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SEMI-INVARIANT SUBMANIFOLDS OF CODIMENSION 3 WITH HARMONIC CURVATURE

By

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§ 0. Introduction.

A Riemannian curvature tensor is said to be *harmonic* if the Ricci tensor R_{ji} satisfies the Codazzi equation, namely, in local coordinates, $R_{jik}=R_{jki}$, where R_{jik} denotes the covariant derivative of the Ricci tensor R_{ji}. Recently Riemannian manifolds with harmonic curvature are studied by A. Derdziński [1], H. Nakagawa and U-H. Ki [4], [5], [6], E. Ômachi [9], M. Umehara [6], [10] and others.

The purpose of the present paper is to study submanifolds with harmonic curvature admitting almost contact metric structure in a Euclidean space and to prove the following:

THEOREM. *Let M be a* $(2n+1)$*-dimensional complete simply connected semi-invariant submanifold in a* $(2n+4)$*-dimensional Euclidean space. If M has harmonic curvature and of constant mean curvature and if the distinguished normal is parallel in the normal bundle, then M is isometric to one of the following spaces;*

$$E^{2n+1},\ S^{2n+1} \quad or \quad S^{2n-r+1}\times E^{r}, \qquad (r\leqq 2n-1).$$

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§ 1. Preliminaries.

Let $\overline{M}$ be a $(2n+4)$-dimensional almost Hermitian manifold covered by a system of coordinate neighborhoods $\{U: X^{A}\}$. Manifolds, submanifolds, geometric objects and mappings discussed in this paper are assumed to be differentiable and of class C^{∞}. Denote by G_{CB} components of the Hermitian metric tensor, and by $F_{B}{}^{A}$ those of the almost complex structure F of $\overline{M}$. Then we have

$$F_{C}{}^{B}F_{B}{}^{A}=-\delta_{C}{}^{A}, \tag{1.1}$$

$$F_{C}{}^{E}F_{B}{}^{D}G_{ED}=G_{CB}, \tag{1.2}$$

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$\delta_C{}^A$ being the Kronecker delta. We use throughout this paper the systems of indices as follows:

$$A, B, C, D, \cdots : 1, 2, \cdots, 2n+4;$$
$$h, i, j, k, \cdots : 1, 2, \cdots, 2n+1.$$

The summation will be used with respect to those systems of indices.

Let M be a $(2n+1)$-dimensional Riemannian manifold covered by a system of coordinate neighborhoods $\{V; Y^h\}$ and immersed isometrically in $\overline{M}$ by the immersion $i: M \to \overline{M}$. In the sequel we identify $i(M)$ with M itself and represent the immersion by

$$X^A = X^A(Y^h). \tag{1.3}$$

We put

$$B_i{}^A = \partial_i X^A, \qquad \partial_i = \partial/\partial Y^i \tag{1.4}$$

and denote by C^A, D^A and E^A three mutually orthogonal unit normals to M. Then denoting by g_{ji} the fundamental metric tensor of M, we have

$$g_{ji} = B_j{}^C B_i{}^B G_{CB} \tag{1.5}$$

since the immersion is isometric.

As to the transformations of $B_i{}^A$, C^A, D^A and E^A by $F_B{}^A$ we have respectively equations of the form

$$F_B{}^A B_i{}^B = f_i{}^h B_h{}^A + u_i C^A + v_i D^A + w_i E^A, \tag{1.6}$$

$$F_B{}^A C^B = -u^h B_h{}^A - \nu D^A + \mu E^A, \tag{1.7}$$

$$F_B{}^A D^B = -v^h B_h{}^A + \nu C^A - \lambda E^A, \tag{1.8}$$

$$F_B{}^A E^B = -w^h B_h{}^A - \mu C^A + \lambda D^A, \tag{1.9}$$

where $f_i{}^h$ is a tensor field of type (1,1), u_i, v_i, w_i 1-forms and λ, μ, ν functions in M, u^h, v^h and w^h being vector fields associated with u_i, v_i and w_i respectively.

Applying the operator F to both sides of (1.6)-(1.9), using (1.1), we find

$$f_i{}^t f_t{}^h = -\delta_i^h + u_i u^h + v_i v^h + w_i w^h, \tag{1.10}$$

$$u_t f_i{}^t = -\nu v_i + \mu w_i, \quad v_t f_i{}^t = \nu u_i - \lambda w_i, \quad w_t f_i{}^t = -\mu u_i + \lambda v_i, \tag{1.11}$$

$$f_t{}^h u^t = \nu v^h - \mu w^h, \quad f_t{}^h v^t = -\nu u^h + \lambda w^h, \quad f_t{}^h w^t = \mu u^h - \lambda v^h, \tag{1.12}$$

$$u_t u^t = 1 - \mu^2 - \nu^2, \quad v_t v^t = 1 - \nu^2 - \lambda^2, \quad w_t w^t = 1 - \lambda^2 - \mu^2, \tag{1.13}$$
$$u_t v^t = \lambda\mu, \quad u_t w^t = \lambda\nu, \quad v_t w^t = \mu\nu.$$

Also, from (1.2), (1.5) and (1.6), we obtain

$$f_j{}^t f_i{}^s g_{ts} = g_{ji} - u_j u_i - v_j v_i - w_j w_i. \tag{1.14}$$

Putting $f_{ji}=f_j{}^t g_{ti}$, we see that $f_{ji}=-f_{ij}$. From (1.12), we can easily see that

$$f_t{}^h p^t=0, \tag{1.15}$$

where

$$p^h=\lambda u^h+\mu v^h+\nu w^h. \tag{1.16}$$

Suppose that the set (f, g, P) of the tensor field of type (1,1), the Riemannian metric tensor g_{ji} and the vector field P^h given by (1.16) defined an almost contact metric structure, that is, in addition to (1.15), the set (f, g, P) satisfies

$$f_i{}^t f_t{}^h=-\delta_i{}^h+P_i P^h, \tag{1.17}$$

$$f_j{}^t f_i{}^s g_{ts}=g_{ji}-P_j P_i, \tag{1.18}$$

$$P_t P^t=1, \tag{1.19}$$

where $P_i=g_{it}P^t$. Then we find from (1.13), (1.16) and (1.19)

$$\lambda^2+\mu^2+\nu^2=1. \tag{1.20}$$

Conversely suppose that the functions λ, μ, ν satisfy (1.20). Then the set (f, g, P) defines an almost contact metric structure [11].

§ 2. Semi-invariant submanifolds of codimension 3.

Let $\bar{M}$ be an almost Hermitian manifold with almost complex structure F. A submanifold M is called a *CR submanifold* of $\bar{M}$ if there exists a differentiable distribution D on M satisfying the following conditions:

(1) D is invariant, that is, $FD_x=D_x$ for each x in M,

(2) the complementary orthogonal distribution $D^\perp$ on M is anti-invariant, that is, $FD_x^\perp\subset N_x$ for each x in M, where N_x denotes the normal space to M at x. In particular, M is said to be *semi-invariant* provided that dim $D^\perp=1$. Then a unit normal vector field in $FD^\perp$ is called the *distinguished normal* to the semi-invariant submanifold. Putting $N^A=\lambda C^A+\mu D^A+\nu E^A$, we can see that

$$\begin{aligned} F_B{}^A B_i{}^B&=f_i{}^h B_h{}^A+P_i N^A \\ F_B{}^A N^B&=-P^h B_h{}^A \end{aligned} \tag{2.1}$$

and that N^A is an intrinsically defined unit normal to M and $\lambda^2+\mu^2+\nu^2=1$ [11]. Moreover the set (f, g, P) admits an almost contact metric structure.

Now suppose that the condition $\lambda^2+\mu^2+\nu^2=1$ is satisfied and take $N^A=\lambda C^A+\mu D^A+\nu E^A$ as C^A. Then we have $\lambda=1$, $\mu=0$, $\nu=0$ and consequently $u^h=P^h$, $v_i=0$, $w_i=0$ because of (1.13) and (1.16). Thus (1.6)-(1.9) reduce respectively to

$$F_B{}^A B_i{}^B=f_i{}^h B_h{}^A+P_i C^A, \tag{2.3}$$

(2.4) $$F_B{}^A C^B = -P^h B_h{}^A,$$

(2.5) $$F_B{}^A D^B = -E^A,$$

(2.6) $$F_B{}^A E^B = D^A.$$

Now denoting by ∇_j the operator of van der Waerden-Bortolotti covariant differentiation with respect to g_{ji}, we have equations of Gauss for M of $\overline{M}$

(2.7) $$\nabla_j B_i{}^A = h_{ji} C^A + k_{ji} D^A + l_{ji} E^A,$$

where h_{ji}, k_{ji}, l_{ji} are the second fundamental tensors with respect to normals C^A, D^A, E^A respectively. The mean curvature vector H^A is given by

(2.8) $$H^A = \frac{1}{2n+1}(hC^A + kD^A + lE^A),$$

where we have put

$$h = g^{ji} h_{ji}, \quad k = g^{ji} k_{ji}, \quad l = g^{ji} l_{ji},$$

g^{ji} being contravariant components of the metric tensor.

The equations of Weingarten are given by

(2.9) $$\nabla_j C^A = -h_j{}^h B_h{}^A + l_j D^A + m_j E^A,$$

(2.10) $$\nabla_j D^A = -k_j{}^h B_h{}^A - l_j C^A + n_j E^A,$$

(2.11) $$\nabla_j E^A = -l_j{}^h B_h{}^A - m_j C^A - n_j D^A,$$

where $h_j{}^h = h_{jt} g^{th}$, $k_j{}^h = k_{jt} g^{th}$, $l_j{}^h = l_{jt} g^{th}$, l_j, m_j and n_j being the third funda mental tensors.

We now assume that $\overline{M}$ is Kaehlerian and differentiate (2.3) covariantly along M and make use of (2.4)-(2.6), we can find

(2.12) $$\nabla_j f_i{}^h = -h_{ji} P^h + h_j{}^h P_i, \qquad \nabla_j P_i = -h_{jt} f_i{}^t,$$

(2.13) $$k_{ji} = -l_{jt} f_i{}^t - m_j P_i, \qquad l_{ji} = k_{jt} f_i{}^t + l_j P_i.$$

From (2.13), we have

(2.14) $$k_{jt} P^t = -m_j, \quad l_{jt} P^t = l_j, \quad k = -m_t P^t, \quad l = l_t P^t.$$

From (2.12)-(2.14), using (1.17)-(1.19) and (2.12)-(2.14), it follows that

(2.15) $$l_t f_i{}^t = k P_i + m_i,$$

(2.16) $$kl + m_t l^t = 0,$$

(2.17) $$k_{jt} l_i{}^t + k_{it} l_j{}^t = -(l_i m_j + m_i l_j),$$

(2.18) $$l_{jt} l_i{}^t - k_{jt} k_i{}^t = l_j l_i - m_j m_i.$$

§ 3. Semi-invariant submanifolds of codimension 3 with harmonic curvature of E^{2n+4}.

Let M be a $(2n+1)$-dimensional semi-invariant submanifold of codimension 3 of an even-dimensional Euclidean space E^{2n+4}. Then equations of Gauss are given by

$$R_{kji}{}^{h}=h_{k}{}^{h}h_{ji}-h_{j}{}^{h}h_{ki}+k_{k}{}^{h}k_{ji}-k_{j}{}^{h}k_{ki}+l_{k}{}^{h}l_{ji}-l_{j}{}^{h}l_{ki}, \tag{3.1}$$

where $R_{kji}{}^{h}$ is the Riemannian curvature tensor of M, those of Codazzi by

$$\nabla_{k}h_{ji}-\nabla_{j}h_{ki}-l_{k}k_{ji}+l_{j}k_{ki}-m_{k}l_{ji}+m_{j}l_{ki}=0, \tag{3.2}$$

$$\nabla_{k}k_{ji}-\nabla_{j}k_{ki}+l_{k}h_{ji}-l_{j}h_{ki}-n_{k}l_{ji}+n_{j}l_{ki}=0, \tag{3.3}$$

$$\nabla_{k}l_{ji}-\nabla_{j}l_{ki}+m_{k}h_{ji}-m_{j}h_{ki}+n_{k}k_{ji}+n_{j}k_{ki}=0, \tag{3.4}$$

and those of Ricci by

$$\nabla_{k}l_{j}-\nabla_{j}l_{k}+h_{k}{}^{t}k_{jt}-h_{j}{}^{t}k_{kt}+m_{k}n_{j}-m_{j}n_{k}=0, \tag{3.5}$$

$$\nabla_{k}m_{j}-\nabla_{j}m_{k}+h_{k}{}^{t}l_{jt}-h_{j}{}^{t}l_{kt}+n_{k}l_{j}-n_{j}l_{k}=0, \tag{3.6}$$

$$\nabla_{k}n_{j}-\nabla_{j}n_{k}+k_{k}{}^{t}l_{jt}-k_{j}{}^{t}l_{kt}+l_{k}m_{j}-l_{j}m_{k}=0. \tag{3.7}$$

Now, we denote the normal components of $\nabla_{j}C$ by $\nabla_{j}^{\perp}C$. The normal vector field C is said to be *parallel* in the normal bundle if $\nabla_{j}^{\perp}C=0$, that is, l_{j} and m_{j} vanish identically.

Throughout this paper we assume that the normal vector field C is parallel in the normal bundle and we denote

$$\begin{aligned} \acute{\nabla}_{k}h_{ji}&=\nabla_{k}h_{ji}, \\ \acute{\nabla}_{k}k_{ji}&=\nabla_{k}k_{ji}-n_{k}l_{ji}, \\ \acute{\nabla}_{k}l_{ji}&=\nabla_{k}l_{ji}+n_{k}k_{ji}. \end{aligned} \tag{3.8}$$

Then we have

$$\acute{\nabla}_{k}h_{ji}{}^{x}=\acute{\nabla}_{j}h_{ki}{}^{x}, \tag{3.9}$$

where $h_{ji}{}^{1}=h_{ji}$, $h_{ji}{}^{2}=k_{ji}$ and $h_{ji}{}^{3}=l_{ji}$.

Differentiating (2.17) and (2.18) covariantly and using $l_{j}=0$, $m_{j}=0$, (3.8) and (3.9), we have

$$k_{jt}(\nabla_{k}l_{i}{}^{t})+l_{jt}(\nabla_{k}k_{i}{}^{t})=0, \qquad k_{jt}(\acute{\nabla}_{k}l_{i}{}^{t})+l_{jt}(\acute{\nabla}_{k}k_{i}{}^{t})=0 \tag{3.10}$$

and

$$k_{jt}(\nabla_{i}k_{k}{}^{t})=l_{jt}(\nabla_{i}l_{k}{}^{t}), \qquad k_{jt}(\acute{\nabla}_{i}k_{k}{}^{t})=l_{jt}(\acute{\nabla}_{i}l_{k}{}^{t}) \tag{3.11}$$

respectively.

In the sequel we assume that the submanifold M with harmonic curvature has constant mean curvature, that is,

$$\nabla_k R_{ji} - \nabla_j R_{ki} = 0, \tag{3.12}$$

and $\|H\|^2 := C_{AB}H^A H^B$ is constant which together with $k=0$ and $l=0$ implies

$$\nabla_k h = 0. \tag{3.13}$$

From Gauss and Codazzi equations and the definition of harmonic curvature it follows that

$$(\nabla_k h_{it})h_j{}^t - (\nabla_j h_{it})h_k{}^t + 2\{(\acute{\nabla}_k k_{it})k_j{}^t - (\acute{\nabla}_j k_{it})k_k{}^t\} = 0,$$

that is,

$$\sum_{x=1}^{3} (\acute{\nabla}_k h_{jt}{}^x) h_i{}^{tx} = \sum_{x=1}^{3} (\acute{\nabla}_k h_{it}{}^x) h_j{}^{tx}, \tag{3.14}$$

because of (3.9) and (3.11). By the Ricci equations (3.5) and (3.6), and $\nabla_j^{\perp} C = 0$, we have

$$h_{jt} h_i{}^{tx} = h_{it} h_j{}^{tx}, \tag{3.15}$$

where $x = 1, 2, 3$. Differentiating (3.15) covariantly and using (3.8), we find

$$(\acute{\nabla}_k h_{it}) h_j{}^{tx} + (\acute{\nabla}_k h_{jt}{}^x) h_i{}^t = (\acute{\nabla}_k h_{jt}) h_i{}^{tx} + (\acute{\nabla}_k h_{it}{}^x) h_j{}^t. \tag{3.16}$$

Transvecting (3.16) with $h_s{}^{jx}$, we have

$$\begin{aligned} &\Sigma_x\{(\acute{\nabla}_k h_{it}) h_s{}^{tx} h_j{}^{sx} - (\acute{\nabla}_k h_{st}) h_i{}^{tx} h_j{}^{sx}\} \\ &= \Sigma_x\{(\acute{\nabla}_k h_{it}{}^x) h_s{}^t h_j{}^{sx} - (\acute{\nabla}_k h_{st}{}^x) h_i{}^t h_j{}^{sx}\}. \end{aligned} \tag{3.17}$$

By the properties (3.14) and (3.15), we have

$$\Sigma_x (\acute{\nabla}_k h_{st}{}^x) h_i{}^t h_j{}^{sx} = \Sigma_x (\acute{\nabla}_k h_{js}{}^x) h_t{}^s h_i{}^{tx}.$$

Transvecting (3.17) with $\nabla_k h_{ij}$ and using this equation, we have

$$\Sigma_x (\acute{\nabla}_k h_{ij})(\acute{\nabla}^k h_{ti}) h_s{}^{tx} h^{jsx} = \Sigma_x (\acute{\nabla}_k h_{ij})(\acute{\nabla}^k h_{st}) h^{itx} h^{jsx}. \tag{3.18}$$

On the other hand, for fixed indices k and x $(\acute{\nabla}_k h_{it}) h_j{}^{tx} - (\acute{\nabla}_k h_{jt}) h_i{}^{tx}$ can be regarded as a square matrix of order $2n+1$. By (3.18) the norm of this matrix with respect to the usual inner product vanishes identically, which implies

$$(\acute{\nabla}_k h_{jt}) h_i{}^{tx} = (\acute{\nabla}_k h_{it}) h_j{}^{tx}. \tag{3.19}$$

The equations (3.16) and (3.19) show

$$(\acute{\nabla}_k h_{jt}{}^x) h_i{}^t = (\acute{\nabla}_k h_{it}{}^x) h_j{}^t \tag{3.20}$$

for any indices x, i, j and k.

Differentiating the first equation of (2.13) and using $m_j = 0$, (2.12), (2.17), (3.8), (3.14) and (3.19), we have

(3.21) $$h_{jt}k_i{}^t=0, \qquad h_{jt}l_i{}^t=0.$$

From (2.18), (3.14) and (3.19), we find

(3.22) $$(\acute{\nabla}_k k_{jt})k_i{}^t=(\acute{\nabla}_k k_{it})k_j{}^t.$$

Differentiating (3.22) covariantly and taking the skew-symmetric part and using (3.7), (3.8), (3.10) and the Ricci identity, we obtain

$$(R_{lkjs}k_t{}^s+R_{lkts}k_j{}^s)k_i{}^t-(R_{lkis}k_t{}^s+R_{lkts}k_i{}^s)k_j{}^t$$
$$=4k_{ks}l_l{}^s k_{jt}l_i{}^t+2\{(\acute{\nabla}_t k_{kj})(\acute{\nabla}^t k_{li})-(\acute{\nabla}_t k_{ki})(\acute{\nabla}^t k_{lj})\}$$

from which, transvecting this with g^{ki} and using (2.17), (2.18), (3.1) and $k_3=0$,

(3.23) $$(\acute{\nabla}_s k_{jt})(\acute{\nabla}^s k_i{}^t)=4(k_{ji})^4+k_2(k_{ji})^2,$$

where $k_2=k_{st}k^{st}$, $k_3=k_{sr}k_t{}^r k^{ts}$, $(k_{ji})^2=k_{jt}k_i{}^t$ and $(k_{ji})^4=k_j{}^t k_t{}^s k_s{}^r k_{ir}$.

From (3.22), using (3.9), we find

(3.24) $$k_j{}^t(\acute{\nabla}_k k_{it})=k_k{}^t(\acute{\nabla}_t k_{ji}).$$

Transvecting (3.24) with $(k_{ji})^2$, using $k_3=0$, we have

$$(k_{ji})^3(\nabla_k k^{ji})=0.$$

If we put $k_4=(k_{ji})^3k^{ji}$, then $\nabla_k k_4=4(k_{ji})^3(\nabla_k k^{ji})$. Hence we have

(3.25) $$\nabla_k k_4=0,$$

that is, k_4 is a constant.

Next, from the equation (3.19), we have

$$(\nabla_k h_{jt})h_i{}^t=(\nabla_k h_{it})h_j{}^t,$$

from which,

$$\nabla_k(h_{ji})^2-\nabla_j(h_{ki})^2=0,$$

namely, $(h_{ji})^2$ is of Codazzi type. Since the mean curvature is constant, we can easily see that

(3.26) $$\nabla_k h_{ji}=0$$

(for detail, see [10]).

On the other hand, from (3.1), we have

$$R_{ji}=hh_{ji}-(h_{ji})^2-2(k_{ji})^2$$

from which,

$$(R_{ji})^2=h^2(h_{ji})^2-2h(h_{ji})^3+(h_{ji})^4+4(k_{ji})^4.$$

Hence we have

(3.27) $$R_2=h^2h_2-2hh_3+h_4+4k_4$$

is constant, because of (3.13), (3.25) and (3.26). And, using the Ricci identity and (3.26), we find

(3.28) $$h(h_{ji})^2 - h_2 h_{ji} = 0.$$

Furthermore, From the Ricci identity, (3.1) and (3.3), we have

(3.29) $$\Delta R_{ji} = h_3 h_{ji} - h(h_{ji})^3.$$

§4. Proof of Theorem.

Let M be a semi-invariant submanifold with harmonic curvature of codimension 3 of an even-dimensional Euclidean space E^{2n+4} such that the distinguished normal C^4 is parallel in the normal bundle. If the submanifold M has contant mean curvature, then we can consider two cases.

Case I: $h=0$

From (3.28), we have

(4.1) $$h_{ji} = 0,$$

from which, using (3.29)

(4.2) $$\Delta R_{ji} = 0.$$

Hence we have

(4.3) $$\nabla_k R_{ji} = 0,$$

because of (3.27). Since $R_{ji} = -2(k_{ji})^2$, using (2.17), (3.8) and (4.3), we have

(4.4) $$k_{jt}(\acute{\nabla}_k k_i{}^t) = 0.$$

From (3.23) and (4.4), we find

$$4(k_{ji})^6 + k_2(k_{ji})^4 = 0,$$

from which

$$k_{ji} = 0, \qquad l_{ji} = 0$$

because of (2.18).

Case II: $h \neq 0$

From (3.28), we have

(4.5) $$(h_{ji})^2 = \lambda h_{ji},$$

where $\lambda = h_2/h$. Substituting (4.5) into (3.29), we have

(4.6) $$\Delta R_{ji} = 0.$$

Hence we have

(4.7) $$\nabla_k R_{ji} = 0,$$

because of (3.27). Since $R_{ji}=hh_{ji}-(h_{ji})^2-2(k_{ji})^2$, using (2.17), (3.8), (3.13) and (4.7), we have

$$k_{jt}(\acute{\nabla}_k k_i{}^t)=0. \tag{4.8}$$

From (3.23) and (4.8), we have

$$4(k_{ji})^6+k_2(k_{ji})^4=0.$$

from which

$$k_{ji}=0, \qquad l_{ji}=0$$

because of (2.18).

Thus we have

LEMMA. *Let M be a semi-invariant submanifold of codimension 3 in E^{2n+4}. If M has harmonic curvature and of constant mean curvature and if the distinguished normal is parallel in the normal bundle, then*

$$(h_{ji})^2=ah_{ji}, \quad k_{ji}=0, \quad l_{ji}=0,$$

where a is constant.

PROOF OF THEOREM.

Let N_x^1 is the first normal space of M for each x in M and is the second fundamental form of M, that is, $N_x^1=\{\alpha(u, v);\ u, v\in N_x\}$, where $T_xE^{2n+4}=M_x\oplus N_x$ and $N_x=\{\xi;\xi\in T_xE^{2n+4}, \xi\perp M_x\}$. If $a=0$, M is totally geodesic and consequently $M=E^{2n+1}$. Next we consider the case of $a\neq0$. In this case, the above lemma yields dim $N_x^1=1$ for each x in M. Moreover the distribution $N^1=\cup_x N_x^1\subset N(M)$ is parallel. Accordingly, a theorem due to J. Erbacher [2], for the reduction of the codimension implies that there exists a $(2n+2)$-dimensional totally geodesic submanifold E^{2n+2} in E^{2n+4} in which M is the hypersurface with parallel second fundamental form. Since M is complete and simply connected, by [8], we have results in Theorem.

Bibliography

[1] Derdziński, A., Compact Riemannian manifolds with harmonic curvature and non-parallel Ricci tensor, Global Differential Geometry and Global Analysis, Lecture notes in Math., Springer, **838** (1979), 126-128.

[2] Erbacher, J., Reduction of the codimension of an isometric immersion, J. Differential Geometry, **5** (1971), 333-340.

[3] Ki, U-H., Eum, S.S., Kim, U.K. and Kim, U.H., Submanifolds of codimension 3 of Kaehlerian manifold (I), J. Korean Math. Soc., **16-2** (1980), 137-153.

[4] Ki, U-H. and Nakagawa, H., Submanifolds with harmonic curvature, Tsukuba J. of Math. **10-2** (1986), 43-50.

[5] Ki, U-H. and Nakagawa, H., Totally real submanifolds with harmonic curvature, to

appear in Kyungpook Math. J.
[6] Ki, U-H., Nakagawa, H. and Umehara, M., On complete hypersurfaces with harmonic curvature, Tsukuba J. of Math., **11** (1987), 61-76.
[7] Ki, U-H. and Pak, J.S., Generic submanifolds of an even-dimensional Euclidean space, J. Diff. Geom., **16** (1981), 293-303.
[8] Nomizu, K. and Smyth, B., A formula of Simons' type and hypersurfaces with constant mean curvature, J. Differential Geometry, **3** (1969), 367-377.
[9] Ômachi, E., Hypersurfaces with harmonic curvature in a space of constant curvature, Kodai Math. J. **9** (1986), 170-174.
[10] Umehara, M., Hypersurfaces with harmonic curvature, Tsukuba J. of Math., **10** (1986), 79-88.
[11] Yano, K. and Ki, U-H., On $(f,g,u,v,w,\lambda,\mu,\nu)$-structure satisfying $\lambda^2+\mu^2+\nu^2=1$, Kōdai Math. Sem. Rep., **29** (1978), 285-307.

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