Seasonality, Household Production and Aggregate Fluctuations

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abstract

Benhabib, Rogerson and Wright demonstrate that introducing home production improves the performance of the stochastic growth model along several dimensions simultaneously. A key reason for the difference between their result and the results in the rest of the literature is that there is a good deal of substitutability between the home sector and the market. In this paper, we examine their results in a model which is augmented to reflect seasonal shifts in preferences and technology. Does the seasonal patterns and business cycle properties of a dynamic general equilibrium model of the U.S. economy that includes an explicit household production sector accord well with the seasonally unadjusted U.S. data? Our answer to this question depends critically on the specification of the depreciation. With seasonal variation in the rate of depreciation, it is quite possible that seasonal variations in preferences and technology alone produce seasonal patterns in nonmarket investment and hours worked in the market. In addition, the model does relatively well in matching standard deviation observed in the data for both hours worked and market consumption as well as the autocorrelation function of the actual U.S. output series. The simulated series for market and nonmarket investment however are too volatile and the model does not always mimic seasonal patterns observed in the actual data for market consumption and market investment. We plan to address these problems in future research.

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1. Introduction

A rapidly emerging literature on real business cycle theory focuses on the relation of household production and aggregate fluctuations. A major motivation for this focus is the realization that the household sector is large, whether measured in terms of the time allocated to home production activities or in terms of the estimated value of home produced output (Benhabib, Rogerson and Wright [1]). Recent advances in measuring the value of home produced output are important factors assisting this modeling effort, thereby imposing more discipline on the model-building exercise. For example, assessing the value of home production, Eisner [4] reports a wide range of estimates, from 20 to 50 percent of GNP. As for the time allocated to home production activities, Hill [7] analyzes the Michigan Time Use Survey, and observes that a married couple spends on average 56.91 hours a week working in the market, 49.1 hours working at home, 67.85 hours in leisure activities, and 162.14 hours in sleep and other activities. In other words, excluding "personal care" which consists mainly of sleep, work at home counts for 28.2% of total discretionary time. Furthermore, using different data, Rupert, Rogerson and Wright [12] point out that home work occupies a relevant part of the agents time, especially of the married women (see also Greenwood, Rogerson and Wright [6], McGrattan, Rogerson and Wright [9]). As for the capital used in the production, Greenwood and Hercowitz, [5] clearly show that the capital used in the home sector, defined as consumer durables plus residential capital, is bigger than the capital used in the market sector, defined as business nonresidential capital. More concretely, they show that the average ratio of the former to the latter is about 1.13 for the period 1954–88 in the U.S. All the facts cited above clearly point at the fact that the home sector is empirically relevant (Perli [10]).

It is apparent the stochastic growth model does not do as well along some dimensions as it does along others. Benhabib, Rogerson and Wright [1] demonstrate that introducing home production improves the performance of the model along all these dimensions simultaneously. A key reason for the difference between this result and the results in the rest of the literature is that there is a good deal of substitutability between the home sector and the market (Benhabib, Rogerson and Wright [1]). The rest of the literature, either explicitly or implicitly, assumes that the behavior of the home sector is approximately independent of the market. This hypothesis is however not in agreement with the data, as a look at the same capital time series in Greenwood and Hercowitz [5] can prove: over the 1954–88 period, investment in household capital is indeed procyclical, and it moves together with business capital. Moreover, there is clear evidence of substitutability between work at home and work in the market. Individuals who are not employed in the market enjoy more leisure, but the difference in leisure with respect to employed individuals is much less than the difference in time spent working in the market (Perli [10], Hill [7] and Rios-Rull [11]). Benhabib, Rogerson and Wright [1] go on to conjecture that when individuals are able to substitute between market and nonmarket production over time, volatility in market activity can arise because of relative productivity differentials between the two sectors, and not just absolute productivity shocks, as is the case in one-sector models. In other words, the size of the fluctuations induced by productivity shocks will depend on the degree to which agents are willing to substitute between home and market commodities (both time and goods) and not just the degree to which they are willing to substitute between these commodities at different dates, as is the case in the standard real business cycle model.

Another standard assumption in many models of current equilibrium business cycle theories is that most macroeconomic time series can be decomposed into seasonal and nonseasonal components and the former can be ignored in the investigation of economic fluctuations. Braun and Evans [2] challenge this traditional explanation. They show that the non-time-separable model predicts most of the seasonal patterns found in aggregate quantity time series. Their model successfully offers predictions across both business and seasonal cycle frequencies. In this paper, we examine the Benhabib, Rogerson and Wright [1] conjecture in a model which is augmented to reflect seasonal shifts in preferences and technology.

Does the seasonal patterns and business cycle properties of a dynamic general equilibrium model of the U.S. economy that includes an explicit household production sector accord well with the seasonally unadjusted U.S. data? Our answer to this question depends critically on the specification of the depreciation. Previous real business cycle studies assume the fixed rate of depreciation. With seasonal variation in the rate of depreciation, it is quite possible that seasonal variations in preferences and technology alone produce seasonal patterns in nonmarket investment and hours worked in the market. The model, however, does not always mimic seasonal patterns observed in the actual data for market consumption and market investment.

The model presented here does relatively well in matching standard deviation observed in the data for hours worked in the market, hours worked at home and market consumption . In contrast, the previous models have equilibria along which these variables are not volatile enough relative to output. In addition the autocorrelation function of simulated series mimics that of the actual U.S. data for output. In general, the performance of this model is comparable to single-sector real business cycle models, but it is insufficient along some dimensions; in particular, the model generates excess volatility for two types of investment. We plan to address these problems in future research.

The rest of the paper is organized as follows. Section 2 presents a simple dynamic general equilibrium model with home production. In Section 3 the perfect foresight seasonal equilibrium path is defined. Section 4 describes parameter values and the approximation methods for solving and simulating the model. Section 5 presents and analyzes the simulation results. The last section concludes.

II. The Model

The purpose of this section is to construct a simple economy that includes an explicit household production sector. The model is then used to investigate the effects of seasonality. Since leisure and hours of nonmarket (home) work have strong seasonal components, to use seasonally unadjusted data is the best way.

There are 2 goods in the economy, a market good and a home-produced or nonmarket good. They are imperfect substitutes for each other. Consumption is an aggregate of market consumption c_m $(c_m \geq 0)$ and nonmarket consumption c_n $(c_n \geq 0)$. Hours of work H $(H \geq 0)$ are divided between hours of work in the market sector h_m $(h_m \geq 0)$ and hours of work in the nonmarket sector h_n $(h_n \geq 0)$. The remainder of the agent's time consists of leisure L, sleep and other activities. We normalize total discretionary time in a period to unity, so that $L = 1 - h_m - h_n$. We suppress time variable for notational economy.

The period utility function is assumed to take the form

(1) $U(c_{mt}, c_{nt}, h_{mt}, h_{nt}) = \frac{b_s}{e}\ln[a_s c_m^e + (1-a_s)c_n^e] + (1-b_s)\ln(1-h_m-h_n)$.

U is continuously differentiable, concave, and $U_1 > 0$, $U_2 > 0$, $U_3 < 0$, $U_4 < 0$. The elasticity of substitution between c_m ($c_m \geq 0$) and c_n ($c_n \geq 0$) is given by 1/(1-e).

Consider an economy made up of a large number of identical representative households each of which seeks to maximize the expected value of

(2) $\sum_{t=0}^{\infty} \beta^t U(c_{mt}, c_{nt}, h_{mt}, h_{nt})$.

The parameter β $(0 < \beta < 1)$ represents the rate at which the agent discounts the future. The household owns capital K, that it can divide at a point in time between market capital k_m and nonmarket capital k_n.

(3) $K = k_m + k_n$.

This amounts to assume that capital can be freely moved between the two sectors. It should be noted that, if there is some depreciation, it is possible to reallocate capital from one sector to another. By choosing not to replace the worn out capital in one sector and putting all new investment in the other, the economy is not necessary to physically move capital to achieve the desired allocation across sectors. The assumption of perfect mobility is made simply for analytical convenience.

Although capital can be freely transformed between its two uses, it may depreciate at different rates in its two uses. Investment in each of the two capital goods is defined residually by

(4m) $i_m = k_{m+} - (1-\delta_{ms})k_m$ (4n) $i_n = k_{n+} - (1-\delta_{ns})k_n$

where k_{m+} and k_{n+} denote $k_{m\,t+1}$ and $k_{n\,t+1}$, and δ_{ms} and δ_{ns} represent the rates of depreciation in the market sector and the nonmarket sector. From (3),(4m), and (4n), we have

(5) $K_+ = (1-\delta_{ns})K + (\delta_{ns} - \delta_{mts})k_m + I$,

where $I = i_m + i_n$ and $K_+ = K_{t+1}$.
The economy-wide resource constraint in per capita terms is

(6) $c_m + i \leq y$.

In both sectors, the production technology is assumed to be Cobb-Douglas and no externalities exist. In the market sector y is produced using k_m and h_m according to

(7m) $y=f_s(h_m,k_m)=\tilde{\gamma}_s k_m^{\theta} h_m^{1-\theta}$.

Here $\tilde{\gamma}_s$ is an exogenous technology parameter.

In the home sector, what is produced is immediately consumed, so that c_{nt} is produced according to

(7n) $c_n=g_s(h_n,k_n)=\tilde{\xi}_s k_n^{\eta} h_n^{1-\eta}$.

Here $\tilde{\xi}_s$ is an exogenous technology parameter. Market consumption is given by

(8) $c_m=f_s(h_m,k_m)-i_m-i_n$.

The exogenous technology parameters $\tilde{\gamma}_s$ and $\tilde{\xi}_s$ are decomposed into deterministic seasonal components, γ_s and ξ_s and the stochastic shocks, $\exp(s_{m,t})$ and $\exp(s_{n,t})$

(9m) $\tilde{\gamma}_s=\gamma_s\exp(s_{m,t})$ (9n) $\tilde{\xi}_s=\xi_s\exp(s_{n,t})$,

where subscript $s\in S$ $(S=\{1,2,3,4\})$ denotes the season.

Finally, $s_{m,t}$ and $s_{n,t}$ are stochastic processes with

(10m) $s_{m,t+1}=\rho_m s_{m,t}+\varepsilon_{m,t}$ (10n) $s_{n,t+1}=\rho_n s_{n,t}+\varepsilon_{n,t}$.

It is assumed that $|\rho_m|<1$ and $|\rho_n|<1$ and that the innovations $\varepsilon_{m,t}$ and $\varepsilon_{n,t}$ are independent and identically distributed, with standard deviations σ_m and σ_n and contemporaneous correlation ν.

III. The Perfect Foresight Seasonal Equilibrium

A perfect foresight seasonal equilibrium is a particular perfect foresight equilibrium path. This path has the characteristic that the value of a variable x in quarter s is always equal to its realization four quarters ago (Braun and Evans [2]). To obtain this path for our model economy, we first remove the uncertainty. Then substitute (4m),(4n),(6),(7m), (7n) and (8) into $\sum_{t=0}^{\infty}\beta^t U(c_{mt},c_{nt},h_{mt},h_{nt})$ and differentiate with respect to $h_{mt},h_{nt}\ k_{mt}$ *and* k_{nt}. Omitting time subscript t, the resulting first order conditions are

(11m) $U_1 f_1+U_3=0$

(11n) $U_2 g_1+U_4=0$

(12m) $U_1(f_2+1-\delta_m)-\frac{1}{\beta}U_{1-}=0$

(12n) $$U_1(1-\delta_n)+U_2 g_2-\frac{1}{\beta}U_{1-}=0,$$

where $f_1=\dfrac{\partial f}{\partial h_m}$, $f_2=\dfrac{\partial f}{\partial k_m}$, $g_1=\dfrac{\partial g}{\partial h_n}$, and $g_2=\dfrac{\partial g}{\partial k_n}$.

The perfect foresight seasonal equilibrium satisfies (11m), (11n), (12m) and (12n). For given parameter values of the model, these 16 equations, together with (3), (4m), (4n), (6), (7m), (7n) and (8) can be solved for 44 unknowns, c_m, c_n, y, i_m, i_n, k_m, k_n, h_m, h_n, K and I. (See Chatterjee and Ravikumar [3] for a formal characterization of existence and uniqueness.)

Since $U_3=U_4=-\dfrac{1-b}{1-h_m-h_n}$, from (11m) and (11n)

(13) $$U_1 f_1=U_2 g_1.$$

Dividing by $U_1 g_1$, one gets the familiar relation that equates the marginal rate of substitution between the two types of consumption to the ratio between the two marginal products of labor. From (12m) and (12n)

(14) $$U_1(1-\delta_n)+U_2 g_2=U_1(f_2+1-\delta_m).$$

Substituting from (13) yields

(15) $$1-\delta_n+\frac{g_2 f_1}{g_1}=f_2+1-\delta_m.$$

Differentiating (7n) with respect to h_n and k_n, and calculating the steady state ratio of g_2 to g_1, we have:

(16) $$\frac{g_2}{g_1}=\frac{\eta}{1-\eta}\frac{h_n}{k_n}.$$

Then from (15)

(17) $$\frac{\eta}{1-\eta}\frac{h_n}{k_n}f_1=f_2+\delta_n-\delta_m.$$

Therefore,

(18) $$\frac{\eta}{1-\eta}h_n = \frac{f_2+\delta_n-\delta_m}{f_1}k_n.$$

Equation (18) will be used in the next section to calibrate the value of η.

Note that the first order conditions imply three relations ((13), (14) and (16)) among contemporaneous variables that can be used to reduce the dimension of the system. Therefore, the attention will be focused on one of the two Euler equations, for example, equation (12m). With the seasonal index s, it is

(19) $$U_{1,s}(f_{2,s}+1-\delta_{m,s})+\frac{1}{\beta}U_{1,s-1}=0 \qquad (s = 1, 2, 3, 4).$$

IV. Calibration

It is a well-known fact that with the exception of a few special cases, stochastic growth models cannot be solved analytically. In this paper, we therefore follow an approach pioneered by Kydland and Prescott [8]. First, substituting $f_s(h_m, k_m)-i_m-i_n$ for c_{ms}, and $g_s(h_n, k_n)$ for c_{ns} in the utility function, we eliminate the nonlinearity in the constraint. Second, we approximate this function by a quadratic in the neighborhood of the model's perfect foresight seasonal equilibrium path. The resulting approximate quadratic functions are

(20) $$U_s(x_s)=u_s(\bar{x}_s)+\alpha_s'(x_s-\bar{x}_s)+(x_s-\bar{x}_s)'Q_s(x_s-\bar{x}_s) \qquad s=1,\dots,4,$$

where $x_s' = [K_s, z_{ms}, z_{ns}, H_s, I_s, k_{ms}]$ and $\bar{x}_s$ denotes the perfect foresight path for x_s. Note that since our data are seasonally unadjusted, the coefficients, $\alpha_s \in R^6$ and a 6 X 6 symmetric matrix Q_s are defined for each s (that is, separate coefficients for each season).

From (5), (10m), and (10n), the law of motion for the state variables $\xi_s' = [K_s, z_{ms}, z_{ns}]$ can be written

(21) $$\xi_{t+1} = A_s\xi_t + B_s\eta_t + \zeta_t,$$

where the seasonal index s equals time index t with a wrap-around time dating convention that s= mod(t-1, 4)+ 1. The ζ_t's are assumed to be independent and distributed with covariance matrix Ω. The variable η_t consists of three control variables (total hours worked H_s, total investment I_s, and capital stock in the market sector k_{ms}). This yields a periodic linear-quadratic dynamic programming problem which can be solved by using a recursive algorithm.

To find the steady state of our economy, first I choose values for the parameters that can be easily calibrated as functions of the average values of k_{ms}, k_{ns}, i_{ms}, i_{ns}, h_{ms} and y_s which are observed in the data. From now on variable names with the seasonal index will denote the perfect foresight seasonal path for the variables. Time indices, however, will be dropped because they are superfluous due to stationality. With the exception of capital stock, all variables in the model are quarterly and seasonally unadjusted, for the period 1964-1989. As for capital, *Fixed Reproducible Tangible Wealth in the United States, 1925-89* offers annual estimates of net stocks in constant-cost valuations by type of equipment and structures. Estimates are as of the end of the year and previous studies interpolate them to quarterly data. In this model, the end of year capital stock corresponds to k_{ms} and k_{ns}, s=1,5,9,... Following the previous literature, we interpolate the stock of capital in nonmarket sector and obtain the rate of depreciation from

$$(22) \qquad \delta_n = k_{n1} / 4(i_{n1} + i_{n2} + i_{n3} + i_{n4}).$$

In contrast, quarterly seasonally unadjusted data on investment can be taken from the National Income and Product Accounts. The evolution of the stocks of capital in market sector is determined recursively by investment in the stock, net of depreciation. In the perfect foresight seasonal equilibrium, taking the fact $k_{m1} = k_{m5} = k_{m9} = \ldots$ .into consideration, we have

$$
\begin{aligned}
&k_{m2} = (1-\delta_{m1})k_{m1} + i_{m1} \\
&k_{m3} = (1-\delta_{m2})k_{m2} + i_{m2} = (1-\delta_{m2})\{(1-\delta_{m1})k_{m1} + i_{m1}\} + i_{m2} \\
(23) \quad &k_{m4} = (1-\delta_{m3})k_{m3} + i_{m3} = (1-\delta_{m3})[(1-\delta_{m2})\{(1-\delta_{m1})k_{m1} + i_{m1}\} + i_{m2}] + i_{m3} \\
&k_{m5} = k_{m1} = (1-\delta_{m4})k_{m4} + i_{m4} \\
&\quad = (1-\delta_{m4})[(1-\delta_{m3})[(1-\delta_{m2})\{(1-\delta_{m1})k_{m1} + i_{m1}\} + i_{m2}] + i_{m3}] + i_{m4}.
\end{aligned}
$$

Since only annual data on capital stock are available, these equations cannot be solved for δ_{m1}, δ_{m2}, δ_{m3}, and δ_{m4}. Instead, k_{m2}, k_{m3} and k_{m4} can be though of as functions of these parameters and the data impose a restriction among δ_{m1}, δ_{m2}, δ_{m3}, and δ_{m4}

$$(24) \qquad \delta_{m4} = 1 - (k_{m1} - i_{m4}) / [(1-\delta_{m3})[(1-\delta_{m2})\{(1-\delta_{m1})k_{m1} + i_{m1}\} + i_{m2}] + i_{m3}].$$

We now proceed to choose parameter values. As in most of the previous literature we use the following specification: θ=0.36, $\rho_m = \rho_n$=0.95 and $\sigma_m = \sigma_n$. The standard deviation of the innovation, common to the two sectors, is set at 0.0015, which yields the same standard deviation of GNP for the model as for the postwar U.S. economy. We experiment with three values of contemporaneous correlation ν; these are 0.1, 0.5 and 0.9. Extensive experimentation with alternative combinations suggests to us that the alterations would be slight.

Choosing values for e is less immediate. On the one hand, using micro (PSID) data, Rupert, Rogerson and Wright [12] find values for e ranging from a minimum of zero for single males to a maximum of either 0.363 or 0.75 for married couples depending on the estimation method. On the other hand, based on aggregate data, McGrattan, Rogerson, and Wright [9] obtain maximum likelihood estimates. Their estimates imply a value of 0.385 for e. In this paper, following Benhabib, Rogerson and Wright [1], we set e at 0.8 in the simulations, implying an elasticity of substitution of 5.

Using a procedure similar to that in Benhabib, Rogerson and Wright [1], we now set the value of η. Using Michigan Time Use Survey, Hill[7] reports market work and homework for an average household consisting of a married couple. The implied ratio is $\sum_{s\in S} h_{ns} \Big/ \sum_{s\in S} h_{ms} = \frac{.28}{.33} = 0.848.$

From (18) we have

$$\text{(25)}\qquad \frac{\eta}{1-\eta}\sum_{s\in S} h_{ns} = \sum_{s\in S} \frac{f_{2s}+\delta_{ns}-\delta_{ms}}{f_{1s}} k_{ns}.$$

Suppose that the values of $\delta_{m1}, \delta_{m2}, \delta_{m3}$, and δ_{m4} are given. The steady state values of k_{m2}, k_{m3} and k_{m4} can be calculated from (23). The values of γ_s, in turn, can be obtained from $y_s = \gamma_s k_m^{\theta} h_m^{1-\theta}$. Then equation (25) can be solved for η. To our knowledge, the estimates on homework has been relied on annual data. Therefore we set the value for η to match the annual data. Since the use of quarterly data is vital to analyze a high-frequency phenomenon like substitution between the two types of labor, one priority for future work includes experimentation based on the high frequency panel data on homework.

The remaining parameters are $\beta, a_s\ b_s$ *and* ξ_s (s = 1, 2, 3, 4). Equation (19) can be written in matrix,

$$\text{(26)}\qquad \begin{bmatrix} x_{1,1} & 1/\beta & 0 & 0 \\ 0 & x_{2,2} & 1/\beta & 0 \\ 0 & 0 & x_{3,3} & 1/\beta \\ 1/\beta & 0 & 0 & x_{4,4} \end{bmatrix} \begin{bmatrix} U_{1,1} \\ U_{1,2} \\ U_{1,3} \\ U_{1,4} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

where, $x_{s,s} = f_{2s} + 1 - \delta_{ms}$.

For the perfect foresight seasonal equilibrium to exist, this equation is required to have nonnegative solutions for $[U_{1,1}, U_{1,2}, U_{1,3}, U_{1,4}]$. Therefore, the determinant of the matrix on the left hand side should be diminished, which implies

$$\text{(27)}\qquad \beta^4 = \prod_{s=1}^{4}(f_{2s} + 1 - \delta_{ms}).$$

Given the values of $\delta_{m1}, \delta_{m2}, \delta_{m3}$, and δ_{m4}, equation (27) can be solved for β. Returning to (26), the solutions are, of course, the same up to a size parameter. Therefore, we obtain the three ratios $U_{1,1}/U_{1,2}$, $U_{1,2}/U_{1,3}$ and $U_{1,3}/U_{1,4}$. Differentiating the utility function (1) with respect to consumption of market good and substituting C_{ns} from (7n), one obtains

$$\text{(28)}\qquad U_{1,s} = \frac{a_s b_s C_{ms}^{e-1}}{a_s C_{mt}^{e} + (1-a_s)C_{nt}^{e}} = \frac{b_s}{C_{ms} + \frac{1-a_s}{a_s}\xi_s k_n^{\eta} h_n^{1-\eta} (\frac{C_{ns}}{C_{ms}})^{e-1}} \quad (s = 1, 2, 3, 4).$$

Dividing by the derivative of the utility function with respect to consumption of nonmarket good yields

$$(29) \qquad \frac{U_{2,s}}{U_{1,s}} = \frac{1-a_s}{a_s}\left(\frac{C_{ns}}{C_{ms}}\right)^{e-1} = \frac{(1-\theta)\gamma_s k_{ms}^{\theta} h_{ms}^{-\theta}}{(1-\eta)\xi_s k_{ns}^{\eta} h_{ns}^{1-\eta}}.$$

Then we have:

$$(30) \qquad \frac{1-a_s}{a_s}\xi_s\left(\frac{C_{ns}}{C_{ms}}\right)^{e-1} = \frac{(1-\theta)\gamma_s k_{ms}^{\theta} h_{ms}^{-\theta}}{(1-\eta)k_{ns}^{\eta} h_{ns}^{1-\eta}}.$$

Substituting (30) into (28) yields

$$(31) \qquad U_{1,s} = \frac{b_s}{C_{ms} + \dfrac{(1-\theta)\gamma_s k_{ms}^{\theta} h_{ms}^{-\theta}}{1-\eta}}.$$

Differentiating the utility function with respect to market work and homework, one obtains

$$(32) \qquad U_{3,s} = U_{4,s} = \frac{b_s - 1}{1 - h_{ms} - h_{ns}}.$$

Substituting (32) and the derivative of the market production function with respect to hours of work in the market sector into (11m) yields

$$(33) \qquad U_{1s}(1-\theta)\gamma_s k_{ms}^{\theta} h_{ms}^{-\theta} + \frac{b_s - 1}{1 - h_{ms} - h_{ns}} = 0 .$$

Substituting for $U_{1,s}$ from (31), we can solve (33) for b_s.

Equation (31) in turn yields the values for U_{1s}'s. The results, however, are not always satisfy (26). Therefore, we resort to numerical techniques and choose the values of $\delta_{m1}, \delta_{m2}, \delta_{m3}$ and δ_{m4} to equate $U_{1,1}/U_{1,2}$, $U_{1,2}/U_{1,3}$ and $U_{1,3}/U_{1,4}$ to the ratios implied by (26). Note that the degree of freedom is only three because of the restriction (24).

Finally, we can solve (28) for $\frac{1-a_s}{a_s}\xi_s$. We cannot identify ξ_s and a_s, but this is reasonable because in the home sector, what is produced is immediately consumed. If we assume, for example, $\xi_s = \gamma_s$ (s = 1,..., 4), we can obtain the values for these parameters separately.

The following table summarizes a parameterization that generates the perfect foresight seasonal equilibrium for the model.

Table Model Parameters

β=.9691, θ=0.36, η=.4422, e= .8, δ_{ns}= .0192 (s = 1,..., 4)
$\rho_m = \rho_n$=0.95 , $\sigma_m = \sigma_n$=.0015.

	I	II	III	IV
γ_s	0.0341	0.0352	0.035	0.0368
δ_{ms}	0.0156	0.023	0.0207	0.0349
b_s	0.5483	0.5108	0.5256	0.4516
a_s	0.4375	0.4539	0.4489	0.4833

Note: The values for a_s's are obtained under the assumption $\xi_s = \gamma_s$ (s = 1,..., 4).

V. Simulation Results

Table 1a and Table 1b list the standard deviations of the key variables for the U.S. economy. As in Braun and Evans [2], we detrend the data by using the log-first-difference filter. The data sources for these series are National Income and Product Accounts and Survey of Current Business. The data are now described briefly. All the variables are divided by population of age greater than or equal to 17. Market consumption is measured using expenditures on nondurable goods and services, excluding the service flow attributed to the housing stock. Home capital is measured by consumer duarables plus residential structures. Business capital is measured using nonresidential structures. Market output is defined as the sum of investment and market consumption, excluding the foreign and government sector. The hours worked in the market are computed as the product of total nonagricultural employment times average hours per week of nonagricultural production workers times 13 weeks per quarter (per capita). Following McGrattan, Rogerson and Wright [9], the discretionary hours are set to 1,134 hours per quarter.

Tables 2-4 contain some summary statistics computed from simulations of the model. The solutions are filtered by the log-first-difference filter. The three sets of columns in these tables present the simulation results for the cases of $\nu = corr(\varepsilon_{mt}, \varepsilon_{nt})$ = 0.1, 0.5 and 0.9. In all cases, the tables include means (Columns 1, 3 and 5) and standard deviations (Columns 2, 4 and 6) of sample distributions. These numbers are based on 50 simulations of 104 periods each. The three cases produce the same general patterns and the fit is good.

Table 2W, Table 2S, Table 2Su and Table 2F report the standard deviations of the simulated series for Quarter I, II, III and IV, respectively. Table 2T presents total variations, i.e., the sum of seasonal and nonseasonal variations. Although the model does relatively well in matching standard deviation observed in the data for aggregate investment, the model's characterization of investment in each of the two capital goods is problematic; the volatility of i_m and i_n is excessive.

Tables 3 report correlations with output for the model's variables. Table 3W, Table3S, Table 3Su and Table 3F contain the results for Quarter I, II, III and IV, respectively. Table 3T includes the total variations. The model's negative correlation between the capital stock and output shown in Table 3T is consistent with the data. Note, however, that it is positive in Tables 3W, 3S, 3Su and 3F. Note also that since the original series for the capital stock are annual, the corresponding correlation cannot be obtained by quarter for the actual U.S. data unless we make some specific assumption on the rate of depreciation.

Now, we turn our attention to the dynamic properties of output. The autocorrelation function of output up to six periods is shown in Tables 4. The first three sets of columns in these tables present the simulation results and the last set includes the results for the postwar U.S. data. In this table, both the actual and simulated series are detrended using the Hodrick-Prescott filter. This is because the series detrended by using the log-first-difference filter show strong negative autocorrelation at lag one. Except for positive autocorrelation from lag two to three, the autocorrelation properties of the simulated series are consistent with those of the smoothed output series for the U.S. post-war data. Table 4 indicates that in the U.S. data output is negatively and slightly autocorrelated at lag two and three.

As pointed out in Benhabib, Rogerson and Wright [1], the statistics generated by the standard real business cycle model differ from the data along the following five dimensions. First, output is less volatile in the model than in the data. Second, in the model, investment is too volatile and consumption is not volatile enough relative to either output or productivity. Third, in the model, hours worked are not volatile enough relative to either output or productivity. Fourth, hours worked and productivity are highly positively correlated in the model but not in the data. Fifth, the two investment series are positively correlated in the data but not in the model (Greenwood J., R. Rogerson, and R. Wright [6]). In this model both hours worked and market consumption are volatile enough, though we share other shortcomings with the standard real business cycle models.

The simulated series and the U.S. actual data has been log first differenced and regressed on four seasonal dummies. Tables 6 reports the coefficients estimates and the R-square using seasonally unadjusted U.S. data. Tables 6a, 6b and 6c report the results for the simulated series. There are thirteen sets of results, since we carry out the estimation for the thirteen key variables of the model. In the first line of each set of results, we include the average seasonal means (Columns 1 through Column 4) and the average R-square of the regressions (Column 5). In the second line of each set we list their standard deviations for 50 draws. Comparisons of the results in Table 6 with those in Tables 6a, 6b and 6c show that the model does capture the seasonal properties of output. In contrasts, the model does not mimic the seasonal movements in consumption, investment and hours. Note, however, that the average R-squares for these variables are small. On the whole, there is little evidence that the use of unadjusted data with seasonal dummies provides better results than using seasonally adjusted data.

VI. Conclusions

In this paper we have presented a real business cycle model with home production, with the key feature that its predictions match the seasonally unadjusted, postwar U.S. data. We showed that it is able to improve the performance of other standard RBC models with respect to the evidence on output. The results are not conclusive, on the other hand, for investment and the labor time spent in market work, in the sense that some statistics indicate that the model improves also with respect to these variables, while some other statistics indicate the opposite. We plan to address these problems in future research. Another possibly interesting line of future research is the exploration of the effects of adjustment costs on the two types of capital on the results. This is an important question, as sluggish adjustment of capital stock resulting from significant adjustment costs may be one factor that could help explain some of the time-series properties of the data. We expect that adjustment costs on the two types of capital in a home production model should extend some of the findings in the previous studies. Overall, it seems to us that the line of research according to which shocks propagate throughout the economy because of substitutability between the market and nonmarket sectors is a promising one.

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Table 1a
Sample Standard Deviations (U.S. Economy 1964.1-1989.4)

	Total	*Winter*	*Spring*	*Summer*	*Fall*
h_m	0.01874	0.03495	0.03141	0.03061	0.02846
y/h_n	0.13719	0.05751	0.029131	0.048078	0.051149
c_m	0.05359	0.02513	0.0274	0.02568	0.03067
y	0.06681	0.03218	0.03817	0.03686	0.04
i_m	0.09641	0.07911	0.0724	0.07771	0.07651
i_n	0.17909	0.1332	0.1407	0.1373	0.1269

Table 1b
Correlation with GNP(U.S. Economy 1964.1-1989.4)

	Total	*Winter*	*Spring*	*Summer*	*Fall*
h_m	0.369633	-0.25357	-0.1027	0.022053	0.148625
y/h_n	0.918209	0.790769	0.686161	0.710637	0.504334
c_m	0.965558	0.715339	0.793492	0.904783	0.892656
i_m	0.893655	0.702145	0.261983	0.507351	0.472647
i_n	0.929455	0.773816	0.713244	0.725144	0.607069

Table 2W Standard Deviations

	$corr(\varepsilon_{mt}, \varepsilon_{nt})$=0.1		$corr(\varepsilon_{mt}, \varepsilon_{nt})$=0.5		$corr(\varepsilon_{mt}, \varepsilon_{nt})$=0.9	
K	0.004	0.0007	0.0042	0.0008	0.0044	0.0008
h_m	0.0645	0.0041	0.065	0.0042	0.0655	0.0042
h_n	0.0697	0.0032	0.0697	0.0035	0.0698	0.0038
y/h_n	0.0593	0.0005	0.0594	0.0006	0.0594	0.0007
H	0.063	0.002	0.0635	0.0022	0.064	0.0025
I	0.2219	0.0126	0.2222	0.0113	0.2224	0.01
k_m	0.0263	0.0036	0.0256	0.0035	0.0249	0.0034
k_n	0.0211	0.0028	0.0208	0.0028	0.0204	0.0028
c_m	0.0823	0.0029	0.0818	0.0027	0.0812	0.0025
c_n	0.0184	0.0022	0.0173	0.0021	0.0161	0.0021
y	0.0333	0.0053	0.0335	0.0053	0.0336	0.0054
i_m	1.5351	0.1928	1.5042	0.1892	1.4735	0.1865
i_n	1.5773	0.1461	1.567	0.1423	1.5561	0.139

Table 3W Correlation with GNP

	$corr(\varepsilon_{mt}, \varepsilon_{nt})$=0.1		$corr(\varepsilon_{mt}, \varepsilon_{nt})$=0.5		$corr(\varepsilon_{mt}, \varepsilon_{nt})$=0.9	
K	0.0277	0.1031	0.0201	0.1018	0.013	0.1008
h_m	0.5423	0.0972	0.5517	0.0929	0.5612	0.0883
h_n	-0.1019	0.1089	-0.0723	0.1042	-0.042	0.0988
y/h_n	-0.0372	0.1107	-0.0491	0.1052	-0.0614	0.0992
H	0.2345	0.1127	0.2557	0.1075	0.277	0.1016
I	0.2523	0.1046	0.2656	0.1014	0.2791	0.0981
k_m	0.9254	0.0284	0.9206	0.0311	0.916	0.0339
k_n	-0.8731	0.0377	-0.8612	0.0407	-0.8488	0.0435
c_m	0.2988	0.1123	0.2896	0.1058	0.2804	0.0989
c_n	-0.6563	0.0939	-0.5829	0.1041	-0.4993	0.1092
y	1	0	1	0	1	0
i_m	-0.2165	0.1538	-0.2087	0.1539	-0.2003	0.1544
i_n	0.1922	0.1144	0.1952	0.1107	0.1982	0.1071

Table 2S Standard Deviations

	$corr(\varepsilon_{mt}, \varepsilon_{nt})$=0.1		$corr(\varepsilon_{mt}, \varepsilon_{nt})$=0.5		$corr(\varepsilon_{mt}, \varepsilon_{nt})$=0.9	
K	0.004	0.0007	0.0042	0.0008	0.0043	0.0008
h_m	0.0404	0.0053	0.0409	0.0054	0.0414	0.0055
h_n	0.0114	0.0016	0.01	0.0014	0.0083	0.0012
y/h_n	0.0044	0.0004	0.0047	0.0004	0.0049	0.0004
H	0.0168	0.0021	0.0179	0.0022	0.0189	0.0025
I	0.0623	0.0074	0.0651	0.0079	0.0677	0.0085
k_m	0.0283	0.004	0.0276	0.0039	0.0269	0.0037
k_n	0.0226	0.0032	0.0222	0.0031	0.0219	0.0031
c_m	0.0287	0.004	0.0274	0.0038	0.0261	0.0036
c_n	0.0163	0.0024	0.0145	0.0021	0.0126	0.0018
y	0.0379	0.0051	0.038	0.0051	0.0381	0.0051
i_m	0.9573	0.1018	0.9353	0.0994	0.9126	0.097
i_n	2.1421	0.1804	2.1355	0.1785	2.1281	0.1768

Table 3S Correlation with GNP

	$corr(\varepsilon_{mt}, \varepsilon_{nt})$=0.1		$corr(\varepsilon_{mt}, \varepsilon_{nt})$=0.5		$corr(\varepsilon_{mt}, \varepsilon_{nt})$=0.9	
K	0.0129	0.112	0.0064	0.1107	0.0003	0.109
h_m	0.998	0.0007	0.9982	0.0006	0.9984	0.0005
h_n	-0.831	0.0655	-0.7803	0.0798	-0.7375	0.0819
y/h_n	-0.5394	0.0815	-0.6079	0.0752	-0.6704	0.0703
H	0.976	0.009	0.9807	0.007	0.9875	0.0041
I	0.9617	0.0103	0.9668	0.0088	0.9728	0.0068
k_m	0.9963	0.0013	0.9967	0.0011	0.9981	0.0005
k_n	-0.96	0.0121	-0.954	0.0137	-0.9483	0.0148
c_m	0.9597	0.0143	0.9583	0.0143	0.9594	0.0128
c_n	-0.9008	0.0395	-0.8758	0.0465	-0.8622	0.0443
y	1	0	1	0	1	0
i_m	-0.4494	0.1402	-0.4525	0.1395	-0.4564	0.1377
i_n	0.2316	0.1187	0.2335	0.1178	0.2354	0.1164

Table 2Su Standard Deviations

	$corr(\varepsilon_{mt}, \varepsilon_{nt})$=0.1		$corr(\varepsilon_{mt}, \varepsilon_{nt})$=0.5		$corr(\varepsilon_{mt}, \varepsilon_{nt})$=0.9	
K	0.7675	0	0.7675	0	0.7675	0
h_m	0.1971	0.0011	0.1973	0.0012	0.1974	0.0012
h_n	0.0267	0.0018	0.0262	0.0018	0.0257	0.0017
y/h_n	5.5012	0	5.5012	0	5.5012	0
H	0.0308	0.0016	0.0315	0.0018	0.0321	0.002
I	4.0542	0.0002	4.0542	0.0002	4.0542	0.0002
k_m	0.2417	0.0004	0.2416	0.0004	0.2415	0.0004
k_n	0.021	0.0032	0.0207	0.0031	0.0203	0.0031
c_m	0.0506	0.0029	0.0499	0.0028	0.0492	0.0026
c_n	0.0165	0.0024	0.015	0.0021	0.0133	0.0018
y	0.0469	0.0126	0.0466	0.0123	0.0465	0.0121
i_m	1.4859	0.2091	1.4494	0.2031	1.4108	0.1972
i_n	1.9583	0.3013	1.9472	0.2864	1.9365	0.2701

Table 3Su Correlation with GNP

	$corr(\varepsilon_{mt}, \varepsilon_{nt})$=0.1		$corr(\varepsilon_{mt}, \varepsilon_{nt})$=0.5		$corr(\varepsilon_{mt}, \varepsilon_{nt})$=0.9	
K	0.1261	0.5301	0.1287	0.5221	0.1324	0.5144
h_m	0.3105	0.5049	0.3166	0.4966	0.3233	0.4885
h_n	-0.0068	0.1059	0.0458	0.1038	0.1	0.0995
y/h_n	0.1248	0.5302	0.1275	0.5221	0.1311	0.5145
H	0.6216	0.1347	0.6421	0.1345	0.6618	0.1345
I	0.1329	0.5296	0.1363	0.5215	0.1407	0.5138
k_m	0.2113	0.5228	0.2124	0.5149	0.2144	0.5072
k_n	-0.7629	0.1551	-0.7623	0.1521	-0.7615	0.1494
c_m	0.6319	0.1395	0.6253	0.1354	0.6178	0.1311
c_n	-0.5343	0.1414	-0.49	0.1389	-0.4429	0.1347
y	1	0	1	0	1	0
i_m	-0.0404	0.211	-0.0348	0.2114	-0.0299	0.2126
i_n	0.2887	0.1308	0.2942	0.1291	0.3001	0.1271

Table 2F Standard Deviations

	$corr(\varepsilon_{mt}, \varepsilon_{nt})$=0.1		$corr(\varepsilon_{mt}, \varepsilon_{nt})$=0.5		$corr(\varepsilon_{mt}, \varepsilon_{nt})$=0.9	
K	0.7816	0.0281	0.7816	0.0284	0.7815	0.0287
h_m	0.2152	0.003	0.2153	0.0029	0.2154	0.0027
h_n	0.1126	0.0059	0.1126	0.0063	0.1126	0.0067
y/h_n	4.6985	0.0714	4.6982	0.0754	4.6975	0.0782
H	0.1036	0.0028	0.104	0.0032	0.1044	0.0035
I	3.1096	0.0766	3.1097	0.0749	3.1099	0.0735
k_m	0.2415	0.0045	0.2413	0.0044	0.2412	0.0044
k_n	0.0335	0.0036	0.033	0.0035	0.0325	0.0035
c_m	0.3236	0.0034	0.3234	0.003	0.3232	0.0027
c_n	0.2143	0.0072	0.2142	0.0072	0.214	0.0072
y	0.0523	0.009	0.0518	0.0089	0.0512	0.0088
i_m	1.0254	0.1651	1.004	0.1598	0.9822	0.1539
i_n	3.4348	0.4053	3.4165	0.3873	3.3982	0.3668

Table 3F Correlation with GNP

	$corr(\varepsilon_{mt}, \varepsilon_{nt})$=0.1		$corr(\varepsilon_{mt}, \varepsilon_{nt})$=0.5		$corr(\varepsilon_{mt}, \varepsilon_{nt})$=0.9	
K	0.0366	0.4069	0.0365	0.4007	0.0351	0.3921
h_m	0.3208	0.3728	0.3243	0.3662	0.327	0.357
h_n	0.0944	0.1411	0.1121	0.1351	0.1303	0.1275
y/h_n	0.0329	0.4065	0.0327	0.4003	0.0312	0.3917
H	0.359	0.1162	0.371	0.11	0.3838	0.1023
I	0.0414	0.4046	0.0411	0.3985	0.0394	0.3901
k_m	0.1627	0.4037	0.1594	0.3978	0.1548	0.3895
k_n	-0.6967	0.1887	-0.6904	0.189	-0.6849	0.1888
c_m	0.2241	0.3707	0.2253	0.3646	0.2256	0.3557
c_n	-0.0365	0.4025	-0.0284	0.3967	-0.0214	0.3886
y	1	0	1	0	1	0
i_m	-0.4702	0.1422	-0.4614	0.1444	-0.4521	0.1463
i_n	0.0709	0.122	0.0606	0.1199	0.0501	0.117

Table 2T Standard Deviations

	$corr(\varepsilon_{mt}, \varepsilon_{nt})=0.1$		$corr(\varepsilon_{mt}, \varepsilon_{nt})=0.5$		$corr(\varepsilon_{mt}, \varepsilon_{nt})=0.9$	
K	0.0039	0.0007	0.0041	0.0008	0.0043	0.0008
h_m	0.0668	0.0036	0.0672	0.0037	0.0676	0.0037
h_n	0.0646	0.0032	0.0645	0.0035	0.0644	0.0038
y/h_n	0.0491	0.0005	0.0491	0.0005	0.0492	0.0006
H	0.0602	0.0016	0.0607	0.0018	0.0611	0.002
I	0.1689	0.0102	0.1692	0.0091	0.1695	0.0077
k_m	0.03	0.0034	0.0292	0.0033	0.0284	0.0032
k_n	0.024	0.0026	0.0236	0.0026	0.0231	0.0026
c_m	0.0876	0.0023	0.0872	0.0021	0.0867	0.0019
c_n	0.0208	0.0022	0.0195	0.0021	0.018	0.002
y	0.039	0.0045	0.039	0.0045	0.039	0.0046
i_m	1.1916	0.1102	1.1643	0.1064	1.1362	0.1021
i_n	2.3856	0.1063	2.3725	0.1031	2.3592	0.0997

Table 3T Correlation with GNP

	$corr(\varepsilon_{mt}, \varepsilon_{nt})=0.1$		$corr(\varepsilon_{mt}, \varepsilon_{nt})=0.5$		$corr(\varepsilon_{mt}, \varepsilon_{nt})=0.9$	
K	-0.0101	0.0542	-0.0162	0.0524	-0.0217	0.0496
h_m	0.7029	0.0565	0.7082	0.0536	0.7135	0.0506
h_n	-0.0202	0.075	0.0087	0.069	0.0377	0.0621
y/h_n	-0.19	0.0785	-0.2017	0.0722	-0.2135	0.0651
H	0.3963	0.0762	0.4138	0.0713	0.4309	0.0662
I	0.2234	0.0731	0.2333	0.0708	0.2433	0.0682
k_m	0.9332	0.0215	0.9307	0.0233	0.9286	0.025
k_n	-0.9033	0.0231	-0.8959	0.0248	-0.8887	0.0262
c_m	0.4491	0.0777	0.4423	0.0713	0.4353	0.0634
c_n	-0.6642	0.0697	-0.6119	0.0776	-0.5547	0.083
y	1	0	1	0	1	0
i_m	-0.3143	0.0596	-0.3084	0.0595	-0.3026	0.0591
i_n	0.1893	0.0314	0.1859	0.031	0.1825	0.0308

Table 4 Autocorrelations of Output

lags	$corr(\varepsilon_{mt}, \varepsilon_{nt})$=0.1		$corr(\varepsilon_{mt}, \varepsilon_{nt})$=0.5		$corr(\varepsilon_{mt}, \varepsilon_{nt})$=0.9		U.S. Economy	
1	0.5629	0.0862	0.565	0.084	0.5669	0.0816	0.6387	0.1057
2	0.3604	0.1079	0.3606	0.1075	0.3609	0.1068	-0.045	0.0986
3	0.1214	0.1172	0.1215	0.1166	0.1213	0.116	-0.1247	0.0441
4	0.0853	0.0913	0.0822	0.0917	0.0788	0.0926	0.8381	0.0448
5	-0.1104	0.0869	-0.1111	0.0866	-0.1121	0.0861	-0.739	0.0998
6	-0.145	0.0856	-0.1459	0.0862	-0.1471	0.0873	-0.0464	0.1068

Note to Table 4: All series are detrended by using Hodrick Prescott Filter.

Table 5 Seasonality U.S. Data

	Winter	Spring	Summer	Fall	R^2
c_m	-0.0803	0.0335	0.0073	0.0561	0.9220
h_m	-0.0119	0.0078	0.0063	0.0030	0.1393
i_m	-0.0881	0.0919	-0.0320	0.0540	0.8262
i_n	-0.2038	0.1776	0.0135	0.0360	0.9042
y	-0.1008	0.0641	0.0032	0.0522	0.9387

Table 6a Seasonality Simulation $corr(\varepsilon_{mt},\ \varepsilon_{nt}) = 0.1$

	Winter	Spring	Summer	Fall	R^2
K	0.0003	-0.0002	0	0.0001	0.0216
	0	0	0	0	0.0001
h_m	0.1451	-0.055	0.0281	-0.119	0.887
	0.0001	0	0	0.0001	0.0003
h_n	0.1748	-0.0702	0.0251	-0.1294	0.9851
	0.0001	0	0	0	0
y/h_n	-0.1324	0.0539	-0.0163	0.0949	0.9992
	0	0	0	0	0
H	0.1585	-0.062	0.0265	-0.1233	0.9839
	0	0	0	0	0
I	-0.4144	0.1777	-0.0456	0.281	0.9615
	0.0004	0.0001	0.0001	0.0003	0.0001
k_m	-0.0157	0.0082	0.0018	0.0052	0.1346
	0.0001	0	0	0.0001	0.012
k_n	0.014	-0.0071	-0.0012	-0.0051	0.145
	0.0001	0	0	0	0.0124
c_m	0.2247	-0.0739	0.0372	-0.1885	0.9765
	0	0	0	0	0
c_n	0.0291	-0.011	0.0066	-0.0243	0.5569
	0.0001	0	0	0	0.0231
y	0.0127	-0.0011	0.0118	-0.0241	0.1608
	0.0001	0	0	0.0001	0.0099
i_m	2.1325	-0.8483	0.6162	-1.9077	0.1427
	1.2503	0.7738	1.3937	2.2485	0.0146
i_n	-1.2825	0.6841	-0.5144	1.1216	0.1509
	0.2719	0.2161	0.4223	0.4425	0.0102

Table 6b Seasonality Simulation $corr(\varepsilon_{mt}, \varepsilon_{nt}) = 0.5$

	Winter	Spring	Summer	Fall	R^2
K	0.0003	-0.0002	0	0.0001	0.0227
	0	0	0	0	0.0001
h_m	0.1455	-0.0551	0.0281	-0.1193	0.8868
	0.0001	0	0	0.0001	0.0003
h_n	0.1748	-0.0702	0.0251	-0.1294	0.9864
	0.0001	0	0	0	0
y/h_n	-0.1325	0.0539	-0.0163	0.095	0.999
	0	0	0	0	0
H	0.1588	-0.0621	0.0266	-0.1235	0.9822
	0	0	0	0	0
I	-0.4135	0.1772	-0.0454	0.2804	0.9598
	0.0003	0.0001	0.0001	0.0003	0.0001
k_m	-0.0156	0.0082	0.0018	0.0052	0.1356
	0.0001	0	0	0.0001	0.0118
k_n	0.0137	-0.007	-0.0013	-0.0049	0.1445
	0.0001	0	0	0	0.0118
c_m	0.2248	-0.0738	0.0371	-0.1886	0.9782
	0	0	0	0	0
c_n	0.0291	-0.011	0.0066	-0.0243	0.5903
	0.0001	0	0	0	0.0228
y	0.013	-0.0012	0.0118	-0.0243	0.1638
	0.0001	0	0	0.0001	0.0089
i_m	2.1594	-0.855	0.6045	-1.9152	0.1438
	1.2257	0.77	1.3427	2.1445	0.0145
i_n	-1.2585	0.6649	-0.4894	1.0918	0.1518
	0.2414	0.2021	0.3794	0.3782	0.0102

Table 6c Seasonality Simulation $corr(\varepsilon_{mt}, \varepsilon_{nt}) = 0.9$

	Winter	Spring	Summer	Fall	R^2
K	0.0002	-0.0002	0	0.0001	0.0235
	0	0	0	0	0.0001
h_m	0.1459	-0.0552	0.0281	-0.1196	0.8866
	0	0	0	0.0001	0.0002
h_n	0.1749	-0.0704	0.0251	-0.1294	0.9878
	0.0001	0	0	0	0
y/h_n	-0.1326	0.054	-0.0163	0.095	0.9988
	0	0	0	0	0
H	0.1591	-0.0623	0.0266	-0.1237	0.9804
	0	0	0	0	0
I	-0.4126	0.1766	-0.0453	0.2798	0.958
	0.0002	0.0001	0.0001	0.0002	0.0001
k_m	-0.0155	0.0082	0.0018	0.0051	0.137
	0.0001	0	0	0.0001	0.0115
k_n	0.0135	-0.007	-0.0013	-0.0047	0.1441
	0	0	0	0	0.0113
c_m	0.2247	-0.0737	0.0372	-0.1886	0.98
	0	0	0	0	0
c_n	0.029	-0.0111	0.0066	-0.0242	0.6293
	0.0001	0	0	0	0.0221
y	0.0133	-0.0012	0.0118	-0.0246	0.1669
	0	0	0	0.0001	0.0079
i_m	2.1989	-0.8525	0.5758	-1.9295	0.1453
	1.19	0.7716	1.2805	1.9921	0.0147
i_n	-1.2359	0.6557	-0.4662	1.0549	0.1563
	0.2171	0.1877	0.3482	0.3296	0.0099