

Eta meson mass and topology in QCD with two light flavors*

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We present results for the mass of the flavor singlet meson calculated on two-flavor full QCD configurations generated by the CP-PACS full QCD project. We also investigate topological charge fluctuations and their dependence on the sea quark mass.

1. INTRODUCTION

Recently considerable progress has been made in the simulation of full QCD [1]. In particular, sea quark effects have been found to lead to a closer agreement of the light meson spectrum with experiment [2,3].

Missing from the calculated spectrum, however, has been the flavor singlet meson η' . Due to the difficulty of the determination of the disconnected contribution, only preliminary lattice results have been available [4–6].

In the first half of this article, we present new results on this problem. Since these are obtained with two flavors of dynamical quark, we call the flavor singlet meson as η and reserve the name η' for the case of $N_f=3$.

The η' meson is expected to obtain a large mass through the connection to instantons. This leads us to an investigation of topology in full QCD, presented in the latter half of this article.

Calculations have been performed on configurations of the CP-PACS full QCD project [2,3]. These have been generated using an RG-improved gauge action and a tadpole-improved SW clover quark action at four different lattice spacings and four values of the sea quark mass corresponding

Table 1

Parameters of lattices used in this calculation.

$N_s^3 \times N_t$	β	$N_s a$ [fm]	a^{-1} [GeV]	m_{PS}/m_V
$12^3 \times 24$	1.8	2.58(3)	0.92(1)	0.81–0.55
$16^3 \times 32$	1.95	2.45(3)	1.29(2)	0.80–0.59
$24^3 \times 48$	2.1	2.60(5)	1.82(3)	0.80–0.58
$24^3 \times 48$	2.2	2.06(6)	2.30(7)	0.80–0.63

to $m_{PS}/m_V \sim 0.8–0.6$. An overview of the simulation parameters is given in Table 1. More details can be found in [2,3].

2. FLAVOR SINGLET MESON

The mass difference Δm between the flavor singlet (η) and non-singlet (π) meson can be extracted from the ratio

$$R(t) = \frac{\langle \eta(t)\eta(0) \rangle_{\text{disc}}}{\langle \eta(t)\eta(0) \rangle_{\text{conn}}} \rightarrow 1 - B \exp(-\Delta m t), \quad (1)$$

where the right hand side indicates the expected behavior at large time separation t .

In this work, the connected propagator was calculated with the standard method. For the disconnected propagator we used two methods. In the first instance, it was calculated using a volume source without gauge fixing (the Kuramashi method)[5]. This measurement was made after

*talk presented by R. Burkhalter at Lattice '99

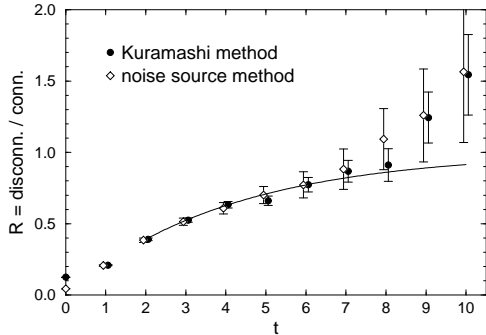


Figure 1. Ratio for $\beta = 1.95$ and $m_\pi/m_\rho = 0.59$. The solid line is from a fit with Eq. 1.

every trajectory in the course of configuration generation for all runs listed in Table 1. As for the second method, we employed a U(1) volume noise source with 10 random noise ensembles for each color and spin combination. This was performed only at $\beta = 1.95$ on stored configurations separated by 10 HMC trajectories.

Figure 1 compares the ratio $R(t)$ for the two methods. We observe that they are consistent with each other but that the error is smaller for the first method. This might be due to the fact that there are 10 times more measurements with it, although binning is made over 50 HMC trajectories for both cases to take into account auto-correlations. In the following we only use data obtained with the first method.

In Fig. 1 we also see that the error of $R(t)$ increases exponentially, which make the determination of Δm via Eq. 1 impossible at large time separations. The data, however, shows the expected behavior already beginning from small t and a fit with Eq. 1 is possible from $t_{min} = 2$. Increasing t_{min} leads to stable results, as can be seen in Fig. 2.

The chiral extrapolation of m_η^2 linear in the quark mass is shown in Fig. 2. Contrary to the pion mass, the flavor singlet meson remains massive in the chiral limit.

Figure 3 shows the η meson mass at all measured lattice spacings after the chiral extrapolation. The scale is set using the ρ meson mass. A linear extrapolation to the continuum limit gives $m_\eta = 863(86)$ MeV. This value lies between the experimental $\eta(547)$ and $\eta'(958)$ masses. We

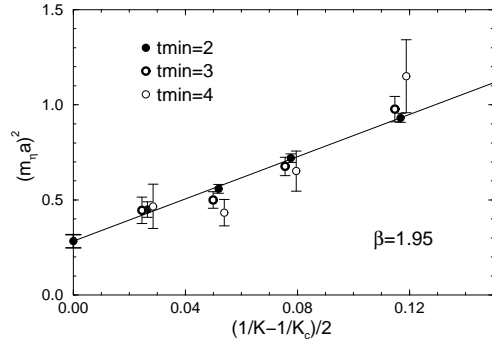


Figure 2. Chiral extrapolation of η meson mass.

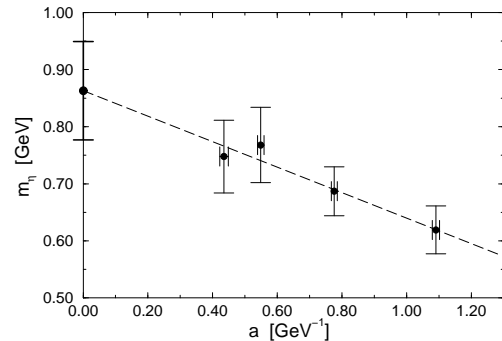


Figure 3. Continuum extrapolation of η mass.

emphasize that a proper comparison with experiment requires the introduction of a third (strange) quark and a mixing analysis.

3. TOPOLOGY

Studies of topology on the lattice have encountered several difficulties. In addition to the ambiguity of defining a lattice topological charge, it was found that topological modes have a very long auto-correlation time in the case of full QCD with the Kogut-Sussind quark action [7].

We employ the field theoretic definition of the topological charge together with cooling. For the charge we use a tree-level improved definition which includes a 1×2 -plaquette, hence the $O(a^2)$ terms are removed for instanton configurations. For the cooling we compare two choices of improved actions, both including a 1×2 -plaquette term: 1) a tree-level Symanzik improved (LW) action and 2) the RG improved Iwasaki action.

Using different actions for cooling can lead to

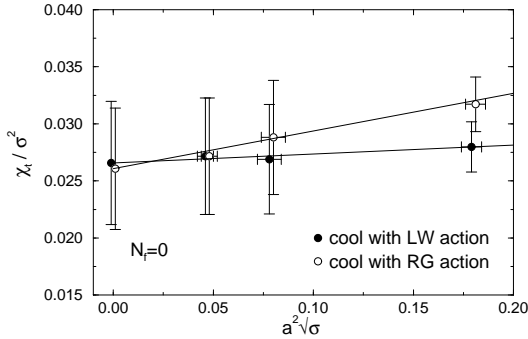


Figure 4. Topological susceptibility for $N_f = 0$.

different values of the topological charge. This ambiguity is only expected to vanish when the lattice is fine enough. We have tested this explicitly by simulating the pure SU(3) gauge theory at three lattice spacings in the range $a \sim 0.2\text{--}0.1$ fm and at a constant size of 1.5 fm. As Fig. 4 shows, the topological susceptibilities $\chi_t = \langle Q^2 \rangle / V$ for the two cooling actions converge to a common value towards the continuum limit. Using $\sqrt{\sigma} = 440$ MeV we obtain $\chi_t = (178(9) \text{ MeV})^4$, in agreement with previous studies [8].

In full QCD we have so far measured the topological charge at $\beta = 1.95$. Figure 5 shows the time history for two quark masses. Auto-correlation times are visibly small even for the smallest quark mass. For the Wilson quark action rather short auto-correlation times have been reported in Ref. [9]. The fact that we find even shorter auto-correlations might be explained by the coarseness of our lattice.

Based on the anomalous flavor-singlet axial vector current Ward identity, one expects the topological susceptibility to vanish in the chiral

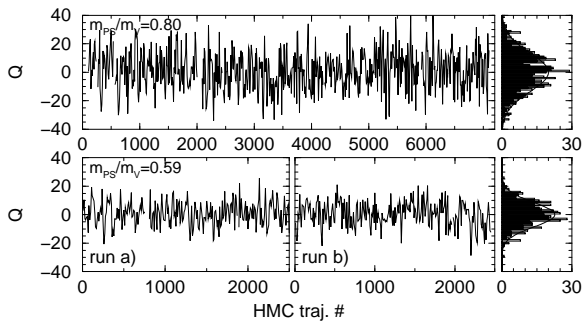


Figure 5. Charge history at $\beta = 1.95$.

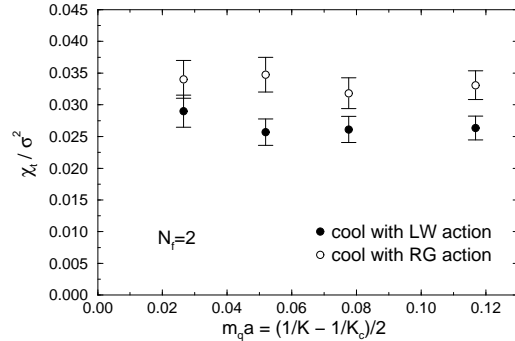


Figure 6. Topological susceptibility at $\beta = 1.95$.

limit. Indeed, Fig. 5 shows the width to be shrinking with the quark mass. The decrease, however, is not sufficient; as we find in Fig. 6, the dimensionless ratio χ_t/σ^2 , with σ calculated for each sea quark mass, does not vary much with the quark mass, and takes a value similar to that for pure gauge theory. To understand the origin of this behavior, more investigations at different lattice spacings will be needed.

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