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REPRESENTATION OF NEAR-RING MORITA CONTEXTS AND RECOGNIZING MORITA NEAR-RINGS

By

Shoji KYUNO and Stefan VELDSMAN

Abstract. Subject to certain faithfulness requirements in a morita context for near-rings, a canonical representation thereof is provided. Necessary and sufficient conditions (using an idempotent element) on a near-ring are given which determine when the near-ring is a morita near-ring.

1. Introduction and preliminaries

In [2] we defined a morita context $\Gamma=(L,G,H,R)$ for near-rings as well as the associated morita near-ring $M_2(\Gamma)$. The examples provided in [3] probably best motivates the reason for defining and investigating these concepts for near-rings (for the ring case, they stood the test of time, see for example Amitsur [1] or Rowen [4]). It is a generalization of one of these examples, which also appeared in [2], in which we are interested here. In fact, in Section 2 we show, subject to some mild faithfulness requirements, that every morita context for near-rings can be embedded in a context of this type.

In the next section we give necessary and sufficient conditions on a near-ring to ensure that it is a morita near-ring. As is usual with matrices or matrix-like structures, this involves idempotents. Firstly we recall some relevant definitions and results from [2]:

All near-rings considered will be right distributive and 0-symmetric. Let R and L be near-rings and let G be a group. G is a *left L-module* if there is a mapping $L\times G\rightarrow G,(x,g)\mapsto xg$ such that $(x_1+x_2)g=x_1g+x_2g$ and $(x_1x_2)g=x_1(x_2)g$ for all $x,x_1,x_2\in L$ and $g\in G$. G is a *right R-module* if there is a mapping $G\times R\rightarrow G,(gr)\mapsto gr$ such that $(g_1+g_2)r=g_1r+g_2r$ and $(gr_1)r_2=g(r_1r_2)$ for all $g,g_1,g_2\in G,r,r_1,r_2\in R$. G is an *L-R-bimodule* if it is both a left L-module and a right R-module for which $(xg)r=x(gr)$ for all $x\in L,g\in G,r\in R$. Strictly speaking we should talk about, for example, a left near-ring L-module G, for even if L is a ring, G is not necessarily a left ring L-

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module. A normal subgroup K of G, G an L-R-bimodule, is an *ideal* of G if $x(g+k)-xg \in K$ and $kr \in K$ for all $x \in L, g \in G, k \in K$, and $r \in R$.

For each $i,j \in N_2 := \{1,2\}$, let Γ_{ij} be a group. The quadruple $\Gamma = (\Gamma_{11}, \Gamma_{12}, \Gamma_{21}, \Gamma_{22})$ is a *near-ring morita context* if for every $i,j,k \in N_2$, there is a function

$$\Gamma_{jk} \times \Gamma_{ki} \to \Gamma_{ji},\ (x,y) \mapsto xy,$$

which satisfies $(a+b)c = ac+bc$ and $(db)e = d(be)$ for all $a,b \in \Gamma_{jk}, c \in \Gamma_{ki}, d \in \Gamma_{ij}$ and $e \in \Gamma_{km}$ where $i,j,k,m \in N_2$.

It is clear that if $\Gamma = (\Gamma_{11}, \Gamma_{12}, \Gamma_{21}, \Gamma_{22})$ is a morita context, then so is $(\Gamma_{22}, \Gamma_{21}, \Gamma_{12}, \Gamma_{11})$; the one being called the dual of the other. For $\Delta_{ij} \subseteq \Gamma_{ij}$ and $\Delta_{jk} \subseteq \Gamma_{jk}$, we define

$$\Delta_{ij}\Delta_{jk} := \{xy \mid x \in \Delta_{ij}, y \in \Delta_{jk}\}$$

and

$$\Delta_{ij} * \Delta_{jk} := \{x(z+y)-xz \mid x \in \Delta_{ij}, y \in \Delta_{jk}, z \in \Gamma_{jk}\}.$$

When necessary, the additive identity of the group Γ_{ij} will be denoted by 0_{ij}, otherwise we just write 0. Since the near-rings Γ_{11} and Γ_{22} are 0-symmetric, $x0_{jk} = 0_{ik}$ for all $x \in \Gamma_{ij}$, for all $i,j,k \in N_2$.

For each $i,j \in N_2$ let $\Delta_{ij} \subseteq \Gamma_{ij}$. The quadruple $\Delta = (\Delta_{11}, \Delta_{12}, \Delta_{21}, \Delta_{22})$ is an *ideal of the morita context* $\Gamma = (\Gamma_{11}, \Gamma_{12}, \Gamma_{21}, \Gamma_{22})$ if each Δ_{ij} is a normal subgroup of $\Gamma_{ij}, \Delta_{ij}\,\Gamma_{jk} \subseteq \Delta_{ik}$ and $\Gamma_{ki} * \Delta_{ij} \subseteq \Delta_{jk}$ for all $i,j,k \in N_2$. In this case we get the quotient morita context

$$\Gamma/\Delta = (\Gamma_{11}/\Delta_{11}, \Gamma_{12}/\Delta_{12}, \Gamma_{21}/\Delta_{21}, \Gamma_{22}/\Delta_{22})$$

where the relevant maps are defined as is usual in the universal algebra:

$$\Gamma_{ij}/\Delta_{ij} \times \Gamma_{jk}/\Delta_{jk} \to \Gamma_{ik}/\Delta_{ik}$$

$$(x+\Delta_{ij}, y+\Delta_{jk}) \mapsto (x+\Delta_{ij})(y+\Delta_{jk}) := xy+\Delta_{ik}.$$

Let Γ and Γ' be two morita contexts. A *morita context homomorphism* from Γ to Γ' is a quadruple $\alpha = (\alpha_{11}, \alpha_{12}, \alpha_{21}, \alpha_{22})$ such that each $\alpha_{ij} : \Gamma_{ij} \to \Gamma'_{ij}$ is a group homomorphism for which $\alpha_{kj}(xy) = \alpha_{ki}(x)\alpha_{ij}(y)$ for $x \in \Gamma_{ki}, y \in \Gamma_{ij}, i,j,k \in N_2$. We say α is an *embedding* (or *injective*) if each α_{ij} is injective and is *surjective* if each α_{ij} is surjective. As usual, if α is both injective and surjective, it is called an *isomorphism*. The *kernel* of α, $\ker\alpha$, is defined by $\ker\alpha = (\ker\alpha_{11}, \ker\alpha_{12}, \ker\alpha_{21}, \ker\alpha_{22})$. It is clear that $\ker\alpha$ is an ideal of the morita context Γ.

For a morita context $\Gamma = (\Gamma_{11}, \Gamma_{12}, \Gamma_{21}, \Gamma_{22})$, the associated morita near-ring $M_2(\Gamma)$ is the subnear-ring of $M_0(\Gamma^+) := \{f : \Gamma^+ \to \Gamma^+ \mid f(0) = 0\}, \Gamma^+$ is the matrix group $\Gamma^+ = \begin{bmatrix} \Gamma_{11} \Gamma_{12} \\ \Gamma_{21} \Gamma_{22} \end{bmatrix}$, generated by the functions

$$s_{ij}^{x} : \Gamma^{+} \to \Gamma^{+}, s_{ij}^{x} \begin{bmatrix} a_{11}\, a_{12} \\ a_{21}\, a_{22} \end{bmatrix} = \begin{bmatrix} b_{11}\, b_{12} \\ b_{21}\, b_{22} \end{bmatrix}$$

where $b_{i1} = xa_{j1}, b_{i2} = xa_{j2}, b_{i_c 1} = 0 = b_{i_c 2}$ (i_c denotes the complement of i in N_2), $x \in \Gamma_{ij}$. For later reference, we recall some useful facilities for doing calculations in $M_2(\Gamma)$:

PROPOSITION 1.1 [2].

(1) $s_{ij}^{x} + s_{ij}^{y} = s_{ij}^{x+y}$

(2) $s_{ij}^{x} + s_{km}^{y} = s_{km}^{y} + s_{ij}^{x}$ if $i \neq k$

(3) $s_{ij}^{x} s_{km}^{y} = \begin{cases} s_{im}^{xy} & \text{if } j = k \\ 0 & \text{if } j \neq k \end{cases}$ *(here, of course,* $0 = s_{ij}^{0} = s_{km}^{0}$*)*

(4) $s_{ij}^{x}(s_{1k_1}^{y_1} + s_{2k_2}^{y_2}) = s_{ik_j}^{xy_j}$

(5) *For any* $U \in M_2(\Gamma)$, $U\begin{bmatrix} a_{11}\, a_{12} \\ a_{21}\, a_{22} \end{bmatrix} = U\begin{bmatrix} a_{11}\, 0 \\ a_{21}\, 0 \end{bmatrix} + U\begin{bmatrix} 0\;\, a_{12} \\ 0\;\, a_{22} \end{bmatrix}$

(6) *For any* $U, V \in M_2(\Gamma)$, $U\begin{bmatrix} a\; 0 \\ b\; 0 \end{bmatrix} + V\begin{bmatrix} 0\; c \\ 0\; d \end{bmatrix} = V\begin{bmatrix} 0\; c \\ 0\; d \end{bmatrix} + U\begin{bmatrix} a\; 0 \\ b\; 0 \end{bmatrix}$

(7) *For* $k \in N_2, C_k := \{s_{1k}^{x_1} + s_{2k}^{x_2} \mid x_i \in \Gamma_{ik}\}$ *is a left invariant subgroup of* $M_2(\Gamma)$

(8) *For* $U \in M_2(\Gamma), U\begin{bmatrix} a_{11}\, a_{12} \\ a_{21}\, a_{22} \end{bmatrix} = \begin{bmatrix} b_{11}\, b_{12} \\ b_{21}\, b_{22} \end{bmatrix}$ *if and only if* $U(s_{1i}^{a_{1i}} + s_{2i}^{a_{2i}}) = s_{1i}^{b_{1i}} + s_{2i}^{b_{2i}}$ *for* $i = 1, 2$. ■

For $U \in M_2(\Gamma)$, it is possible that U may be expressed in more than one way as a combination of a finite number of sums and products of the functions s_{ij}^{x}. The *weight* of U, written as $w(U)$, is the minimum number of s_{ij}^{x}'s which can appear in a representation of U.

2. Representation of a morita context

For a near-ring morita context $\Gamma = (L, G, H, R)$, G is a right R-module. Let

$$M_R(G) := \{f : G \to G \mid f(gr) = f(g)r \text{ for all } g \in G, r \in R\}$$

and

$$M_R(G, R) := \{f : G \to R \mid f(gr) = f(g)r \text{ for all } g \in G, r \in R\}.$$

Both these sets of functions are groups with respect to pointwise addition. The former is in fact a 0-symmetric near-ring with identity. As in [2], Example

1.2(3),

$$\Gamma^{\#} := (M_R(G), G, M_R(G,R), R)$$

is a morita context for near-rings with respect to:

$M_R(G) \times G \to G, (f,g) \mapsto fg := f(g)$

$R \times M_R(G,R) \to M_R(G,R), (r,f) \mapsto rf : G \to R, (rf)(g) := rf(g)$

$M_R(G,R) \times M_R(G) \to M_R(G,R), (f,f') \mapsto ff' := f \circ f'$

$G \times M_R(G,R) \to M_R(G), (g,f) \mapsto gf : G \to G, (gf)(g') := gf(g')$ and

$M_R(G,R) \times G \to R, (f,g) \mapsto fg := f(g)$.

There are natural maps $\alpha_{11} : L \to M_R(G)$ and $\alpha_{21} : H \to M_R(G,R)$ given by

$$\alpha_{11}(x) = \alpha_{11}^x : G \to G, \alpha_{11}^x(g) := xg \text{ and}$$

$$\alpha_{21}(h) = \alpha_{21}^h : G \to R, \alpha_{21}^h(g) := hg$$

with

$$ker\, \alpha_{11} = (0:G)_L := \{x \in L \mid xG = 0\} \text{ and}$$

$$ker\, \alpha_{21} = (0:G)_H := \{h \in H \mid hG = 0\}.$$

If we let $\alpha_{12} : G \to G$ and $\alpha_{22} : R \to R$ be the identity mappings, then $\alpha = (\alpha_{11}, \alpha_{12}, \alpha_{21}, \alpha_{22}) : \Gamma \to \Gamma^{\#}$ is a morita context homomorphism. Hence we have

PROPOSITION 2.1. *$\alpha : \Gamma \to \Gamma^{\#}$ is an embedding if and only if $(0:G)_L = 0$ and $(0:G)_H = 0$.* ■

PROPOSITION 2.2. *$\alpha : \Gamma \to \Gamma^{\#}$ is an isomorphism if and only if the following conditions are satisfied:*

(i) *L has an identity*

(ii) *$(0:G)_L = 0$ and $(0:G)_H = 0$*

(iii) *For every $f \in M_R(G,R)$, there is an $h \in H$ (depending on f) such that $hg = f(g)$ for all $g \in G$.*

(iv) *For every $f \in M_R(G)$, there is an $x \in L$ (depending on f) such that $xg = f(g)$ for all $g \in G$.*

PROOF. If α is an isomorphism, then $\alpha_{11} : L \to M_R(G)$ is an isomorphism. Since $M_R(G)$ has an identity, so does L. The remainder of the proof follows from Proposition 2.1 and the fact that $\alpha_{11} : L \to M_R(G)$ is surjective iff for every $f \in M_R(G)$ there is an $x \in L$ such that $\alpha_{11}^x = f$, i.e. $xg = f(g)$ for all $g \in G$. A similar argument takes care of (iii). ■

The conditions in Proposition 2.2 can be realized if, for example , L has an identity, the right (respt. left) L-module H (respt. G) is unital and $L = GH := \{gh \mid g \in G, h \in H\}$.Indeed, if 1 is the identity of L, then $1 = g_0 h_0$ for some $g_0 \in G, h_0 \in H$. If $xG = 0$ $(x \in L)$, then $x = x1 = (xg_0)h_0 = 0$; hence $(0:G)_L = 0$. If $hG = 0 (h \in H)$, then $h = h1 = (hg_0)h_0 = 0$ and thus $(0:G)_H = 0$. For $f \in M_R(G,R)$, let $f(g_0) = r_0$.Then $x := r_0 h_0 \in H$ and for every $g \in G, f(g) = f(1g) = f(g_0(h_0 g)) = f(g_0)(h_0 g) = r_0(h_0 g) = (r_0 h_0)g = xg$. A similar argument shows that (iv) is also satisfied.

Not every morita context may have the faithfulness required in Proposition 2.1, but it has at least a homomorphic image which does. For the morita context $\Gamma = (L, G, H, R) = (\Gamma_{11}, \Gamma_{12}, \Gamma_{21}, \Gamma_{22})$ let $\Delta_{11} = (0:G)_L$, $\Delta_{12} = 0$, $\Delta_{21} = (0:G)_H$ and $\Delta_{22} = 0$. Then $\Delta = (\Delta_{11}, \Delta_{12}, \Delta_{21}, \Delta_{22})$ is an ideal of Γ.
Let $\beta: \Gamma \to \Gamma/\Delta := (\Gamma_{11}/\Delta_{11}, \Gamma_{12}/\Delta_{12}, \Gamma_{21}/\Delta_{21}, \Gamma_{22}/\Delta_{22})$ be the canonical morita context homomorphism. Then

$(0:\Gamma_{12}/\Delta_{12})_{\Gamma_{11}/\Delta_{11}} = 0$ and $(0:\Gamma_{12}/\Delta_{12})_{\Gamma_{21}/\Delta_{21}} = 0.$

3. Recognizing morita near-rings

Let A be a near-ring with an identity 1. For an idempotent $e \in A$, let $e_1 = e$ and let $e_2 = 1 - e$. For $i = 1,2$, let $D_i = \{e_1 a e_i + e_2 b e_i \mid a, b \in A\}$ and let S be the subnear-ring of A generated by $\{e_i a e_j \mid 1 \le i, j \le 2, a \in A\}$.

PROPOSITION 3.1. *Let A be a near-ring with identity. Then A is isomorphic to a morita near-ring $M_2(\Gamma)$ for some morita context $\Gamma = (\Gamma_{11}, \Gamma_{12}, \Gamma_{21}, \Gamma_{22})$ where Γ_{11} and Γ_{22} are near-rings with identity (all modules in Γ are unital) if and only if A contains a distributive idempotent e for which the following holds:*

(i) $ea + (1-e)b = (1-e)b + ea$ *for all* $a, b \in A$

(ii) $(0:D_1)_A \cap (0:D_2)_A = 0$

(iii) $S = A$.

PROOF. Suppose $A \cong M_2(\Gamma)$. Let I be the identity of $M_2(\Gamma)$. Then $I = s_{11}^1 + s_{22}^1$ (we use 1 to denote both the identity of Γ_{11} and Γ_{22}). Let $e = e_1 = s_{11}^1$. Then e is a distributive idempotent and $s_{11}^1 U + (I - s_{11}^1)V = s_{11}^1 U + s_{22}^1 V = s_{22}^1 V + s_{11}^1 U = (I - s_{11}^1)V + s_{11}^1 U$ for all $U, V \in M_2(\Gamma)$. Using properties 1.1(7) and (4), we have

$$D_i = \{s_{11}^1 U s_{ii}^1 + s_{22}^1 V s_{ii}^1 \mid U, V \in M_2(\Gamma)\}$$
$$= \{s_{1i}^{a_1} + s_{2i}^{a_2} \mid a_j \in \Gamma_{ji}\} \text{for } i = 1,2.$$

Hence, if $UD_i = 0$ for $i = 1,2$, then

$$U\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} = U\begin{bmatrix} a_{11} & 0 \\ a_{21} & 0 \end{bmatrix} + U\begin{bmatrix} 0 & a_{12} \\ 0 & a_{22} \end{bmatrix} = U(s_{11}^{a_{11}} + s_{21}^{a_{21}})\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + U(s_{12}^{a_{12}} + s_{22}^{a_{22}})\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = 0$$

for all $a_{jk} \in \Gamma_{jk}, j,k \in N_2$. Thus $U = 0$.

Finally, $\{s_{ii}^1 U s_{jj}^1 \mid U \in M_2(\Gamma), i,j \in N_2\} = \{s_{ij}^a \mid a \in \Gamma_{ij}, i,j \in N_2\}$ and so $S = M_2(\Gamma)$. Conversely, let $e_1 = e$ be a distributive idempotent of A which satisfies conditions (i), (ii) and (iii). Then $e_2 := 1 - e$ is idempotent. Furthermore, it is easily seen that e_2 is distributive by using condition (i). Note also $e_1 e_2 = 0 = e_2 e_1$. For each $i,j \in N_2$, let $\Gamma_{ij} = e_i A e_j$. Clearly Γ_{ij} is a subgroup of A and if the mappings

$$\Gamma_{ij} \times \Gamma_{jk} \to \Gamma_{ik} \text{ are defined by } (x,y) \mapsto xy,$$

we obtain a near-ring morita context $\Gamma = (\Gamma_{11}, \Gamma_{12}, \Gamma_{21}, \Gamma_{22})$. Each near-ring Γ_{ii} has an identity e_i and all Γ_{ii}-modules (left or right) are unital, $i = 1,2$. Define $\theta : M_2(\Gamma) \to A = S$ *by* $\theta(U) = u$ where $u \in S$ is obtained from $U \in M_2(\Gamma)$ by replacing each s_{ij}^x present in U by x. At the outset we have to verify that θ is well-defined. We first need two remarks:

(1) If $x \in \Gamma_{ij} = e_i A e_j$, then $x = e_i a e_j$ for some $a \in A$ and thus $x = e_i x e_j$.

(2) If $U \in M_2(\Gamma)$ and $U(s_{1i}^{a_1} + s_{2i}^{a_2}) = s_{1i}^{b_1} + s_{2i}^{b_2}$, then $u(a_1 + a_2) = b_1 + b_2$:We will substantiate this claim by induction on $w(U)$. If $w(U) = 1$, then $U = s_{jk}^x$ for some $x \in \Gamma_{jk}$. Thus $\theta(U) = u = x$ and $U(s_{1i}^{a_1} + s_{2i}^{a_2}) = s_{jk}^x(s_{1i}^{a_1} + s_{2i}^{a_2}) = s_{ji}^{xa_k}$. Now $u(a_1 + a_2) = x(a_1 + a_2) = e_j x e_k (e_1 a_1 e_i + e_2 a_2 e_i) = e_j x e_k e_k a_k e_i = x a_k$. Suppose the result holds for all $V \in M_2(\Gamma)$ with $w(V) < m, m \geq 2$. If $w(U) = m$, then $U = U_1 + U_2$ or $U = U_1 U_2$ where $U_1 U_2 \in M_2(\Gamma)$ with $w(U_i) < m, i = 1,2$. Suppose $U_1(s_{1i}^{a_1} + s_{2i}^{a_2}) = (s_{1i}^{b_1} + s_{2i}^{b_2})$, $U_2(s_{1i}^{a_1} + s_{2i}^{a_2}) = (s_{1i}^{c_1} + s_{2i}^{c_2})$ and $U_1(s_{1i}^{c_1} + s_{2i}^{c_2}) = (s_{1i}^{d_1} + s_{2i}^{d_2})$. If $U = U_1 + U_2$ then $U(s_{1i}^{a_1} + s_{2i}^{a_2}) = s_{1i}^{b_1 + c_1} + s_{2i}^{b_2 + c_2}$ and $u(a_1 + a_2) = (u_1 + u_2)(a_1 + a_2) = b_1 + b_2 + c_1 + c_2 = b_1 + c_1 + b_2 + c_2$ since $b_2 + c_1 = e_2 b_2 e_i + e_1 c_1 e_i = e_1 c_1 e_i + e_2 b_2 e_i = c_1 + b_2$. If $U = U_1 U_2$, then $U(s_{1i}^{a_1} + s_{2i}^{a_2}) = s_{1i}^{d_1} + s_{2i}^{d_2}$ and $u(a_1 + a_2) = u_1 u_2 (a_1 + a_2) = u_1(c_1 + c_2) = d_1 + d_2$.

We now show that θ is well-defined. Suppose $U, V \in M_2(\Gamma)$ with $U = V$. For $i,j \in N_2$, let $a_{ij} \in \Gamma_{ij}$. Suppose

$$U\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} = V\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}.$$

By property 1.1(8),

$$U(s_{1i}^{a_1} + s_{2i}^{a_2}) = V(s_{1i}^{a_1} + s_{2i}^{a_2}) = s_{1i}^{b_1} + s_{2i}^{b_2} \text{for } i \in N_2.$$

From (2) above,

$$u(a_{1i} + a_{2i}) = v(a_{1i} + a_{2i}), \text{ i.e.}$$

$$(u - v)(e_1 a_{1i} e_i + e_2 a_{2i} e_i) = (u - v)(a_{1i} + a_{2i}) = 0$$

and so $u - v \in (0:D_1)_A \cap (0:D_2)_A = 0$. Hence $u = v$ and $\theta(U) = \theta(V)$. Thus θ is well-defined and clearly it is a near-ring homomorphism. For any $u \in A = S$, u is a finite combination of sums and products of $e_i a e_j$'s, $a \in A$. By replacing each $e_i a e_j$ in u by $s_{ij}^{e_i a e_j}$ we obtain an element U of $M_2(\Gamma)$ for which $\theta(U) = u$. Thus θ is surjective. Finally we show that θ is injective. Suppose $u = \theta(U) = 0$ for $U \in M_2(\Gamma)$. For all $i, j \in N_2$ and $a_{ij} \in \Gamma_{ij}$ if $U\begin{bmatrix} a_{11} a_{12} \\ a_{21} a_{22} \end{bmatrix} = \begin{bmatrix} b_{11} b_{12} \\ b_{21} b_{22} \end{bmatrix}$, then $U(s_{1i}^{a_{1i}} + s_{2i}^{a_{2i}}) = s_{1i}^{b_{1i}} + s_{2i}^{b_{2i}}$ and so $0 = u(a_{1i} + a_{2i}) = b_{1i} + b_{2i}$ for $i \in N_2$. Thus for all $i, j \in N_2, 0 = e_j(b_{1i} + b_{2i}) = e_j b_{ji} = b_{ji}$, hence $U = 0$. ■

Let us remark that if A is a ring, then any idempotent $e \in A$ satisfies the conditions of the previous result and A is isomorphic to the morita ring $\begin{bmatrix} eAe & eA(1-e) \\ (1-e)Ae & (1-e)A(1-e) \end{bmatrix}$; of course,

$$A = eAe + eA(1-e) + (1-e)Ae + (1-e)A(1-e)$$

is just the Peirce decomposition of A.

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Department of Mathematics,
Tohoku Gakuin University,
Tagajo 985,
Japan.

Department of Mathematics,
University of Port Elizabeth,
P.O. Box 1600,
Port Elizabeth 6000,
South Africa.