

GAPS BETWEEN COMPACTNESS DEGREE AND COMPACTNESS DEFICIENCY FOR TYCHONOFF SPACES

By

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1. Introduction.

In this paper we assume that all spaces are Tychonoff. For a space X , $\dim X$ denotes the Čech-Lebesgue dimension of X (see [3]).

J. de Groot proved that a separable metrizable space X has a metrizable compactification αX with $\dim(\alpha X \setminus X) \leq 0$ iff X is rim-compact (see [4]). A space X is *rim-compact* if each point of X has arbitrarily small neighborhoods with compact boundary. Modified the concept of rim-compactness, he defined the *compactness degree* of a space X , $\text{cmp } X$, inductively, as follows.

A space X satisfies $\text{cmp } X = -1$ iff X is compact. If n is a non-negative integer, then $\text{cmp } X \leq n$ means that each point of X has arbitrarily small neighborhoods U with $\text{cmp } \text{Bd } U \leq n - 1$. We put $\text{cmp } X = n$ if $\text{cmp } X \leq n$ and $\text{cmp } X \not\leq n - 1$. If there is no integer n for which $\text{cmp } X \leq n$, then we put $\text{cmp } X = \infty$.

By the *compactness deficiency* of a Tychonoff space (resp. a separable metrizable space) X we mean the least integer n such that X has a compactification (resp. a metrizable compactification) αX with $\dim(\alpha X \setminus X) = n$. We denote this integer by $\text{def}^* X$ (resp. $\text{def } X$). We allow n to be ∞ .

Thus, with this terminology, J. de Groot's result above asserts that $\text{cmp } X \leq 0$ iff $\text{def } X \leq 0$ for every separable metrizable space X . The general problem whether $\text{cmp } X \leq n$ iff $\text{def } X \leq n$ for arbitrary separable metrizable space X has been known as J. de Groot's conjecture, and was unsolved for a long time.

However, in 1982 R. Pol [7] constructed a separable metrizable space X such that $\text{cmp } X = 1$ and $\text{def } X = 2$. In the class of separable metrizable spaces, another example X with the property that $\text{cmp } X \neq \text{def } X$ seems to be still unknown but Pol's example above.

On the other hand, in the class of Tychonoff spaces, M. G. Charalambous [1] has already constructed a space X such that $\text{cmp } X = 0$ and $\text{def}^* X = n$ for each $n = 1, 2, \dots, \infty$. J. van Mill [6] has constructed a Lindelöf space X such that $\text{cmp } X = 1$ and $\text{def}^* X = \infty$.

In this paper we construct a countably compact space X such that $\text{cmp } X = m$ and

def* $X = n$ for $m, n \in \mathbb{N} \cup \{\infty\}$ with $m < n$.

2. Lemmas and the main result.

We begin with the following inductive conception, which is closely related to $\text{cmp } X$.

DEFINITION 2.1. For a subset A of a space X we define

- $\text{ind } (A, X) = -1$ iff A is empty,
 $\text{ind } (A, X) \leq n$ iff each point of A has arbitrarily small neighborhoods
 U in X with $\text{ind } (\text{Bd}_X U \cap A, X) \leq n - 1$,
 $\text{ind } (A, X) = n$ iff $\text{ind } (A, X) \leq n$ and $\text{ind } (A, X) \not\leq n - 1$,
 $\text{ind } (A, X) = \infty$ iff $\text{ind } (A, X) \not\leq n$ for all n .

The following lemma readily follows from induction.

LEMMA 2.2. For a closed subset A of a space X $\text{cmp } A \leq \text{cmp } X$.

LEMMA 2.3. Let $A \subset B \subset X \subset Y$. Then

- (1) $\text{ind } (A, X) \leq \text{ind } (B, X)$,
- (2) $\text{ind } (A, X) \leq \text{ind } (A, Y)$.

PROOF. (1) is easy by induction.

(2). We proceed by induction on $\text{ind } (A, Y) = n$. Obviously, (2) holds for $n = -1$. Let $n \geq 0$ and assume that (2) holds for every k with $k < n$. Suppose that $\text{ind } (A, Y) = n$. For each $x \in A$ and each neighborhood U of x in X there are neighborhoods U' and V' of x in Y such that $U = U' \cap X$, $V' \subset U'$ and $\text{ind } (\text{Bd}_Y V' \cap A, Y) \leq n - 1$. Let $V = V' \cap X$. The induction hypothesis implies that $\text{ind } (\text{Bd}_Y V' \cap A, X) \leq \text{ind } (\text{Bd}_Y V' \cap A, Y) \leq n - 1$. Since $\text{Bd}_X V \cap A \subset \text{Bd}_Y V' \cap A$, by (1), we have $\text{ind } (\text{Bd}_X V \cap A, X) \leq n - 1$. Hence we have $\text{ind } (A, X) \leq n$, therefore $\text{ind } (A, X) \leq \text{ind } (A, Y)$.

For every space X we set $R(X) = \{x \in X \mid x \text{ has no neighborhood with compact closure}\}$.

LEMMA 2.4. For every space X we have $\text{cmp } X \leq \text{ind } (R(X), X) + 1$.

PROOF. We shall apply induction with respect to $\text{ind } (R(X), X) = n$. Obviously, the lemma holds for $n = -1$. Let $n \geq 0$ and assume that the lemma holds for every k with $k < n$. Suppose that $\text{ind } (R(X), X) = n$. We shall prove that $\text{cmp } X \leq n + 1$. To prove this, we only consider points of $R(X)$, because $X \setminus R(X)$ is locally compact and open in X . Let $x \in R(X)$ and U a neighborhood of x in X . Then we take a neighborhood V of x in X such that $V \subset U$ and $\text{ind } (\text{Bd}_X V \cap R(X), X) \leq n - 1$. Since $R(\text{Bd}_X V) \subset \text{Bd}_X V \cap R(X)$, by lemma 2.3, we have $\text{ind } (R(\text{Bd}_X V), \text{Bd}_X V) \leq \text{ind } (\text{Bd}_X V \cap R(X), X) \leq n - 1$. By induction hypothesis, we have $\text{cmp } \text{Bd}_X V \leq n$. Hence we have $\text{cmp } X \leq n + 1$.

As usual, an ordinal α is the space of all ordinals less than α with order topology. For each ordinal α we denote by $[0, \alpha]$ the long segment for α . That is, $[0, \alpha] = (\alpha \times [0, 1)) \cup \{\alpha\}$ as the set, where $[0, 1)$ is the half-open unit interval, with order topology with respect to an order $<$ as follows; for $(\beta, t), (\gamma, s) \in \alpha \times [0, 1)$ $(\beta, t) < (\gamma, s)$ iff $(\beta < \gamma)$ or $(\beta = \gamma$ and $t < s)$ and for all $(\beta, t) \in \alpha \times [0, 1)$ $(\beta, t) < \alpha$.

Then the space $[0, \alpha]$ is compact and connected. For ordinals $\alpha_i, 1 \leq i \leq n$, we have $\dim \prod_{i=1}^n [0, \alpha_i] = \text{ind } \prod_{i=1}^n [0, \alpha_i] = n$. For any points $\beta, \gamma \in [0, \alpha]$ with $\beta < \gamma$ we define $[\beta, \gamma] = \{\delta \in [0, \alpha] \mid \beta \leq \delta \leq \gamma\}$. Similarly, we define $[\beta, \gamma), (\beta, \gamma]$ and (β, γ) .

LEMMA. 2.5. *Let $m \geq 1$ and $Y_m = (\omega_1 \times [0, \omega_1]^{m+1}) \cup (\{(\omega_1, \omega_1, \omega_1)\} \times [0, \omega_1]^{m-1})$ be the subspace of $(\omega_1 + 1) \times [0, \omega_1]^{m+1}$. Then $\text{cmp } Y_m = m$.*

PROOF. Since $R(Y_m) = \{(\omega_1, \omega_1, \omega_1)\} \times [0, \omega_1]^{m-1}$, we have $\text{ind } (R(Y_m), Y_m) = m - 1$. By Lemma 2.4, $\text{cmp } Y_m \leq m$. Thus we only show that $\text{cmp } Y_m \geq m$. We proceed by induction on m .

Step 1. Suppose that $m = 1$.

Let $\{y\} = R(Y_1) = \{(\omega_1, \omega_1, \omega_1)\}$ and $U = ((\omega_1 + 1) \times (1, \omega_1]^2) \cap Y_1$. Then U is a neighborhood of y in Y_1 . Assume that there is a neighborhood V of y in Y_1 such that $V \subset U$ and $\text{Bd } V$ is compact. Let $p: (\omega_1 + 1) \times [0, \omega_1]^2 \rightarrow \omega_1 + 1$ be the projection. Then we have $p(\text{Bd } V) \subset \omega_1$. Since $p(\text{Bd } V)$ is compact, we can take an ordinal $\alpha < \omega_1$ such that $p(\text{Bd } V) \subset \alpha$. On the other hand, there is an ordinal $\beta < \omega_1$ such that $(\gamma, \omega_1, \omega_1) \in V$ for every γ with $\beta < \gamma < \omega_1$. Pick up an ordinal γ with $\max\{\alpha, \beta\} < \gamma < \omega_1$. Then $\gamma \notin p(\text{Bd } V)$, $(\gamma, 0, 0) \notin V$ and $(\gamma, \omega_1, \omega_1) \in V$. This contradicts the connectedness of $\{y\} \times [0, \omega_1]^2$. Thus $\text{Bd } V$ is not compact for every neighborhood V of y in Y_1 with $V \subset U$. Hence $\text{cmp } Y_1 = 1$.

Step 2. Assume that $\text{cmp } Y_k = k$ for every k with $k < m$.

Let $Z = ((\omega_1 + 1) \times [0, \omega_1]^m \times [0, 1]) \cap Y_m$, $U = ((\omega_1 + 1) \times [0, \omega_1]^m \times [0, 1/2]) \cap Y_m$ and $x = (\omega_1, \omega_1, \dots, \omega_1, 0)$. Then Z is closed in Y_m and U is a neighborhood of x in Z . For each neighborhood V of x in Z with $V \subset U$ we set

$$t = \sup \{s \in [0, 1] \mid (\omega_1, \omega_1, \dots, \omega_1, s) \in V\}.$$

Let $p: (\omega_1 + 1) \times [0, \omega_1]^{m+1} = (\omega_1 + 1) \times \prod_{i=1}^{m+1} [0, \omega_1]_i \rightarrow [0, \omega_1]_{m+1}$ be the projection and $A = p(\{(\omega_1, \omega_1, \dots, \omega_1)\} \times [0, 1]) \cap V$. For each $x \in (\{(\omega_1, \omega_1, \dots, \omega_1)\} \times [0, 1]) \cap V$ we take $\alpha_{ix} < \omega_1, i = 0, 1, \dots, m$, and an open subset U_x of $[0, 1]$ such that $x \in V_x = ((\omega_1 + 1) \setminus \alpha_{0x}) \times \prod_{i=1}^m [\alpha_{ix}, \omega_1] \times U_x \cap Z \subset V$. Since $(\{(\omega_1, \omega_1, \dots, \omega_1)\} \times [0, 1]) \cap V$ is Lindelöf, we can take a countable subset $\{x(n) \mid n \in \mathbb{N}\}$ such that $\{V_{x(n)} \mid n \in \mathbb{N}\}$ covers $(\{(\omega_1, \omega_1, \dots, \omega_1)\} \times [0, 1]) \cap V$. Let $\alpha_i = \sup \{\alpha_{ix(n)} \mid n \in \mathbb{N}\}$ for each $i = 0, 1, \dots, m$. Then $\alpha_i < \omega_1$ and $(((\omega_1 + 1) \setminus \alpha_0) \times \prod_{i=1}^m [\alpha_i, \omega_1] \times A) \cap Z \subset V$. Let $W = Z \setminus \text{Cl}_Z V$. Then, similarly, we can take an ordinal $\beta_i < \omega_1$ for each $i = 0, 1, \dots, m$ such that $(((\omega_1 + 1) \setminus \beta_0) \times \prod_{i=1}^m [\beta_i, \omega_1] \times B) \cap Z \subset W$, where $B = p(\{(\omega_1, \omega_1, \dots, \omega_1)\} \times [0, 1]) \cap W$. Let us set $\gamma_i = \max\{\alpha_i, \beta_i\}$ for each $i = 0, 1, \dots, m$. Then $(((\omega_1 + 1) \setminus \gamma_0) \times \prod_{i=1}^m [\gamma_i, \omega_1] \times \{t\}) \cap Z$ is homeomorphic to

Y_{m-1} and contained in $\text{Bd}_Z V$ as a closed subset. By Lemma 2.2, we have $\text{cmp Bd}_Z V \geq m-1$, therefore $\text{cmp } Y_m \geq \text{cmp } Z \geq m$. Hence we have $\text{cmp } Y_m = m$. This completes the proof of Lemma 2.5.

Let $n \geq 2$ and $Z_n = \Pi_{i=2}^{n+1} [0, \omega_i] \setminus \{(\omega_2, \omega_3, \dots, \omega_{n+1})\}$ be the subspace of $\Pi_{i=2}^{n+1} [0, \omega_i]$.

Since $\Pi_{i=2}^{n+1} [0, \omega_i]$ is pseudocompact, by Glicksberg's theorem, we have $\beta \Pi_{i=2}^{n+1} [0, \omega_i] = \Pi_{i=2}^{n+1} [0, \omega_i]$, where βY is the Stone-Čech compactification of a space Y . Thus $\beta Z_n = \Pi_{i=2}^{n+1} [0, \omega_i]$. Namely, Z_n has the only compactification $\Pi_{i=2}^{n+1} [0, \omega_i]$.

LEMMA. 2.6. *Let X contain Z_n as a closed subset. Then for every perfect image Y of X we have $\dim Y \geq n$.*

PROOF. Let $f: X \rightarrow Y$ be a perfect surjection and $\beta f: \beta X \rightarrow \beta Y$ the Stone extension of f . Then $\text{Cl}_{\beta X} Z_n$ is a compactification of Z_n . As described above, $\text{Cl}_{\beta X} Z_n$ is homeomorphic to $\Pi_{i=2}^{n+1} [0, \omega_i]$. Let $z = (\omega_2, \omega_3, \dots, \omega_{n+1})$. Then $\text{Cl}_{\beta X} Z_n = Z_n \cup \{z\}$ and $z \in \beta X \setminus X$.

Claim 1. For each $i = 2, 3, \dots, n+1$, there is an ordinal $\alpha_i < \omega_i$ such that $\beta f(A_i) \cap \beta f(B_i) = \emptyset$, where

$$A_i = \Pi_{j=2}^{i-1} [\alpha_j, \omega_j] \times \{\alpha_i\} \times \Pi_{j=i+1}^{n+1} [\alpha_j, \omega_j]$$

and

$$B_i = \Pi_{j=2}^{i-1} [\alpha_j, \omega_j] \times \{\omega_i\} \times \Pi_{j=i+1}^{n+1} [\alpha_j, \omega_j].$$

Proof of Claim 1. Since f is perfect, $\beta f(z) \notin \beta f(Z_n)$ (see [3, 3.7.15]). Thus for each $\alpha < \omega_j$ we take an ordinal $\alpha_i^j(\alpha) < \omega_i$ such that

$$\beta f(\Pi_{i=2}^{i-1} [\alpha_i^j(\alpha), \omega_i] \times \{\alpha\} \times \Pi_{j=i+1}^{n+1} [\alpha_j^j(\alpha), \omega_j]) \cap \beta f(\Pi_{i=2}^{j-1} [\alpha_i^j(\alpha), \omega_i] \times \{\omega_j\} \times \Pi_{j=i+1}^{n+1} [\alpha_i^j(\alpha), \omega_i]) = \emptyset.$$

Let $\alpha_i^j = \sup \{\alpha_i^j(\alpha) \mid \alpha < \omega_j\}$. If $j < i$, then $\alpha_i^j < \omega_i$.

We define, by downward induction on i , an ordinal

$$\alpha_i = \max \{\alpha_i^2, \dots, \alpha_i^{i-1}, \alpha_i^{i+1}(\alpha_{i+1}), \dots, \alpha_i^{n+1}(\alpha_{n+1})\}.$$

Then $\alpha_i < \omega_i$ for each $i = 2, 3, \dots, n+1$. Since $\alpha_i \geq \alpha_i^j(\alpha_j)$, we have $\beta f(A_i) \cap \beta f(B_i) = \emptyset$.

Claim 2. $\dim Y \geq n$.

Proof of Claim 2. Assume that $\dim Y = \dim \beta Y < n$. Since $\beta f(A_i)$ and $\beta f(B_i)$ are disjoint closed subsets of βY for each $i = 2, 3, \dots, n+1$, we take a partition L_i in βY between $\beta f(A_i)$ and $\beta f(B_i)$ such that $\bigcap_{i=2}^{n+1} L_i = \emptyset$ (cf. [2, 3.3.6]). Let $X' = \Pi_{i=2}^{n+1} [\alpha_i, \omega_i]$ and $M_i = \beta f^{-1}(L_i) \cap X'$. Then M_i is a partition in X' between A_i and B_i such that $\bigcap_{i=2}^{n+1} M_i = \emptyset$. Since X' is compact, for each $i = 2, 3, \dots, n+1$, we take a finite collection $\{\beta_i^j \mid 0 \leq j \leq m_i\}$ of ordinals such that

- (1) $\alpha_i = \beta_i^0 < \dots < \beta_i^{j-1} < \dots < \beta_i^{m_i} = \omega_i$,
- (2) $\bigcap_{i=2}^{n+1} \text{St}(M_i, \mathcal{A}) = \emptyset$, $\text{St}(M_i, \mathcal{A}) \cap \text{St}(A_i, \mathcal{A}) = \emptyset$ and $\text{St}(M_i, \mathcal{A}) \cap \text{St}(B_i, \mathcal{A}) = \emptyset$,

where $\mathcal{A} = \{\prod_{i=2}^{n+1} [\beta_i^{j(i)-1}, \beta_i^{j(i)}] \mid (j(2), \dots, j(n+1)) \in \prod_{i=2}^{n+1} \{1, \dots, m_i\}\}$.

Then for each $i=2, 3, \dots, n+1$, there is a continuous mapping $f_i: [\alpha_i, \omega_i] \rightarrow [0, 1] = I_i$ such that $f_i(\beta_i^j) = j/m_i$ and $f_i([\beta_i^{j-1}, \beta_i^j]) = [(j-1)/m_i, j/m_i]$. Let $g = \prod_{i=2}^{n+1} f_i: \prod_{i=2}^{n+1} [\alpha_i, \omega_i] \rightarrow \prod_{i=2}^{n+1} I_i$ be the product mapping defined by $g((t_i)_{i=2}^{n+1}) = (f_i(t_i))_{i=2}^{n+1}$. Since $\text{St}(M_i, \mathcal{A})$ is a partition in X' between A_i and B_i , there are disjoint open subsets U_i and V_i of X' such that $A_i \subset U_i$, $B_i \subset V_i$ and $X' \setminus \text{St}(M_i, \mathcal{A}) = U_i \cup V_i$. Let $K_i^j = I_2 \times \dots \times I_{i-1} \times \{j\} \times I_{i+1} \times \dots \times I_{n+1}$ for each $i=2, 3, \dots, n+1$ and each $j=0, 1$. Let $N_i = g(X' \setminus U_i) \cap g(X' \setminus V_i)$ for each $i=2, 3, \dots, n+1$. Then N_i is a partition in $\prod_{i=2}^{n+1} I_i$ between K_i^0 and K_i^1 , and $\bigcap_{i=2}^{n+1} N_i = \emptyset$. This is a contradiction (cf. [2, 1.8.1]).

Now we construct a space, which is mentioned in the introduction.

EXAMPLE. 2.7. For $m, n \in N \cup \{\infty\}$ with $m < n$ there exists a countably compact space X such that $\text{cmp } X = m$ and $\text{def}^* X = n$.

Case 1. $n \in N$.

Let $X = (\omega_1 \times \prod_{i=2}^{n+1} [0, \omega_i]) \cup (\{(\omega_1, \omega_1, \omega_1)\} \times [0, \omega_1]^{m-1} \times \{(\omega_1, \omega_1, \dots, \omega_1)\}) \cup \{(\omega_1, \omega_2, \dots, \omega_{n+1})\}$ be the subspace of $(\omega_1 + 1) \times \prod_{i=2}^{n+1} [0, \omega_i]$.

It is easy to see that X is countably compact.

Since $R(X) = (\{(\omega_1, \omega_1, \omega_1)\} \times [0, \omega_1]^{m-1} \times \{(\omega_1, \omega_1, \dots, \omega_1)\}) \cup \{(\omega_1, \omega_2, \dots, \omega_{n+1})\}$, we have $\text{ind}(R(X), X) = m - 1$. By Lemma 2.4, we have $\text{cmp } X \leq m$. Since X contains Y_m as a closed subspace, by Lemmas 2.2 and 2.5 we have $\text{cmp } X \geq m$. Hence $\text{cmp } X = m$.

Next, since $\beta X = (\omega_1 + 1) \times \prod_{i=2}^{n+1} [0, \omega_i]$, we have $\dim(\beta X \setminus X) = n$. Thus $\text{def}^* X \leq n$. For each compactification αX of X there is a perfect surjection $f: \beta X \setminus X \rightarrow \alpha X \setminus X$, and $\beta X \setminus X$ contains a closed subset homeomorphic to Z_n . Thus, by Lemma 2.6, we have $\dim(\alpha X \setminus X) \geq n$. Hence $\text{def}^* X = n$.

Case 2. $n = \infty$.

Let $X = (\omega_1 \times \prod_{i=2}^{\infty} [0, \omega_i]) \cup (\{(\omega_1, \omega_1, \omega_1)\} \times [0, \omega_1]^{m-1} \times \{(\omega_1, \omega_1, \dots)\}) \cup \{(\omega_1, \omega_2, \dots)\}$ be the subspace of $(\omega_1 + 1) \times \prod_{i=2}^{\infty} [0, \omega_i]$. Then, similarly, X is countably compact, $\text{cmp } X = m$ and $\text{def}^* X = n$.

3. Statements.

We define $\text{Cmp } X$ of a space X by the following; $\text{Cmp } X = 0$ if $\text{cmp } X \leq 0$, and for $n \geq 1$, $\text{Cmp } X \leq n$ if each closed subset of X has arbitrarily small neighborhoods U with $\text{Cmp } \text{Bd } U \leq n - 1$. $\text{Cmp } X$ was defined by J. de Groot for the case X is separable and metrizable.

It can be prove that $\text{cmp } X \leq \text{Cmp } X$ for every space X and $\text{cmp } X \leq \text{Cmp } X \leq \text{def } X$ for every separable metrizable space X (see [4]). For Pol's example X in [7] shows that $\text{cmp } X = 1$ and $\text{Cmp } X = \text{def } X = 2$. Thus it is unknown whether there is a separable metrizable space X with $\text{Cmp } X < \text{def } X$. It would be interesting to have a separable metrizable space X such that $\text{cmp } X = k$, $\text{Cmp } X = m$ and $\text{def } X = n$ for $k, m, n \in N \cup \{\infty\}$

with $k \leq m \leq n$.

Obviously, $\text{def}^* X \leq \text{def} X$ for every separable metrizable space X . We do not know whether there is a separable metrizable space X with $\text{def}^* X < \text{def} X$ as well as the value of $\text{def}^* X$ for Pol's example X in [7].

In Example 2.7 we have constructed a space X with $\text{cmp} X = m$ and $\text{def}^* X = n$ for $m, n \in N \cup \{\infty\}$ with $m < n$. However, in general, $\text{cmp} X$ need not be less than or equal to $\text{def}^* X$ [5, VII.25]. It would be interesting to have a space X such that $\text{cmp} X = k$, $\text{Cmp} X = m$ and $\text{def}^* X = n$ for $k, m, n \in N \cup \{\infty\}$ with $k \leq m$.

Added in proof. The author constructed a separable metrizable space X such that $\text{def} X - \text{com} X = n$ for each $n \in N$. Thus in the class of separable metrizable spaces the gap between $\text{cmp} X$ and $\text{def} X$ can be arbitrarily large.

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