Phase structure of lattice QCD at finite temperature for 2+1 flavors of Kogut-Susskind quarks

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We report on a study of the finite-temperature chiral transition on an $N_t=4$ lattice for 2+1 flavors of Kogut-Susskind quarks. We find the point of physical quark masses to lie in the region of crossover, in agreement with results of previous studies. Results of a detailed examination of the $m_{u,d}=m_s$ case indicate vanishing of the screening mass of the $\sigma$ meson at the end point of the first-order transition.

1. Introduction

An important issue in finite-temperature lattice QCD is the determination of the nature of the chiral phase transition for a realistic spectrum of light up and down quarks and a heavier strange quark. Despite its importance, past studies of this “2+1” case have been few. For the Kogut-Susskind action, all of them have been made around 1990\cite{1,2,3}.

It was found in these studies that the chiral phase transition changes from a first-order transition to a crossover as the strange quark mass $m_s$ increases beyond a critical value $m^c_s$ for a fixed degenerate up and down quark mass $m_{u,d}$, in agreement with predictions of an effective $\sigma$ model of QCD\cite{4,5}. Results were also obtained\cite{2,3} which indicate the physical point of quark masses to lie in the region of crossover on the $(m_{u,d},m_s)$ plane. However, these results were based on simulations made at only a few sets of quark masses. Clearly a more extensive study is called for to have a full understanding of the phase structure in the 2-parameter space of $(m_{u,d},m_s)$. Here we report first results from our recent effort toward this goal.

An interesting suggestion from a $\sigma$ model analysis\cite{5} is that the second-order transition expected at the critical strange quark mass $m^c_s$ is in the Ising universality class, with the massless mode provided by the $\sigma$ meson. A novel feature of our work is a study of the screening mass of $\sigma$ to examine this point.

Our simulations are performed for the temporal lattice size $N_t = 4$. An $8^3 \times 4$ lattice is employed to make a survey of the phase structure varying $\beta$, $m_{u,d}$ and $m_s$. A detailed investigation is then made along the flavor SU(3) symmetric line $m_{u,d}=m_s$ by another series of simulations on a $16 \times 8^3 \times 4$ and a $16^3 \times 4$ lattice. For each parameter set, $(1-2) \times 10^3$ trajectories of unit length are generated by the hybrid R algorithm.

2. Phase diagram on the $(m_{u,d},m_s)$ plane

We show the result of our phase diagram analysis on an $8^3 \times 4$ lattice in Fig.\cite{1}. At $m_{u,d} = m_s = 0.01$ a clear two-state signal is obtained by a comparison of runs with a hot and a cold start.
The value of the critical mass $m^c$ is to extrapolate the gap of the chiral condensate $\Delta \langle \bar{\psi} \psi \rangle$ toward larger $m$ where it vanishes. Employing the form $\Delta \langle \bar{\psi} \psi \rangle \propto (m^c - m)^{1/2}$ predicted by the mean-field analysis of the $\sigma$ model, we find $m^c \approx 0.034$. If we use a naive linear extrapolation, we obtain $m^c \approx 0.049$. A similar value $m^c \approx 0.045$ was previously reported by a linear extrapolation applied to old results.

In the region of crossover $m > m^c$, we expect the peak height of the chiral susceptibility $\chi_m$ to develop a singular behavior $\chi_m \propto (m - m^c)^{-z}$ as $m \to m^c$. We calculate $\chi_m$ for $m \geq 0.04$ with the histogram reweighting method. Assuming $m^c = 0.034$, we fit the peak height to the form above. A reasonable fit with $\chi^2/df = 1.05$ is obtained with the value of the exponent $z = 0.67(3)$, which is comparable to the Ising value $z \approx 0.79$ and the mean-field value $2/3$.

In order to examine the screening mass $M_\sigma$ of the $\sigma$ meson, we employ $U(1)$ random source and no gauge fixing to evaluate the two quark loop contribution of the $\sigma$ propagator. Good results are obtained for the full $\sigma$ propagator with this method as illustrated in Fig. 3. The quark mass dependence of $\pi$ and $\sigma$ screening masses for $m \leq 0.03$, where we find a first-order transition, is plotted in Fig. 3(a). We observe that $M^2_\sigma$ decreases toward zero as $m$ increases toward the critical value, both on the confined and the deconfined side of the transition, in contrast to $M^2_\pi$ which increases.

3. Results along the $m_{u,d} = m_s$ line

We now discuss results along the line $m_{u,d} = m_s \equiv m$. Given our observation of a two-state signal at $m = 0.01, 0.025$, one way to estimate...
Figure 3. $\pi$ and $\sigma$ propagators for $m = 0.1$ at the transition point $\beta=5.25$.

Assuming a linear quark mass dependence, $M_2^2 \propto m^c - m$, predicted by the mean-field analysis of the $\sigma$ model, we obtain $m^c = 0.034(3)$ in the confining phase and $m^c = 0.031(3)$ in the deconfining phase. These values are consistent with each other, and are also in agreement with the estimate from a square root extrapolation of the gap of $\langle \bar{\psi}\psi \rangle$ discussed above. These results indicate vanishing of the $\sigma$ mass at the critical quark mass as suggested by the $\sigma$ model\cite{5}.

Results for larger quark masses ($m \geq 0.04$), where a crossover behavior is seen, is shown in Fig. 3(b). While $M_2^2$ decreases toward smaller values of $m$, the variation is too mild to attempt an independent estimate of $m_c$. An interesting point which requires clarification is that $M_2^2$ stays considerably small compared to $M_2^\pi$ even for large quark masses.

4. Conclusions and future work

Our study with the Kogut-Susskind action supports the previous conclusion with this action that there is no finite-temperature phase transition for three flavors of quarks with physical masses. This means that a discrepancy with the conclusion from the Wilson action\cite{7} still remains.

We also find a strong indication that the screening mass of $\sigma$ vanishes at the end point of the first-order transition along the line $m_{u,d} = m_s$.

We plan to extend analyses carried out here to the $m_{u,d} \neq m_s$ case to further explore the real-world QCD chiral transition.

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