

A REMARK ON R. POL'S THEOREM CONCERNING A-WEAKLY INFINITE-DIMENSIONAL SPACES

By

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For notations and relevant definitions we refer to [1].

THEOREM (MA). *There is no universal space in the class of all metrizable separable A-weakly infinite-dimensional spaces.*

R. Pol proved this theorem in [1] under *CH*. The proof we shall give is similar with the one given in [1] but a little more direct.

LEMMA 1. *Let $S \subset I^\omega$ be a countable union of zero-dimensional subsets. If $C \subset I^\omega$ satisfies that for any open neighbourhood U of S $|C \setminus U| < \mathfrak{c}$, then $C \cup S$ is A-weakly infinite-dimensional.*

The proof is parallel to the proof of Lemma 1 in [1], noting that in I^ω every subset with cardinality less than \mathfrak{c} is zero-dimensional.

LEMMA 2 (MA). *Let $\{G_\alpha : \alpha < \lambda\}$ be a family of open neighbourhoods of Σ in I^ω and $\lambda < \mathfrak{c}$, where $\Sigma = \{x \in I^\omega : \text{all but finitely many coordinates of } x \text{ are equal to zero}\}$. Then there exist positive numbers $a_i \in I (i \in \omega)$ such that $\bigcup_i [0, a_i] \subset \bigcap \{G_\alpha : \alpha < \lambda\}$. Therefore, if $E \subset I^\omega$ can be embedded in an A-weakly infinite-dimensional space, then $\bigcap \{G_\alpha : \alpha < \lambda\} \setminus E \neq \emptyset$.*

PROOF. Let $\mathcal{B} = \{[0, 1/n] : n > 0\}$. We define $\mathbf{P} = \{(a, b) : a \text{ is a finite sequence in } \mathcal{B} \text{ \& } b \in [\lambda]^{<\omega}\}$ and for any $(a', b'), (a, b) \in \mathbf{P}$, where $a = (I_0, I_1, \dots, I_n)$ and $a' = (I'_0, I'_1, \dots, I'_{n'})$, $(a', b') \leq (a, b)$ iff $b' \supset b$, $n \leq n'$, $I_i = I'_i$ for any $i \leq n$ and if $n < n'$, $\prod_{i \leq n'} I'_i \times \prod_{i > n'} I \subset \bigcap \{G_\alpha : \alpha \in b\}$. It is obvious that \leq is a partial order on \mathbf{P} . Since all of first components of elements of \mathbf{P} are countable, \mathbf{P} is *ccc* (in fact σ -centred).

Let $D_\alpha = \{(a, b) \in \mathbf{P} : \alpha \in b\}$ and $F_n = \{(a, b) : \text{the length of } a \text{ is larger than } n\}$. It is easily seen that D_α is dense in \mathbf{P} for any $\alpha < \lambda$. Now we want to

show that F_n is dense for any $n \in \omega$. Take any $(a, b) \in \mathbf{P}$. If the length of a is larger than n , then $(a, b) \in F_n$. So we suppose that $a = (I_0, I_1, \dots, I_m)$, where $m < n$. Since $\bigcap \{G_\alpha : \alpha \in b\}$ is an open neighbourhood of Σ , we can find $a' = (I_0, \dots, I_m, I_{m+1}, \dots, I_n)$ such that $\prod_{i \leq n} I_i \times \prod_{i > n} I_i \subset \bigcap \{G_\alpha : \alpha \in b\}$. Therefore, $(a', b) \leq (a, b)$ and $(a', b) \in F_n$.

By MA, we have a filter \mathbf{G} in \mathbf{P} such $\mathbf{G} \cap D_\alpha \neq \emptyset$ $\mathbf{G} \cap F_n \neq \emptyset$ for any $\alpha < \lambda$ and $n < \omega$. Let $\bigcup \{a : \text{there is a } (a, b) \in \mathbf{G}\} = \{I_n : n \in \omega\}$. Then $\prod_{n < \omega} I_n \subset \bigcap \{G_\alpha : \alpha < \lambda\}$.

PROOF OF THEOREM. Let $E \subset I^\omega$ be any A -weakly infinite-dimensional space. Let $\{(H_\alpha, h_\alpha) : \alpha < c\}$ be the family of all pairs such that H_α is a G_δ -set in I^ω containing Σ and $h_\alpha : H_\alpha \rightarrow I^\omega$ is an embedding which maps Σ onto a subset of E . Let $\{G_\alpha : \alpha < c\}$ be all of the open sets which contain Σ . Take $x_\alpha \in \bigcap \{G_\beta : \beta \leq \alpha\} \setminus h_\alpha^{-1}(E)$. Then by an argument paralleled to the one in the end of [1], we have $M = \Sigma \cup \{x_\alpha : \alpha < c\}$ can not be embedded in E .

REMARK 3. It is easily seen from the proof of Lemma 2 that the theorem is true under $\text{MA}_{\sigma\text{-centred}}$, i. e. $p=c$, which is strictly weaker than MA.

The author wishes to express his gratitude to Professor Y. Kodama and all of the members in his seminar for their stimulating discussions.

References

- [1] Pol, R., A remark on A -weakly infinite-dimensional spaces, *Topology and its applications* 13 (1982), 97-101.

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