TSUKUBA J. MATH.
Vol. 12 No. 2 (1988), 489–495

HOMEOMORPHISMS OF ZERO-DIMENSIONAL SPACES

By

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Dedicated to Professor Yukihiro Kodama on his 60th birthday

1. Introduction.

All spaces considered in this paper are assumed to be compact and metrizable.

Let φ be a homeomorphism from a space (X, d) onto itself. Then φ is *expansive* if there is $c>0$ such that for every $x, y\in X$ with $x\neq y$ there is $n\in \boldsymbol{Z}$ for which $d(\varphi^{n}(x), \varphi^{n}(y))>c$. Given $\delta>0$, a sequence $\{x_{i}: i\in \boldsymbol{Z}\}$ is a *δ-pseudo-orbit* of φ if $d(\varphi(x_{i}), x_{i+1})<\delta$ for every $i\in \boldsymbol{Z}$. Given $\varepsilon>0$, a sequence $\{x_{i}: i\in \boldsymbol{Z}\}$ is *ε-traced* by a point $y\in X$ if $d(\varphi^{i}(y), x_{i})<\varepsilon$ for every $i\in \boldsymbol{Z}$. We say that φ has the *pseudo orbit tracing property* (abbrev. P.O.T.P.) if for every $\varepsilon>0$ there is $\delta>0$ such that every δ-pseudo-orbit of φ can be ε-traced by some point of X.

For a space (X, d) we denote by $\mathcal{H}(X)$ the space of all homeomorphisms of X with the metric $\tilde{d}(\varphi, \phi)=\max\{d(\varphi(x), \phi(x)): x\in X\}$ for every $\varphi, \phi\in \mathcal{H}(X)$. Let $\mathcal{E}(X)=\{\varphi\in \mathcal{H}(X): \varphi$ is expansive$\}$ and $\mathcal{P}(X)=\{\varphi\in \mathcal{H}(X): \varphi$ has P.O.T.P.$\}$.

In Section 3 we are concerned with the Cantor set C. The Cantor set C is the unique zero-dimensional infinite group. N. Aoki [1] proved that every group automorphism of C has P.O.T.P. M. Sears [6] proved that $\mathcal{E}(C)$ is dense in $\mathcal{H}(C)$, constructing a dense subset $\mathcal{A}$ of $\mathcal{E}(C)$ in $\mathcal{H}(C)$. M. Dateyama [3] proved that $\mathcal{P}(C)$ is dense in $\mathcal{H}(C)$, constructing a dense subset $\mathcal{B}$ of $\mathcal{P}(C)$ in $\mathcal{H}(C)$. However, for the sets $\mathcal{A}$ and $\mathcal{B}$ above we have $\mathcal{A}\cap\mathcal{B}=\phi$. So it is unknown whether the set $\mathcal{E}(C)\cap\mathcal{P}(C)$ of all expansive homeomorphisms with P.O.T.P. of C is dense in $\mathcal{H}(C)$. In Section 3 we shall prove the following theorem.

THEOREM 1. *The set of all expansive homeomorphisms with* P.O.T.P. *of the Cantor set C is dense in $\mathcal{H}(C)$.*

We know [6] that $\mathcal{E}(C)$ is of first category. So $\mathcal{E}(C)\cap\mathcal{P}(C)$ is also of first category.

The convergent sequence is another standard zero-dimensional space, classed with the Cantor set. In Section 4 we shall prove the following theorem.

Received October 22, 1987

THEOREM 2. *Let* $S=\{0, 1, 1/2, 1/3, \cdots\}$. *Then*

(a) *the set of all expansive homeomorphisms of* S *is dense in* $\mathcal{H}(S)$,

(b) *the set of all homeomorphisms with* P. O. T. P. *of* S *is dense in* $\mathcal{H}(S)$,

(c) S *has no expansive homeomorphism with* P. O. T. P.

In Section 5 we shall construct a zero-dimensional space having no expansive homeomorphism.

2. Preliminaries.

Let $D^{\boldsymbol{Z}}=\prod\{D_i : i\in\boldsymbol{Z}\}$, where $D_i=\{0, 1\}$ for every $i\in\boldsymbol{Z}$. We define the metric d on $D^{\boldsymbol{Z}}$ by

$$d(x, y)=\begin{cases} 1/\min\{|k| : x_k\neq y_k\} & \text{if } x_0=y_0 \\ 2 & \text{if } x_0\neq y_0 \end{cases}$$

for every $x=(x_i)$, $y=(y_i)\in D^{\boldsymbol{Z}}$.

Obviously, $(D^{\boldsymbol{Z}}, d)$ is homeomorphic to the Cantor set. For a homeomorphism of a compact metrizable space X it is clear that both expansiveness and P. O. T. P. do not depend on the choice of metrics on X. Thus we may regard $(D^{\boldsymbol{Z}}, d)$ as the Cantor set.

For every $i, j\in\boldsymbol{Z}$ with $i\leqq j$ we put $D(i, j)=\prod\{D_k : i\leqq k\leqq j\}$ and for every $f\in D(i, j)$ we put $c^+(f)=j$ and $c^-(f)=i$. We define the order $\leqq$ on $\cup\{D(i, j) : i, j\in\boldsymbol{Z}$ with $i\leqq j\}\cup D^{\boldsymbol{Z}}$ as follows: $f\leqq g$ if and only if one of the following conditions holds; (1) $f=g$, (2) $f\in D(i, j)$, $g\in D(k, l)$, $k\leqq i$, $j\leqq l$ and $f_m=g_m$ for every m, $i\leqq m\leqq j$, (3) $f\in D(i, j)$, $g\in D^{\boldsymbol{Z}}$ and $f_m=g_m$ for every m, $i\leqq m\leqq j$, where $f=(f_i, f_{i+1}, \cdots, f_j)$ for $f\in D(i, j)$ and $f=(\cdots, f_{-1}, f_0, f_1, \cdots)$ for $f\in D^{\boldsymbol{Z}}$. For every $f\in D(i, j)$ and any $n\in\boldsymbol{N}$ with $i\leqq -n$ and $n\leqq j$ (or for every $f\in D^{\boldsymbol{Z}}$ and any $n\in\boldsymbol{N}$) we put $f_{|n}=(f_{-n}, f_{-n+1}, \cdots, f_n)\in D(-n, n)$. For every $f\in D(i, j)$ we put $A_f=p_{ij}^{-1}(f)$, where $p_{ij}: D^{\boldsymbol{Z}}\to D(i, j)$ is the projection.

If a space X is the union of a pairwise disjoint collection $\{X_\lambda : \lambda\in\Lambda\}$ of open-and-closed subsets of X, then we represent X as $X=\bigoplus\{X_\lambda : \lambda\in\Lambda\}$.

3. Proof of Theorem 1.

Let $\psi: D^{\boldsymbol{Z}}\to D^{\boldsymbol{Z}}$ be a homeomorphism and $\varepsilon>0$. We shall construct an expansive homeomorphism φ with P. O. T. P. such that $\tilde{d}(\psi, \varphi)=\max\{d(\psi(x), \varphi(x)) : x\in D^{\boldsymbol{Z}}\}<\varepsilon$.

We take $k, n\in\boldsymbol{N}$ such that $1/k<\varepsilon$ and $d(\psi(x), \psi(y))<1/k$ for every $x, y\in D^{\boldsymbol{Z}}$ with $d(x, y)<1/n$.

Claim 1. For every $f\in D(-n, n)$ there are $h(f)\in D(-k, k)$ and $g(f)\in D(-l_1, l_2)$ for some $l_1, l_2\in \boldsymbol{N}$, $i=1, 2$, satisfying the following three conditions;

(a) $D^{\boldsymbol{Z}}=\bigoplus\{A_{g(f)}: f\in D(-n, n)\}$,

(b) $\psi(A_f)\subset A_{h(f)}$,

(c) $h(f)\leqq g(f)$.

Proof of Claim 1. From $\operatorname{diam} A_f<1/n$ it follows that $\operatorname{diam}\psi(A_f)<1/k$. Since $D^{\boldsymbol{Z}}=\bigoplus\{A_h: h\in D(-k, k)\}$ and $d(A_h, A_{h'})\geqq 1/n$ for every $h, h'\in D(-k, k)$ with $h\neq h'$, there is $h(f)\in D(-k, k)$ such that $\psi(A_f)\subset A_{h(f)}$. For every $h\in D(-k, k)$ list $\{f\in D(-n, n): h(f)=h\}$ as $\{f_{hi}: 1\leqq i\leqq p_h\}$. For every i, $1\leqq i\leqq p_h$, we take $g_{hi}\geqq h$ such that $A_h=\bigoplus\{A_{g_{hi}}: 1\leqq i\leqq p_h\}$. Let us set $g(f_{hi})=g_{hi}$ for every $h\in D(-k, k)$ and any i, $1\leqq i\leqq p_h$. Then $g(f)$ and $h(f)$ have all the required properties.

Next, we shall construct a homeomorphism $\varphi: D^{\boldsymbol{Z}}\to D^{\boldsymbol{Z}}$. For every $x\in D^{\boldsymbol{Z}}$ we define $\varphi(x)$ as follows.

Let $f=x_{|n}\in D(-n, n)$ and $g(f)\in D(-l_1, l_2)$.

Case 1. $l_1+l_2\geqq 2n$ and $l_2\geqq n$.

Let us set

$$(\varphi(x))_i=\begin{cases}(g(f))_i & \text{if } -l_1\leqq i\leqq l_2\\ x_{i+1} & \text{if } l_2+1\leqq i\\ x_{i+l_1+l_2+2} & \text{if } n-l_1-l_2-1\leqq i\leqq -l_1-1\\ x_{i-2n+l_1+l_2+1} & \text{if } i\leqq n-l_1-l_2-2\end{cases}$$

and

$$M^+(f)=1 \quad\text{and}\quad M^-(f)=-2n+l_1+l_2+1.$$

Case 2. $l_1+l_2<2n$ and $l_1\leqq n$.

Let us set

$$(\varphi(x))_i=\begin{cases}(g(f))_i & \text{if } -l_1\leqq i\leqq l_2\\ x_{i+1} & \text{if } i\leqq -n-2\\ x_{i+2n+2} & \text{if } -n-1\leqq i\leqq -l_1-1\\ x_{i+2n-l_1-l_2+1} & \text{if } l_1+1\leqq i\end{cases}$$

and

$$M^+(f)=2n-l_1-l_2+1 \quad\text{and}\quad M^-(f)=1.$$

Case 3. otherwise, i.e. ($l_1+l_2\geqq 2n$ and $l_2<n$) or ($l_1+l_2<2n$ and $l_1>n$). In this case we have $l_2<n<l_1$. Let us set

$$(\varphi(x))_i=\begin{cases}(g(f))_i & \text{if} \quad -l_1\leqq i\leqq l_2\\ x_{i+n-l_2} & \text{if} \quad l_2+1\leqq i\\ x_{i+l_1-n} & \text{if} \quad i\leqq -l_1-1\end{cases}$$

and

$$M^+(f)=n-l_1 \quad \text{and} \quad N^-(f)=l_1-n\,.$$

Then it is obvious that $\varphi_{|A_f}: A_f\to A_{g(f)}$ is a homeomorphism. By (a), φ is a homeomorphism from D^Z onto itself. Let us set $m=\max\{-c^-(g(f)),\ c^+(g(f)): f\in D(-n, n)\}$.

By the construction of φ the following claim is easily seen.

Claim 2. Let $x, y\in D^Z$ with $d(x, y)=1/k\leqq 1/2m$.

(i) If $x_k\neq y_k$, then $d(\varphi(x), \varphi(x))=1/l$ and $x_l\neq y_l$, where $l=k-M^+(x_{|n})$.

(ii) If $x_{-k}\neq y_{-k}$, then $d(\varphi^{-1}(x), \varphi^{-1}(y))=1/l$ and $x_{-l}\neq y_{-l}$, where $l=k-M^-(x_{|n})$.

By Claim 2, $1/2m$ is an expansive constant for φ. Thus φ is expansive.

To prove that φ has P.O.T.P. we need the following mappings α and β.

For every $f\in\cup\{D(i, j): i, j\in \boldsymbol{Z}$ with $i\leqq -n$ and $n\leqq j\}$ let us set

$$\alpha(f)=\max\{g: g<\varphi(h) \text{ for every } h\in D^Z \text{ with } f<h\}\,.$$

For every $g\in\cup\{D(i, j): i, j\in \boldsymbol{Z}$ with $i\leqq -m$ and $m\leqq j\}$ let us set

$$\beta(g)=\max\{f: f<\varphi^{-1}(h) \text{ for every } h\in D^Z \text{ with } g<h\}\,.$$

We shall show that φ has P.O.T.P.

Let $\varepsilon_1>0$. We take $\delta=1/N$ such that $1/N<\min\{\varepsilon_1, 1/2m\}$. Let $\{x^i: i\in \boldsymbol{Z}\}$ be a δ-pseudo-orbit of φ. Let $K(-1)=-N-1$. By induction on $0\leqq i\in \boldsymbol{Z}$, we choose $K(i)$ and $y_j\in D_j$ for every j, $K(i-1)<j\leqq K(i)$, satisfying the following conditions:

(d) $K(i-1)<K(i)$,

(e) $c^+(\alpha^i(y^i))=N$,

(f) $\alpha^i(y^i)_{|N}=x^k_{|N}$,

where $y^i=(y_{-N}, y_{-N+1}, \cdots, y_{K(i)})\in D(-N, K(i))$.

In case $i=0$, let $K(0)=N$ and for every j, $-K(-1)<j\leqq K(0)$, let $y_j=x^0_j$. Assume that $K(i)$ and y_j, $K(i-1)<j\leqq K(i)$, are chosen such that the above conditions hold. Let us set $K(i+1)=K(i)+M^+(\alpha^i(y^i)_{|n})$ and $y_j=x^{i+1}_{j+N-K(i+1)}$ for every j, $K(i)<j\leqq K(i+1)$. It is easy to check that all induction hypothesis are satisfied. Let $L(1)=N+1$. By induction on $0\geqq i\in \boldsymbol{Z}$, similarly as above, we choose $L(i)$ and $y_j\in D_j$ for every j, $L(i)\leqq j<L(i+1)$, satisfying the following conditions:

(g) $L(i)<L(i+1)$,

(h) $c^{-}(\beta^{-i}(y^{i}))=-N$,

(i) $\beta^{-i}(y^{i})_{|N}=x^{k}_{|N}$,

where $y^{i}=(y_{L(i)}, y_{L(i)+1}, \cdots, y_{N})\in D(L(i), N)$. Let us set $y=(\cdots, y_{-1}, y_{0}, y_{1}, \cdots)\in D^{Z}$. Then for every $i\geqq 0$ we have $\varphi^{i}(y)>\alpha^{i}(y^{i})$ and $\alpha^{i}(y^{i})_{|N}=x^{i}_{|N}$. This implies that $\varphi^{i}(y)_{|N}=x^{i}_{|N}$ and therefore we have $d(\varphi^{i}(y), x^{i})<1/N<\varepsilon_{1}$. For every $i\leqq 0$ we have $\varphi^{i}(y)>\beta^{-i}(y^{i})$ and $\beta^{-i}(y^{i})_{|N}=x^{i}_{|N}$. This implies that $\varphi^{i}(y)_{|N}=x^{i}_{|N}$ and therefore we have $d(\varphi^{i}(y), x^{i})<1/N<\varepsilon_{1}$. Hence $\{x^{i}: i\in \boldsymbol{Z}\}$ is ε_{1}-traced by y. Therefore φ has P.O.T.P.

We show that $\tilde{d}(\varphi, \psi)<\varepsilon$. By the construction of φ, $\varphi((A_{f}))=A_{g(f)}$ for every $f\in D(-n, n)$. For every $x\in D^{Z}$, we have $x\in A_{f}$ for some $f\in D(-n, n)$. Thus, by (c), we have $\varphi(x)\in\varphi(A_{f})=A_{g(f)}\subset A_{h(f)}$. On the other hand, by (b), we have $\psi(x)\in\psi(A_{f})\subset A_{h(f)}$. From diam $A_{h(f)}=1/(k+1)<\varepsilon$ it follows that $d(\varphi(x), \psi(x))<\varepsilon$. Hence we have $\tilde{d}(\varphi, \psi)<\varepsilon$. Theorem 1 has been proved.

4. Proof of Theorem 2.

Let d be the Euclidean metric on $S=\{0, 1, 1/2, 1/3, \cdots\}$. Note that a mapping $\varphi: S\to S$ is a homeomorphism if and only if φ is one-to-one, onto and $\varphi(0)=0$. For every $n\in\boldsymbol{N}$ we set $S_{n}=\{1/(n-1), 1/(n-2), \cdots, 1\}$.

(a) Let $\psi\in\mathcal{H}(S)$ and $\varepsilon_{0}>0$. We construct $\varphi\in\mathcal{E}(S)$ such that $\tilde{d}(\varphi, \psi)<\varepsilon_{0}$. To do this, we take $n\in\boldsymbol{N}$ with $1/n<\varepsilon_{0}$. For every $m\in\boldsymbol{N}$, $m<n$, we take $x_{m}\in S$ such that $\psi(x_{m})=1/m$. Let $l=\max\{1/x_{m}: m<n\}+1$. For every $k\in\boldsymbol{N}$, $k\geqq l$, let us set

$$\varphi(1/k)=\begin{cases} 1/(k-2) & \text{if } k=l+2i \text{ for some } i\in\boldsymbol{N} \\ 1/(k+2) & \text{if } k=l+2i-1 \text{ for some } i\in\boldsymbol{N} \\ 1/(l+1) & \text{if } k=l. \end{cases}$$

For every $m\in\boldsymbol{N}$, $m<n$, let us set $\varphi(x_{m})=1/m$ $(=\psi(x_{m}))$. Let $\varphi(0)=0$, and for every $x\in S_{l}-\{x_{m}: m<n\}$ let $\varphi(x)$ be an element of $S_{l}-S_{n}$ such that $\varphi(x)\neq\varphi(x')$ for every $x, x'\in S_{l}-\{x_{m}: m<n\}$ with $x\neq x'$. Then φ is one-to-one, onto and $\varphi(0)=0$. Thus $\varphi\in\mathcal{H}(S)$. By the construction of φ, it is obvious that $\tilde{d}(\varphi, \psi)\leqq 1/n<\varepsilon_{0}$. Let $c=1/(2l^{2}+2l)$. Note that $U_{c}(1/l)=\{1/l\}$. We show that c is an expansive constant for φ. Let $x, y\in S$ with $x\neq y$. We may assume that $x\neq 0$. If $x\in S_{l}$, then $d(x, y)>c$. If $x\notin S_{l}$, then $\varphi^{i}(x)=1/l$ for some $i\in\boldsymbol{Z}$, and therefore $d(\varphi^{i}(x), \varphi^{i}(y))>c$. Hence we have $\varphi\in\mathcal{E}(S)$.

(b) Let $\psi\in\mathcal{H}(S)$ and $\varepsilon_{0}>0$. We costruct $\varphi\in\mathcal{P}(S)$ such that $\tilde{d}(\varphi, \psi)<\varepsilon_{0}$. Let n, l and x_{m}, $m<n$, be as in (a). For every $x\in S_{l}$ let $\varphi(x)$ be as in (a).

For every $x\in S-S_l$ let $\varphi(x)=x$. Then, similarly as in (a), we have $\varphi\in\mathscr{H}(S)$ and $\bar{d}(\varphi, \psi)<\varepsilon_0$. To prove that φ has P.O.T.P. let $\varepsilon_1>0$. Take $k\in\boldsymbol{N}$ with $1/k<\min\{\varepsilon_1, 1/l\}$. Let $\delta=1/(k^2+k)$. Note that $U_\delta(1/j)=\{1/j\}$ for every $j\in\boldsymbol{N}$, $j\leqq k$. It suffices that every δ-pseudo-orbit of φ can be ε_1-traced by some point of S. Let $\{y_i: i\in\boldsymbol{Z}\}$ be a δ-pseudo-orbit of φ. If $y_0\in S-S_k$, then $y_i\leqq 1/n<\varepsilon_1$ for every $i\in\boldsymbol{Z}$. Thus $\{y_i: i\in\boldsymbol{Z}\}$ is ε_1-traced by y_0. If $y_0\in S_k$, then $y_i=\varphi^i(y_0)$ for every $i\in\boldsymbol{Z}$. Thus $\{y_i: i\in\boldsymbol{Z}\}$ is ε_1-traced by y_0. Hence φ has P.O.T.P.

(c) Let $\varphi\in\mathscr{E}(S)$ with an expansive constant c. It is enough to prove that $\varphi\notin\mathscr{P}(S)$. We take $n\in\boldsymbol{N}$ with $1/n<c$. Assume that $1/m$ is a periodic point for every $m\in\boldsymbol{N}$, $m<n$. Then $\cup\{\mathrm{Orb}(1/m): m<n\}$ is finite, where $\mathrm{Orb}(x)=\{\varphi^i(x): i\in\boldsymbol{Z}\}$. Pick up a point $x\in S-(\cup\{\mathrm{Orb}(1/m): m<n\}\cup\{0\})$. Then we have $\mathrm{Orb}(x)\subset S-S_n$, therefore $d(\varphi^i(x), \varphi^i(0))\leqq 1/n<c$ for every $i\in\boldsymbol{Z}$. This is a contradiction. Take $m<n$ such that $1/m$ is not a periodic point. Let $\varepsilon=1/(m^2+m)$. For every $\delta>0$ we can take $l\in\boldsymbol{N}$ such that $\varphi^{l-1}(1/m)<\delta$ and $\varphi^{-l}(1/m)<\delta$, because $\lim_{i\to\infty}\varphi^i(1/m)=0$ the $\lim_{i\to\infty}\varphi^{-i}(1/m)=0$. Let us set

$$y_{2kl+j}=\begin{cases}\varphi^j(1/m) & \text{if } 0\leqq j\leqq l-1\\ \varphi^{j-2l}(1/m) & \text{if } l\leqq j\leqq 2l\end{cases}$$

Then $\{y_i: i\in\boldsymbol{Z}\}$ is a δ-pseudo-orbit of φ. Assume that $\{y_i: i\in\boldsymbol{Z}\}$ is ε-traced by $y\in S$. Since $U_\varepsilon(1/m)=\{1/m\}$ and $y_{2kl}=1/m$ for every $k\in\boldsymbol{Z}$, we have $\varphi^{2kl}(y)=1/m$ for every $k\in\boldsymbol{Z}$. This implies that $1/m$ is a periodic point. This is a contradiction. Hence S has no expansive homeomorphism with P.O.T.P.

5. A zero-dimensional space having no expansive homeomorphism.

S. Fujii [4] proved that a space X is zero-dimensional if and only if the identity mapping id_X has P.O.T.P. So every zero-dimensional space has at least one homeomorphism with P.O.T.P. We know ([2], or see [5]) that the unit interval has no expansive homeomorphism. However, as far as the author knows it is unknown whether there is a zero-dimensional space having no expansive homeomorphism. In this section we construct such a space X. Note that the space X above is contained in the Cantor set, because the Cantor set is universal for the class of zero-dimensional spaces.

Let $C\subset[0, 1]$ be the Cantor set and $S=\{0, 1, 1/2, \cdots\}$ a convergent sequence. Let $X_n=(C\oplus S^n)/\{0, 0_n\}$ be the quotient space obtained by identifying $\{0, 0_n\}$ to a point x_n, where $0\in C$ and $0_n=(0, 0, \cdots, 0)\in S^n$, for every $n\in\boldsymbol{N}$, and let $X_0=\{x_0\}$ be a one-point space. Let $X=\cup\{X_n: n\in\boldsymbol{N}\cup\{0\}\}$. We give X a topology as follows. Let $\mathscr{B}(x)=\{U: U$ is a neighborhood of x in $X_n\}$ for every $x\in X$,

$n\in \boldsymbol{N}$, and $\mathscr{B}(x_0)=\{\cup\{X_i: j\leqq i\}\cup X_0: j\in \boldsymbol{N}\}$. Then $\{\mathscr{B}(x): x\in X\}$ is a neighborhood system. Obviously the space X with the topology generated by $\{\mathscr{B}(x): x\in X\}$ is compact, metrizable and zero-dimensional. Next we show that X has no expansive homeomorphism. To do this let φ be a homeomorphism of X. The point x_n is the only point that has arbitrarily small neighborhoods containing a set homeomorphic to the Cantor set, a set homeomorphic to S^n, and no set homeomorphic to S^{n+1}. Therefore we have $\varphi(x_n)=x_n$ for every $n\in \boldsymbol{N}$. Thus φ has infinitely many fixed points. Hence φ is not expansive.

After I finished writing an early version of this paper, I knew that T. Shimomura [7] also proved Theorem 1, independently.

References

[1] Aoki, N., The spliting of zero-dimensional automorphisms and its application, Colloq. Math. **49** (1985), 161-173.

[2] Bryant, B.F., Unstable self-homeomorphisms of a compact space, Vanderbilt University, Thesis, 1954.

[3] Dateyama, M., Homeomorphisms with the pseudo orbit tracing property of the Cantor set, Tokyo J. Math. **6** (1983), 287-290.

[4] Fujii, S., Distal homeomorphisms with the pseudo orbit tracing property and total disconnectedness, Preprint.

[5] Jakobsen, J.F. and Utz, W.R., The nonexistence of expansive homeomorphisms on a closed 2-cell, Pacific J. Math. **10** (1960), 1319-1321.

[6] Sears, M., Expansive self-homeomorphisms of the Cantor set, Math. System Theory **6** (1972), 129-132.

[7] Shimomura, T., The pseudo-orbit tracing property and expansiveness on Cantor set, Preprint.

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