ), where $U:=Me'$, $V:=Me$ and ϕ is the multiplication map $\phi: u\otimes q \to uq$ (for $u\in U$ and $q\in DA$). In the sequel, these two categories will always be identified.

Observe that $\bar{A}$ is a QF-3 algebra [18]; indeed, $e'\bar{A}$ and $\bar{A}e$ are, respectively, a right and a left minimal faithful $\bar{A}$-module. Observe also that the trivial extension $T(A)=A\ltimes DA$ is the subalgebra of $\bar{A}$ consisting of all the matrices $\begin{bmatrix} a & 0 \\ q & b \end{bmatrix}$ such that $a=b$. Our main objective will be to study the relations between the categories of finite-dimensional right modules $\mathrm{mod}\,\bar{A}$, $\mathrm{mod}\,A$ and $\mathrm{mod}\,T(A)$.

1.2. We shall, from now on, assume that A is a quotient of a finite-dimensional hereditary algebra, that is to say, that the ordinary quiver Q_A of A has no oriented cycles. We shall denote by $1, 2, \cdots, n$ the vertices of Q_A and by $e_1, e_2 \cdots e_n$ the corresponding primitive orthogonal idempotents, which we assume to be admissibly ordered (that is to say, such that $e_jAe_i\neq 0$ implies $i\leqq j$). We shall let $S(i)$ denote the simple module corresponding to the vertex $i\in(Q_A)_0$, $P(i)$ and $I(i)$ denote respectively its projective cover and injective envelope. In order to distinguish between the two copies of A given respectively by $e\bar{A}e$ and $e'\bar{A}e'$, we shall denote the first one by A, and the second one by A'. Accordingly, Q'_A will denote the quiver of A', i' the vertex of Q'_A corresponding to

$i\in(Q_A)_0$ and e_i' the corresponding idempotent.

The ordinary quiver $Q_{\bar{A}}$ of the algebra $\bar{A}$ may now be constructed as follows. Clearly, Q_A and Q_A' are both full connected subquivers of $Q_{\bar{A}}$, and every vertex of $Q_{\bar{A}}$ is a vertex of either Q_A or Q_A'. Also, there is an arrow $i'\to j$ whenever $\operatorname{rad}(e_i'\bar{A}e_j)/\operatorname{rad}^2(e_i'\bar{A}e_j)\neq 0$. Observe that $e_i'\bar{A}e_j=D(e_jAe_i)$ and therefore if $e_jAe_i\neq 0$, there is a non-zero path in $Q_{\bar{A}}$ from i' to j. Also, since $e_i'\bar{A}\simeq D(\bar{A}e_i)$, each $P(i')_{\bar{A}}$ is projective-injective, and its socle is just $S(i)$. On the other hand, every $P(i)_{\bar{A}}$ has its support lying in Q_A, thus is a projective A-module. Dually, $I(i')_{\bar{A}}$ has its support lying completely in Q_A' and is an injective A'-module.

For our purposes, another description of $Q_{\bar{A}}$ will be needed. First, we recall the following developments from [13]: consider the matrix algebra:

$$\hat{A}=\begin{bmatrix} \ddots & & & & 0 \\ \ddots & A_{m-1} & & & \\ & Q_m & A_m & & \\ & & Q_{m+1} & A_{m+1} & \\ 0 & & & \ddots & \ddots \end{bmatrix}$$

where matrices have only finitely many non-zero entries, $A_m=A$ and $Q_m={}_ADA_A$ for all $m\in\boldsymbol{Z}$, all the remaining entries are zero and multiplication is induced from the canonical maps $A\underset{A}{\otimes}DA\simeq DA$, $DA\underset{A}{\otimes}A\simeq DA$ and the zero maps $DA\underset{A}{\otimes}DA\to 0$. Let ν be the automorphism of $\hat{A}$ induced by the identity maps $A_{m+1}\to A_m$, $Q_{m+1}\to Q_m$. Then $\hat{A}|\nu\simeq T(A)$. An $\hat{A}$-module consists of a family $(U_m,\phi_m)_{m\in\boldsymbol{Z}}$ of A-modules U_m and A-linear maps $\phi_m: U_m\otimes DA\to U_{m-1}$ such that, for all $m\in\boldsymbol{Z}$,

$$\phi_{m-1}(\phi_m\otimes 1)=0\,.$$

An $\hat{A}$-linear map $f:(U_m,\phi_m)_{m\in\boldsymbol{Z}}\to(U_m',\phi_m')_{m\in\boldsymbol{Z}}$ consists of a family of A-linear maps $(f_m: U_m\to U_m')_{m\in\boldsymbol{Z}}$ such that, for all $m\in\boldsymbol{Z}$,

$$f_{m-1}\phi_m=\phi_m'(f_m\otimes 1)\,.$$

We shall let, as in [13], $\operatorname{Mod}\hat{A}$ (respectively, $\operatorname{mod}\hat{A}$) denote the category of $\hat{A}$-modules $(U_m,\phi_m)_{m\in\boldsymbol{Z}}$ such that $\dim_k U_m<\infty$ for all $m\in\boldsymbol{Z}$ (respectively, $\dim_k(\bigoplus_{m\in\boldsymbol{Z}}U_m)<\infty$). Then ν induces an automorphism of $\operatorname{Mod}\hat{A}$, and the subcategory $\operatorname{Mod}^{\nu}\hat{A}$ of $\operatorname{Mod}\hat{A}$ consisting of the ν-invariant modules and ν-invariant morphisms is equivalent to $\operatorname{mod} T(A)$ by the functor which maps the $T(A)$-module M on the $\hat{A}$-module $(U_m,\phi_m)_{m\in\boldsymbol{Z}}$ such that $U_m=M$ (considered as an A-module) for all m, and ϕ_m is induced by the action of DA on M [13].

Clearly, $\bar{A}$ is identified to the quotient algebra of $\hat{A}$ defined by the surjection,

$$\hat{A} \longrightarrow \begin{bmatrix} A_0 & 0 \\ Q_1 & A_1 \end{bmatrix}$$

and therefore $Q_{\bar{A}}$ is identified to the full subquiver of $Q_{\hat{A}}$ defined by the vertices: $\{(i, 0) \mid i \in (Q_A)_0\}$ and $\{(i, 1) \mid i \in (Q_A)_0\}$ (in our previous notation, $(i, 0)$ is i and $(i, 1)$ is i').

1.3. Since the trivial extension $T(A)$ is a subalgebra of $\bar{A}$, the inclusion map $T(A) \to \bar{A}$ defines a functor $F: \operatorname{mod} \bar{A} \to \operatorname{mod} T(A)$ (by restriction of the scalars) as follows: for an $\bar{A}$-module (U_A, V_A, ϕ), the $T(A)$-module $M := F(U, V, \phi)$ has the A-module structure of $U_A \oplus V_A$, and the multiplication of $(u, v) \in M$ by $q \in DA$ is given by:

$$(u, v)q = (0, \phi(u \otimes q))$$

Thus, for $(u, v) \in M$ and $\begin{bmatrix} a & 0 \\ q & a \end{bmatrix} \in T(A)$:

$$(u, v)\begin{bmatrix} a & 0 \\ q & a \end{bmatrix} = (ua, va + \phi(u \otimes q))$$

We define in the same way the action of F on the morphisms: if $f = (g, h)$: $(U, V, \phi) \to (U', V', \phi')$ is an $\bar{A}$-linear map, we put $F(f) := g \oplus h : U \oplus V \to U' \oplus V'$ as an A-linear map, the compatibility of this definition with the multiplication by elements of DA follows from the fact that $h\phi = \phi'(g \otimes 1)$.

We shall now give another description of the functor F. Let E be the canonical embedding functor of $\operatorname{mod} \bar{A}$ in $\operatorname{mod} \hat{A}$ (which is obtained by "extending by zeros"): it is full, exact, preserves indecomposable modules and their composition lengths. We also have a functor $\hat{F}: \operatorname{mod} \hat{A} \to \operatorname{mod} T(A)$ (denoted Φ in [13]) which is full, exact, preserves indecomposable modules and their composition lengths and also Auslander-Reiten sequences and irreducible maps: it is the composition of the functor $\operatorname{mod} \hat{A} \to \operatorname{Mod}^{\nu} \hat{A}$ given by $M \to \bigoplus_{m \in \mathbf{Z}} \nu^m M$, $f \to \bigoplus_{m \in \mathbf{Z}} \nu^m f$ (for M, N in $\operatorname{mod} \hat{A}$ and $f \in \operatorname{Hom}_{\hat{A}}(M, N)$) and the equivalence $\operatorname{Mod}^{\nu} \hat{A} \xrightarrow{\sim} \operatorname{mod} T(A)$ described in (1.2). We shall prove:

LEMMA $F = \hat{F} \circ E$.

PROOF. Indeed, for an $\bar{A}$-module (U, V, ϕ), $E(U, V, \phi)$ is the $\hat{A}$-module $(W_m, \psi_m)_{m \in \mathbf{Z}}$ defined by:

$$W_0=V,\quad W_1=U,\quad W_m=0 \quad \text{for } m\neq 0,\, 1,$$

and

$$\psi_1=\phi : U\otimes DA \longrightarrow V,\quad \psi_m=0 \quad \text{for } m\neq 1.$$

This module is mapped on the module $(W'_m, \psi'_m)_{m\in Z}$ in $\mathrm{Mod}^{\nu}\hat{A}$ which is such that $W'_m=U_A\oplus V_A$ for all $m\in \boldsymbol{Z}$, and $\psi'_m: W'_m\otimes DA\rightarrow W'_{m-1}$ is defined, for all $m\in \boldsymbol{Z}$, by:

$$\psi'_m((u, v)\otimes q)=(0, \phi(u\otimes q))$$

(for $(u, v)\in U\oplus V$ and $q\in DA$). Finally, $(W'_m, \psi'_m)_{m\in Z}$ is mapped on the $T(A)$-module whose A-module structure is that of $U_A\oplus V_A$, and where the action of DA on $U\oplus V$ is induced by the mapping ψ'_m. Thus, if $(u, v)\in U\oplus V$, $a\in A$ and $q\in DA$:

$$(u, v)\begin{bmatrix} a & 0 \\ q & a \end{bmatrix}=(ua, va+\phi(u\otimes q)).$$

That is, F and $\hat{F}\circ E$ coincide on the objects. It is easily checked that they coincide also on the morphisms, and hence $F=\hat{F}\circ E$.

COROLLARY *The functor F preserves the indecomposable modules and their composition lengths.*

1.4. We shall now give a sufficient condition for $\bar{A}$ to be simply connected:

PROPOSITION *Let A be a basic, connected, iterated tilted algebra of Dynkin type, then $\bar{A}$ is simply connected.*

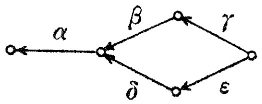
PROOF. Let A be a basic, connected, iterated tilted algebra of Dynkin type, then, by [2], the trivial extension $T(A)$ is representation-finite. The existence of a functor $F: \mathrm{mod}\,\bar{A}\rightarrow \mathrm{mod}\,T(A)$ which preserves indecomposable modules and composition lengths implies that the indecomposable $\bar{A}$-modules must have bounded length. Since $\bar{A}$ is connected, it follows from [3] that it is representation-finite.

Observe that, by (1.2), $\bar{A}$ is a quotient of a finite-dimensional hereditory algebra. On the other hand, A is simply connected [1], hence it satisfies the condition (S) of [6]: that is to say the indecomposable projective A-modules have separated radicals. By the construction of $Q_{\bar{A}}$, this implies that those indecomposable projective $\bar{A}$-modules which are also projective in $\mathrm{mod}\,A$ have separated radicals. Now the remaining indecomposable projective $\bar{A}$-modules are also injective, their radicals are indecomposable and hence separated. Thus $\bar{A}$ satisfies the condition (S), and is therefore simply connected.

Remarks and Examples It is possible that $\bar{A}$ be representation-finite (and even simply connected) even though A may not be iterated tilted of Dynkin type. Consider indeed the following example: let $\wedge(n, s)$ $(n>s)$ denote the algebra given by the quiver:

$$1 \xleftarrow{\alpha_1} 2 \xleftarrow{\alpha_2} 3 \longleftarrow \cdots \qquad \cdots \longleftarrow n-1 \xleftarrow{\alpha_{n-1}} n$$

bound by $\alpha_i\alpha_{i+1}\cdots\alpha_{i+s+1}=0$ $(1\leqq i\leqq n-s)$ [15]. Then the algebra $A=\wedge(9, 3)$ is easily checked to be iterated tilted of Euclidean type $\tilde{\boldsymbol{E}}_8$, but $\bar{A}$ is representation-finite and in fact simply connected (for, there is a full exact embedding [16] of $\mathrm{mod}\,\bar{A}$ into the module category over the algebra $T_2(\wedge(11, 3))$ of all two by two lower triangular matrices with coefficients in $\wedge(11, 3)$, and $T_2(\wedge(11, 3))$ is representation-finite by [15]). In general, however, if A is iterated tilted of Euclidean type, $\bar{A}$ is not representation-finite, for instance, the algebra B given by the quiver:



bound by $\alpha\beta\gamma=0$ and $\beta\gamma=\delta\varepsilon$ is tilted of type $\tilde{\boldsymbol{D}}_4$, but $\bar{B}$ is of tame representation type.

2. The main results

2.1. For an algebra C, we shall denote by τ_C (or simply τ, if no ambiguity may arise) its Auslander-Reiten translation DTr, and by Γ_C its Auslander-Reiten quiver. We shall identify indecomposable C-modules with their isomorphism classes, thus with the corresponding vertices of Γ_C. Recall from [5] that a path $M_0\to M_1\to\cdots\to M_t$ in Γ_C is called *sectional* if $M_i\not\cong\tau M_{i+2}$ for any $0\leqq i\leqq t-2$. A connected subquiver of Γ_C in which every path is sectional is called a *subsection.* A subsection $\mathcal{S}$ is called a *section* if for any irreducible map $M\to N$ with $M\in\mathcal{S}$, either $N\in\mathcal{S}$ or $\tau N\in\mathcal{S}$. Thus, if a section contains an indecomposable summand of the radical of an indecomposable projective, it must contain that projective.

From now on, we shall always assume that A is a basic, connected, tilted algebra of Dynkin type Δ. This implies, by Proposition (1.4), that $\bar{A}$ is simply connected. We shall also assume that $\Gamma_{\bar{A}}$ is given the partial order induced by the arrows: thus $M\leqq N$ means that there exists an oriented path from M to N.

Let now $\mathcal{S}_-$ be the full subquiver of $\Gamma_{\bar{A}}$ consisting of those indecomposable $\bar{A}$-modules M such that there exists an oriented path from M to an indecomposable projective A-module, and moreover every such path is sectional. Clearly,

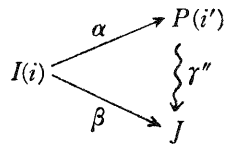
$\mathcal{S}_-$ is connected and is a subsection of $\Gamma_{\bar{A}}$. In the same way, we let $\mathcal{S}'_+$ be the subsection of $\Gamma_{\bar{A}}$ consisting of all the indecomposable $\bar{A}$-modules N such that there is an oriented path from an indecomposable injective A'-module to N, and every such path is sectional. Our first objectives will be to prove that $\mathcal{S}_- < \mathcal{S}'_+$ and that $\mathcal{S}_-$ and $\mathcal{S}'_+$ are isomorphic to complete slices in Γ_A and $\Gamma_{A'}$ respectively.

2.2. LEMMA *For every $i \in (Q_A)_0$, we have $P(i')_{\bar{A}} > \mathcal{S}_-$ and $P(i')_{\bar{A}} < \mathcal{S}'_+$.*

PROOF. Assume first that $i \in (Q_A)_0$ is such that $P(i') \leqq \mathcal{S}_-$. Without loss of generality, we may suppose that the radical of $P(i')_{\bar{A}}$ is the indecomposable injective A-module $I(i)_A$: indeed, the minimal elements among the indecomposable projective-injective $\bar{A}$-modules are such that their radicals are indecomposable injective A-modules (corresponding to the strong sinks: see (1.2) and [13]). Then, since $P(i') \leqq \mathcal{S}_-$, there exists an oriented path in $\Gamma_{\bar{A}}$ from $P(i')$ to an indecomposable projective A-module $P(j)_A$. Now, rad $P(i')_{\bar{A}} = I(i)_A$ and hence we have a path in $\Gamma_{\bar{A}}$:

$$\gamma : I(i)_A \longrightarrow P(i')_{\bar{A}} \longrightarrow \cdots \longrightarrow P(j)_A .$$

The restriction γ' of γ to mod A gives a path in Γ_A from $I(i)_A$ to $P(j)_A$. But A is a tilted algebra of Dynkin type, hence γ' must be a sectional path in Γ_A which, in particular, must factor through an indecomposable summand J_A of $I(i)/S(i)$. Now J is also an indecomposable $\bar{A}$-module, hence must lie on γ. But then we have in $\Gamma_{\bar{A}}$ a situation:



where α and β are arrows, and γ'' a non-trivial path, and this is impossible by [17], Corollary (6). The proof of the second assertion is similar.

COROLLARY (1) (i) *If $M \leqq \mathcal{S}_-$, then the support Supp M of M is contained in Q_A.*

(ii) *If $N \geqq \mathcal{S}'_+$, then the support Supp N of N is contained in Q'_A.*

PROOF. We shall only prove (i), since the proof of (ii) is similar. If $i \in (Q_A)_0$ is such that $\mathrm{Hom}_{\bar{A}}(P(i'), M) \neq 0$, then $P(i') \leqq M$. Since $M \leqq \mathcal{S}_-$, this implies $P(i') \leqq \mathcal{S}$, which is impossible by the previous lemma.

COROLLARY (2) $\mathcal{S}_- \leqq \mathcal{S}'_+$

PROOF. Indeed, if this is not the case, there exist $M \in \mathcal{S}'_+$ and $N \in \mathcal{S}_-$ such that $M \leqq N$. Since $N \in \mathcal{S}_-$, we have $M \leqq \mathcal{S}_-$ and then Supp $M \subseteqq Q_A$. On the other hand, $M \in \mathcal{S}'_+$ implies Supp $M \subseteqq Q'_A$. This is a contradiction since $Q_A \cap Q'_A = \phi$.

2.3. Let now B be a representation-finite tilted algebra (but not necessarily of Dynkin type), and $\mathcal{S}$ be an arbitrary complete slice of Γ_B. If there exists in $\mathcal{S}$ a sink M_B which is not projective, we can replace M by τM and every irreducible map f of codomain M and domain on $\mathcal{S}$ by σf, thus obtaining a new complete slice of Γ_B. Repeating this process as many times as necessary, we ultimately reach a complete slice $\mathcal{L}$ of Γ_B which is characterised by the fact that all its sinks are projective. By construction, $\mathcal{L} \leqq \mathcal{S}$ for every complete slice $\mathcal{S}$ of Γ_B. $\mathcal{L}$ will be called the *left extremal slice* of Γ_B. Dually, we can define the *right extremal slice* $\mathfrak{R}$ to be the complete slice of Γ_B which has all its sources injective. Another characterisation of the extremal slices is as follows:

LEMMA. (i) *$M_B > \mathfrak{R}$ if and only if pd $M > 1$.*
(ii) *$M_B < \mathcal{L}$ if and only if id $M < 1$.*

PROOF of (i) If $M_B > \mathfrak{R}$, then $\tau M \geqq \mathfrak{R}$, and, since $\mathfrak{R}$ is a complete slice, there exists an epimorphism $\bigoplus_{R \in \mathfrak{R}} R \to \tau M$. In particular, for some source I of $\mathfrak{R}$, we have $\mathrm{Hom}_B(I, \tau M) \neq 0$. But I is injective, hence pd $M > 1$.

Conversely, if pd $M > 1$, then $\mathrm{Hom}_B(DB, \tau M) \neq 0$ and there exists an indecomposable injective B-module I_B such that $\mathrm{Hom}_B(I, \tau M) \neq 0$. Since $I \geqq \mathfrak{R}$, we have $\tau M \geqq \mathfrak{R}$ and hence $M > \mathfrak{R}$.

Let us denote by $[\mathcal{L}, \mathfrak{R}]$ the full connected subquiver of Γ_B consisting of those M_B such that $\mathcal{L} \leqq M \leqq \mathfrak{R}$ (that is, $[\mathcal{L}, \mathfrak{R}]$ consists of those vertices of Γ_B lying on a complete slice). Also, let $T(B)$ denote the trivial extension $B \ltimes DB$. We have:

COROLLARY. *Let B be a tilted algebra of Dynkin type, then $[\mathcal{L}, \mathfrak{R}]$ is the maximal full connected subquiver of Γ_B to be embedded fully in $\Gamma_{T(B)}$.*

PROOF. This follows at once from the previous lemma and [12], Theorem (6).

2.4. PROPOSITION. (i) *$\mathcal{S}_-$ is the left extremal slice of Γ_A.*
(ii) *$\mathcal{S}'_+$ is the right extremal slice of $\Gamma_{A'}$.*

PROOF. of (i) It follows from Corollary (2.2.1) that every module on $\mathcal{S}_-$ is an A-module. To prove that $\mathcal{S}_-$ is a complete slice in Γ_A, let us start by proving that no indecomposable projective A-module is a proper successor of $\mathcal{S}_-$ and no indecomposable injective A-module is a proper predecessor of $\mathcal{S}_-$. The first assertion being clear by construction, let $I(i)_A$ be an indecomposable injective A-module such that $I(i)<\mathcal{S}_-$ in $\Gamma_{\bar{A}}$. We may again, without loss of generality, suppose that $I(i)$ is minimal among the indecomposable injective A-modules, and then $I(i)=\operatorname{rad} P(i')_{\bar{A}}$. But in this case, $I(i)<\mathcal{S}_-$ implies that $P(i')_{\bar{A}}\leqq\mathcal{S}_-$ which contradicts Lemma (2.2).

It follows that $\mathcal{S}_-$ contains at least one representative from each τ-orbit of indecomposable A-modules. In fact, $\mathcal{S}_-$ being a subsection of $\Gamma_{\bar{A}}$, but also of Γ_A (because the support of each predecessor of $\mathcal{S}_-$ lies inside Q_A) contains at most one, and hence exactly one representative of each τ-orbit of indecomposable A-modules. By construction, $\mathcal{S}_-$ is convex and it certainly does not contain oriented cycles (because $\bar{A}$ is simply connected). Therefore, $\mathcal{S}_-$ is a complete slice in Γ_A. Since, by construction, all the sinks in $\mathcal{S}_-$ are indecomposable projective A-modules, $\mathcal{S}_-$ is in fact the left extremal slice of Γ_A.

2.5. Let us now denote by $\Gamma=[\mathcal{S}_-, \mathcal{S}'_+]$ the full connected subquiver of $\Gamma_{\bar{A}}$ consisting of those indecomposable $\bar{A}$-modules M such that $\mathcal{S}_-\leqq M\leqq\mathcal{S}'_+$. By Lemma (2.2), all the indecomposable projective-injective $\bar{A}$-modules lie in Γ. Also, by Proposition (2.4), the underlying graph of the subsections $\mathcal{S}_-$ and $\mathcal{S}'_+$ is Δ. In the sequel, we shall call *Δ-subsection* of a translation quiver any subsection whose underlying graph is Δ.

Recall that the surjection $\hat{A}\to\bar{A}$ induces an embedding $\Gamma_{\bar{A}}\to\Gamma_{\hat{A}}$ which is not full in general.

LEMMA. *Γ is the maximal full connected subquiver of $\Gamma_{\bar{A}}$ such that the embedding $\Gamma\to\Gamma_{\bar{A}}\to\Gamma_{\hat{A}}$ is full.*

PROOF. We first observe that a module in Γ which is not projective-injective can only be projective in $\operatorname{mod}\bar{A}$ if it belongs to $\mathcal{S}_-$, and can only be injective in $\operatorname{mod}\bar{A}$ if it belongs to $\mathcal{S}'_+$ (by Proposition (2.4)). Since $\mathcal{S}_-$ and $\mathcal{S}'_+$ are complete slices of Γ_A and $\Gamma_{A'}$ respectively, they are fully embedded in $\Gamma_{\hat{A}}$ (by [13] or [9]).

Let now M be a source in $\mathcal{S}_-$. In particular, M cannot be an injective $\bar{A}$-module. We have two cases to consider: if M is not an indecomposable injective A-module, $\tau_{\bar{A}}^{-1}M=\tau_A^{-1}M$. If we replace M by $\tau_{\bar{A}}^{-1}M$ and every irreducible map f of domain M and codomain on $\mathcal{S}_-$ by $\sigma_{\bar{A}}^{-1}f$, we obtain a new Δ-subsection $\mathcal{S}_1$ of $\Gamma_{\bar{A}}$ which is also a complete slice of Γ_A and is therefore fully embedded in $\Gamma_{\hat{A}}$. If, on the other hand, M is an indecomposable injective A-module $I(i)_A$, the section of $\Gamma_{\bar{A}}$ containing $\mathcal{S}_-$ contains also the projective-injective module $P(i')_{\bar{A}}$ which is such that rad $P(i')_{\bar{A}}=I(i)_A$. We thus replace M by $\tau_{\bar{A}}^{-1}M=P(i')/S(i)$ and every irreducible map f of domain M and codomain on $\mathcal{S}_-$ by $\sigma_{\bar{A}}^{-1}f$. We obtain in this way a complete slice $\mathcal{S}_1$ in the tilted algebra S_i^+A [13] of type Δ, where S_i^+A is the algebra whose ordinary quiver is the full connected subquiver of $Q_{\bar{A}}$ determined by i' and $Q_A\backslash\{i\}$ with the inherited relations. In particular, the Δ-subsection $\mathcal{S}_1$ is again fully embedded in $\Gamma_{\hat{A}}$. Observe that $P(i')$ is mapped in the process on an indecomposable projective-injective $\hat{A}$-module. Applying again the same considerations to $\mathcal{S}_1$, we obtain a new Δ-subsection $\mathcal{S}_2$ which is also fully embedded in $\Gamma_{\hat{A}}$. Inductively, we find a sequence of Δ-subsections:

$$\mathcal{S}_-\leqq\mathcal{S}_1\leqq\mathcal{S}_2\leqq\cdots$$

which have the property that the sections they determine with the indecomposable projective-injective $\bar{A}$-modules are fully embedded in $\Gamma_{\hat{A}}$. This process stops at $\mathcal{S}_t$, where $\mathcal{S}_t$ is such that all its sources are indecomposable injective $\bar{A}$-modules (and hence A'-modules), that is to say, $\mathcal{S}_t=\mathcal{S}'_+$. This completes the proof that the embedding $\Gamma\to\Gamma_{\hat{A}}$ is full. The maximality assertion follows from Corollary (2.3) and Proposition (2.4).

COROLLARY (1) *The embedding $\Gamma_{\bar{A}}\to\Gamma_{\hat{A}}$ is full if and only if A is hereditary.*

PROOF. Indeed, it follows from the lemma that this embedding is full if and only if $\Gamma=\Gamma_{\bar{A}}$ and this is the case if and only if $\mathcal{S}_-$ consists of projective A-modules and $\mathcal{S}'_+$ consists of injective A'-modules. By construction, both of these conditions are equivalent to the condition that A be hereditary.

COROLLARY (2) *Let $\mathcal{S}_+$ and $\mathcal{S}'_-$ denote respectively the right extremal slice of Γ_A and the left extremal slice of $\Gamma_{A'}$, then $\mathcal{S}_-\leqq\mathcal{S}_+<\mathcal{S}'_-\leqq\mathcal{S}'_+$.*

PROOF. It follows from Corollary (2.3) and the previous lemma that $[\mathcal{S}_-,\mathcal{S}_+]\subseteq\Gamma$ and hence $\mathcal{S}_-\leqq\mathcal{S}_+\leqq\mathcal{S}'_+$. Similarly, $\mathcal{S}_-\leqq\mathcal{S}'_-\leqq\mathcal{S}'_+$. Since the support of every predecessor of $\mathcal{S}_+$ lies entirely in Q_A and the support of every successor of $\mathcal{S}'_-$ lies entirely in Q'_A, we have $\mathcal{S}_+<\mathcal{S}'_-$.

2.6. Let now $\mathcal{A}$ denote the additive subcategory of mod $\bar{A}$ generated by the indecomposable $\bar{A}$-modules lying in Γ, and let F' be the restriction to $\mathcal{A}$ of the functor F of (1.3), that is to say, F' is the composition of the embedding $\mathcal{A}\to$ mod $\bar{A}$ and of the functor F: mod $\bar{A}\to$mod $T(A)$.

THEOREM. *The functor F': $\mathcal{A}\to$mod $T(A)$ preserves the indecomposable* *modules, their composition lengths, the Auslander-Reiten sequences and the irreducible maps. Considered as a mapping* $\Gamma \to \Gamma_{T(A)}$, *it is surjective.*

PROOF. Let us recall that the functor $\hat{F}: \mathrm{mod}\,\hat{A} \to \mathrm{mod}\,T(A)$ preserves the indecomposable modules, their composition lengths, the Auslander-Reiten sequences and the irreducible maps. On the other hand, considered as a mapping $\Gamma_{\hat{A}} \to \Gamma_{T(A)}$, it is surjective, in fact, $\Gamma_{\hat{A}}$ is connected and $\Gamma_{\hat{A}}/\nu \simeq \Gamma_{T(A)}$ [13]. It suffices thus to prove, by Lemma (2.5), that Γ contains two Δ-subsections which belong to the same fibre of a complete slice in Γ_A considered as a full connected subquiver in $\Gamma_{T(A)}$. Now we have just seen that $\mathcal{S}_+ < \mathcal{S}'_+$ and that $[\mathcal{S}_+, \mathcal{S}'_+] \subseteq \Gamma$ is fully embedded in $\Gamma_{\hat{A}}$. But $\mathcal{S}_+$ and $\mathcal{S}'_+$ are respectively the right extremal slices of A and $A' = \nu^{-1}A$ (see (1.2)). Therefore, they correspond under the automorphism of $\Gamma_{\hat{A}}$ defined by $\nu: \mathcal{S}'_+ = \nu^{-1}\mathcal{S}_+$. In particular, they belong to the same fibre.

The above theorem allows us to describe the fundamental domains for the representation-finite trivial extension algebra $T(A)$. Recall that Larrión and Salmerón [14] have proved that, if A is a representation-finite, connected, finite-dimensional k-algebra such that Γ_A does not contain oriented cycles, then the universal cover [8] $\tilde{\Gamma}_A$ of Γ_A contains a full subtranslation quiver Σ which is isomorphic to the Auslander-Reiten quiver of a simply connected algebra, and which contains at least one point from each fibre of the covering morphism $\tilde{\Gamma}_A \to \Gamma_A$. Σ is then called a fundamental domain for A. To extend this result to the case of the representation-finite trivial extension algebra $T(A)$, we define a *fundamental domain* (respectively, an *exact fundamental domain*) for $T(A)$ to be a full connected subquiver of $\Gamma_{\hat{A}}$ which contains at least one point (respectively, exactly one point) of each fibre of the map $\Gamma_{\hat{A}} \to \Gamma_{T(A)}$ and which is also a full connected subquiver of the Auslander-Reiten quiver of a simply connected algebra. It follows from Lemma (2.5) and Theorem (2.6) that Γ is a fundamental domain for $T(A)$, maximal inside $\Gamma_{\bar{A}}$. Moreover, Corollary (2.5.1) implies that Γ is in fact equal to the Auslander-Reiten quiver of the simply connected algebra $\bar{A}$ if and only if A is hereditary. The exact fundamental domains are constructed as follows: let $\mathcal{S}$ be an arbitrary complete slice in Γ_A considered as a Δ-subsection of $\Gamma_{\bar{A}}$ (in particular, $\mathcal{S}_- \leqq \mathcal{S} \leqq \mathcal{S}_+$). Then there exists a unique Δ-subsection $\mathcal{S}'$ which is such that $\mathcal{S}' = \nu^{-1}\mathcal{S}$. In fact, $\mathcal{S}' = \tau_{\bar{A}}^{-m_\Delta}\mathcal{S}$, where m_Δ denotes the Coxeter number of the graph Δ minus one, thus $m_{A_n} = n$, $m_{D_n} = 2n-3$, $m_{E_6} = 11$, $m_{E_7} = 17$ and $m_{E_8} = 29$. Hence the exact fundamental domains are precisely the half-open intervals of the forms $[\mathcal{S}, \mathcal{S}'[$ and $]\mathcal{S}, \mathcal{S}']$. It also follows from the proof of the theorem that $\Gamma_{T(A)}$ is obtained from one of these intervals by identifying the two Δ-subsections $\mathcal{S}$ and $\mathcal{S}'$.

We then deduce a simple combinatorial description of $\Gamma_{\bar{A}}$: let $\mathcal{S}$ be an arbitrary complete slice of Γ_A, it embeds fully in $\Gamma_{\hat{A}}$, we shall let $\mathcal{S}_0$ denote its image in $\Gamma_{\hat{A}}$ and put $\mathcal{S}_0' := \tau_{\hat{A}}^{-m_\Delta}\mathcal{S}_0$; Γ_A is then constructed by glueing the full connected subquiver of Γ_A consisting of the predecessors (respectively, successors) of $\mathcal{S}$ to the left (respectively, to the right) of $[\mathcal{S}_0, \mathcal{S}_0']$ identifying $\mathcal{S}$ with $\mathcal{S}_0$ (respectively, $\mathcal{S}_0'$).

For a representation-finite algebra C, let $n(C)$ denote the number of isomorphism classes of indecomposable C-modules. We have:

COROLLARY. $n(\bar{A})=n(T(A))+n(A)$. *Consequently,* $n(\bar{A})\geqq 3n(A)$, *and equality holds if and only if A is hereditary.*

PROOF. Let $\mathcal{S}$ be a complete slice in Γ_A, considered as a full connected subquiver of $\Gamma_{\bar{A}}$, and put $\mathcal{S}'=\tau_{\bar{A}}^{-m_\Delta}\mathcal{S}$. Then an indecomposable $\bar{A}$-module M either lies in $[\mathcal{S}, \mathcal{S}']$, in which case it is associated to a unique isomorphism class of an indecomposable $T(A)$-module, or else, if $M\notin[\mathcal{S}, \mathcal{S}']$, it must satisfy one of the following two conditions: either $M_{\bar{A}}<\mathcal{S}$, or $M_{\bar{A}}\geqq\mathcal{S}'$. In the first case, $M_{\bar{A}}$ is in fact an indecomposable A-module (because $\mathcal{S}\leqq\mathcal{S}_+$) which strictly precedes the complete slice $\mathcal{S}$, and in the second $M_{\bar{A}}$ is an indecomposable A'-module (because $\mathcal{S}'\geqq\mathcal{S}'_-$) which lies on $\mathcal{S}'=\nu^{-1}\mathcal{S}$ or succeeds it. But in this latter case, M is associated to a unique indecomposable A-module lying on $\mathcal{S}$, or succeeding to it. This proves the first assertion. The second follows from the first and [20], Theorem (2.12).

2.7. Let now Γ be a simply connected translation quiver, we shall denote by l_Γ the length function on Γ[8]. Recall from [11] that a *slice* in Γ is a full convex subquiver $\mathcal{S}$ such that, if $x\leqq\mathcal{S}$, then $\mathcal{S}$ contains precisely one vertex from the τ-orbit of x. Observe that this is a more general concept than that of complete slice. We may now state our next theorem.

THEOREM. *Let B be a basic, connected, finite-dimensional k-algebra. Then there exists a tilted algebra A of Dynkin type Δ such that $B\simeq\bar{A}$ if anly if Γ_B is simply connected and contains a slice $\mathcal{S}$ of underlying graph Δ such that:*

(1) *All projective B-modules which are not injective are predecessors of $\mathcal{S}$.*

(2) *All projective-injective B-modules lie between $\mathcal{S}$ and $\tau^{-m_\Delta}\mathcal{S}$.*

(3) *All injective B-modules which are not projective are successors of $\tau^{-m_\Delta}\mathcal{S}$.*

PROOF. We first check the necessity of the conditions. If $B=\bar{A}$, for A tilted of Dynkin type Δ, then Γ_B is simply connected by Proposition (1.4). The other conditions follow from (2.4) and (2.6).

Conversely, assume that B satisfies the stated conditions. Observe first that $\mathcal{S}$ is connected, since it has Δ for underlying graph. Let P_B be the direct sum of the indecomposable projective B-modules which are not injective, and let $A=\operatorname{End} P_B$. We claim that A is a tilted algebra of type Δ. It follows from (1) and (2) that every indecomposable B-module which precedes $\mathcal{S}$ has its support completely contained in A, and consequently, the full connected subquiver of Γ_B consisting of those B-modules which are predecessors of $\mathcal{S}$ is fully embedded in Γ_A. On the other hand, by hypothesis, $\mathcal{S}$ is convex, does not contain oriented cycles (because Γ_B is simply connected) and contains one representative from the τ-orbit of each of its predecessors. Since every projective A-module is a predecessor of $\mathcal{S}$ in Γ_B, hence in Γ_A, $\mathcal{S}$ contains one representative from the τ-orbit of each of the indecomposable projective A-modules. Now A is representation-finite, and has no oriented cycles in its Auslander-Reiten quiver, hence it follows that $\mathcal{S}$ is a complete slice in Γ_A and A is indeed a tilted a tilted algebra of Dynkin type Δ.

By the necessity part of the theorem, the algebra $\bar{A}$ satisfies also the stated conditions. We claim that $\Gamma_{\bar{A}}$ and Γ_B are isomorphic translation quivers. We first observe that, as shown above, the full connected subquiver of Γ_A consisting of those indecomposable A-modules which precede $\mathcal{S}$ is fully embedded in both $\Gamma_{\bar{A}}$ and Γ_B. We shall denote by $p(\mathcal{S}_{\bar{A}})$ and $p(\mathcal{S}_B)$ its respective images, and by $\mathcal{S}_{\bar{A}}$ and $\mathcal{S}_B$ the respective images of the slice $\mathcal{S}$ of Γ_A in $\Gamma_{\bar{A}}$ and Γ_B. Thus, there is a translation quiver isomorphism $f: p(\mathcal{S}_B)\to p(\mathcal{S}_{\bar{A}})$, and $l_{\Gamma_B}(x)=l_{\Gamma_{\bar{A}}}(f(x))$ for each $x\leqq\mathcal{S}_B$. Next we consider the two intervals $[\mathcal{S}_B, \tau^{-m_\Delta}\mathcal{S}_B]$ of Γ_B and $[\mathcal{S}_{\bar{A}}, \tau^{-m_\Delta}\mathcal{S}_{\bar{A}}]$ of $\Gamma_{\bar{A}}$. It follows from (2.5) and [9], §3 that $[\mathcal{S}_{\bar{A}}, \tau^{-m_\Delta}\mathcal{S}_{\bar{A}}]$ is isomorphic, as a translation quiver, to one full period of the configuration of $\boldsymbol{Z}\Delta$ associated to the $\mathcal{S}$-section algebra A, which is stable under the action of τ^{-m_Δ}. Now every module in the open interval $[\mathcal{S}_B, \tau^{-m_\Delta}\mathcal{S}_B]$ which is projective or injective is in fact projective-injective, therefore this open interval is a union of sections formed by parallel Δ-subsections together with the projective-injective modules. On the other hand, the position of each projective-injective is in fact uniquely determined by the length function. Thus $[\mathcal{S}_B, \tau^{-m_\Delta}\mathcal{S}_B]$ is also (again by [9], §3) isomorphic to one full period of the configuration of $\boldsymbol{Z}\Delta$ associated to A. This extends f to a translation quiver isomorphism from $[\mathcal{S}_B, \tau^{-m_\Delta}\mathcal{S}_B]$ to $[\mathcal{S}_{\bar{A}}, \tau^{-m_\Delta}\mathcal{S}_{\bar{A}}]$, and $l_{\Gamma_B}(\tau^{-m_\Delta}x)=l_{\Gamma_B}(x)=l_{\Gamma_{\bar{A}}}(f(x))=l_{\Gamma_{\bar{A}}}(f(\tau^{-m_\Delta}x))$ for each x on $\mathcal{S}_B$. Finally, since no projective modules are successors of $\tau^{-m_\Delta}\mathcal{S}_B$, f can be extended to a translation quiver isomorphism $f: \Gamma_B \xrightarrow{\sim} \Gamma_{\bar{A}}$ (indeed, the remaining parts of these translation quivers are uniquely determined by the values of the respective length functions on $\tau^{-m}\Delta S_B$ and $\tau^{-m}\Delta S_{\bar{A}}$ and these are equal). Since our algebras are simply connected, this implies that $B \simeq \bar{A}$.

2.8 REMARKS. (1) The above results are no longer true if A is assumed to be an iterated tilted algebra of Dynkin type, but not tilted. For instance, if A is the iterated tilted algebra of type $\boldsymbol{A}_6$ given by the quiver:

$$1 \xleftarrow{\alpha_1} 2 \xleftarrow{\alpha_2} 3 \xleftarrow{\alpha_3} 4 \xleftarrow{\alpha_4} 5 \xleftarrow{\alpha_5} 6$$

bound by $\alpha_i\alpha_{i+1}=0 (1 \leqq i \leqq 4)$, then $\Gamma_{\bar{A}}$ has no $\boldsymbol{A}_6$-subsection.

(2) We may generalise the above results in the following way: let $A^{(t)}$ denote the (finite-dimensional) quotient algebra of $\hat{A}$ defined by:

$$A^{(t)}=\begin{pmatrix} A_0 & & & & 0 \\ Q_1 & A_1 & & & \\ & Q_2 & A_2 & & \\ & & \ddots & \ddots & \\ 0 & & & Q_t & A_t \end{pmatrix}$$

(thus, $\bar{A}=A^{(1)}$). Then, if A is a basic, connected, iterated tilted algebra of Dynkin type, $A^{(t)}$ is simply connected. Also, if A is moreover assumed to be tilted, we can describe, just as above, the Auslander-Reiten quiver of $A^{(t)}$ which then contains t exact fundamental domains for $T(A)$.

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